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Extracts from the Register of Copyrights

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दिनांक/Dated:08/02/2024

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5.	कृति का शीर्षक Title of the work	:	SUBJECT PPT ON HEAT & MASS TRASNFER
6.	कृति की भाषा Language of the work	:	ENGLISH
7.	रचयिता का नाम, पता और राष्ट्रीयता तथा यदि रचयिता की मृत्यु हो गई है, तो मृत्यु की तिथि Name, address and nationality of the author and if the author is deceased, date of his decease	:	ROHIT JADHAO , PCET'S NUTAN MAHARASHTRA INSTITUTE OF ENGINEERING AND TECHNOLOGY, PUNE; SAMARTH VIDYA SANKUL, VISHNUPURI, TALEGAON DABHADE, MAHARASHTRA 410507-410507 INDIAN MANGESH KALE , PCET'S NUTAN MAHARASHTRA INSTITUTE OF ENGINEERING AND TECHNOLOGY, PUNE; SAMARTH VIDYA SANKUL, VISHNUPURI, TALEGAON DABHADE, MAHARASHTRA-410507 INDIAN SPANDAN WAGHMARE , PCET'S NUTAN MAHARASHTRA INSTITUTE OF ENGINEERING AND TECHNOLOGY, PUNE; SAMARTH VIDYA SANKUL, VISHNUPURI, TALEGAON DABHADE, MAHARASHTRA-410507 INDIAN
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Registrar of Copyrights

Under Administrative Support
Pimpri Chinchawad Education Trust (PCET)
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Nutan Maharashtra Inst. of Engg. & Tech.

"SUBJECT PPT ON HEAT & MASS TRASNFER"

Nutan Maharashtra Institute of Engineering & Technology
Talegaon Dabhade

UNIT NO: 01 TO 06

CREATED BY

Prof. Rohit R. Jadhao

Assistant Professor

Contents

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- Modes of Heat Transfer

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3. Radiations.

- Heat Conduction
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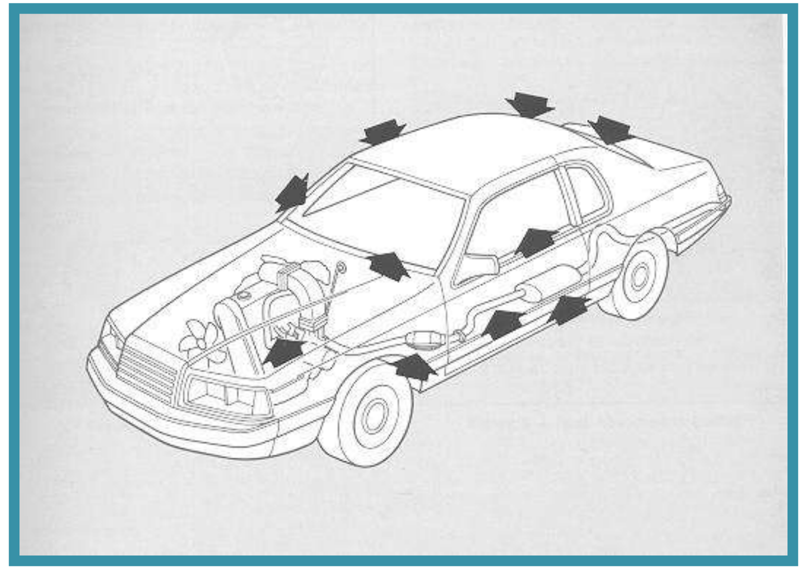


Heat Energy & Heat Transfer

- In this lecture, we are going to study about Heat Energy & HT
- <u>So, let us see first, What is Energy ?</u> Energy is the capacity to do work. Some examples of energies are mechanical energy, chemical energy, electrical energy, nuclear energy and so on
- <u>And, What is Heat Energy?</u> It is a form of energy in transit, the driving force for which, is the temp difference
- It means, whenever there is temp difference between the two bodies, heat energy will flow. When they attain equilibrium of temp , heat flow will stop
- <u>In the study of Heat Transfer:</u> We are concerned, as to HOW and AT WHAT RATE heat transfer takes place. HT is based on the Law of Thermodynamics, which states that heat flows from a body at higher temp to a body at lower temp. Conversely, heat can not flow from a body at lower temp to a body at higher temp, unless & until some external device, like heat pump, is employed

Why should we study Heat Transfer ?

- You will not find a single field of engineering , where knowledge of Heat Transfer is not applied.
- Knowledge of HT is always required while designing an eqpt. Let us take up some examples, branch-wise

Mech Engg: Engines, R &A/C, use of insulations, cooling/heating of bodies for heat treatment etc

Elect Engg: Cooling of motors, transformers, current carrying wires, etc

Chemical Engg: We have to take care of energy produced during chemical reactions, heating or cooling of chemicals for ctions to take place, etc

clear Engg: Conversion of fission/fusion energy to tricity, cooling of nuclear reactors etc

Importance of Heat Transfer

Electronics: Cooling of ICs, electronic devices etc

Computer Engg: Cooling of chips, electronic cct, etc

Civil Engg: Curing of cement in buildings/dams etc

Hydraulics: Generation of electricity from hydraulic energy from dams

Bio-technology: Ripening of fruits, processing of biomaterial

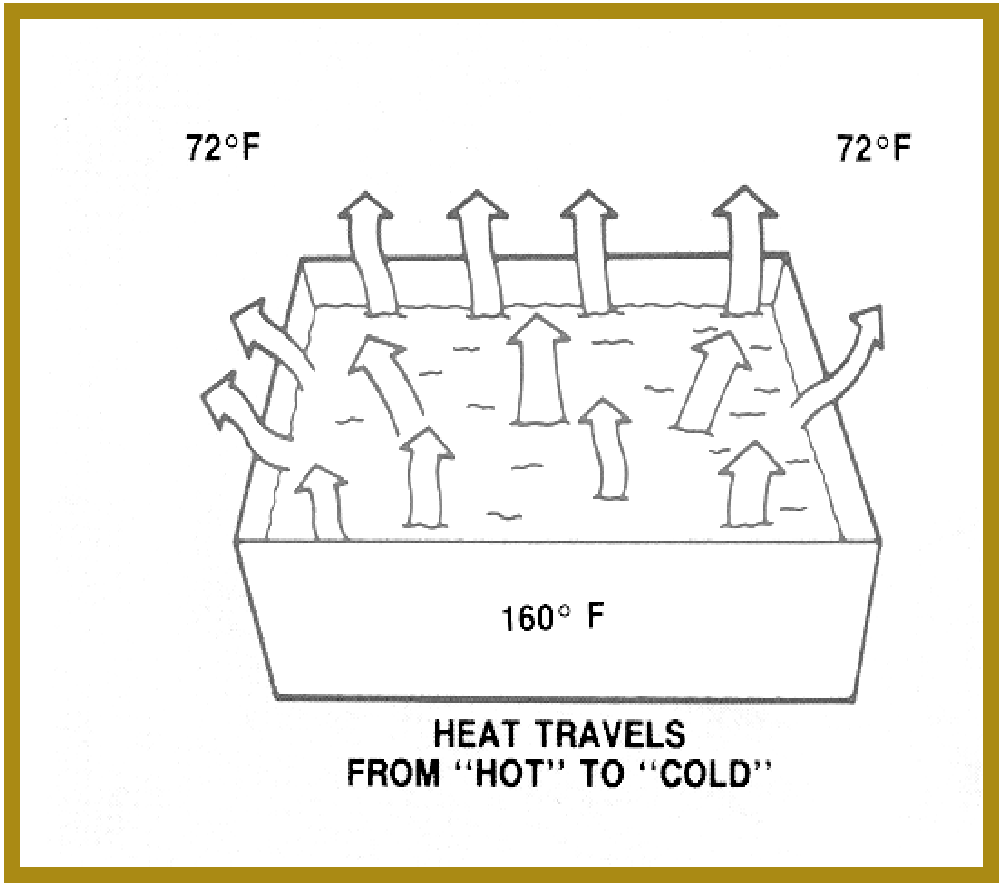
Space Engg: Space applications/ cooling of space vehicles

Aircraft Engg: Aircraft applications

name an equipment and you find that the knowledge
eat transfer is required

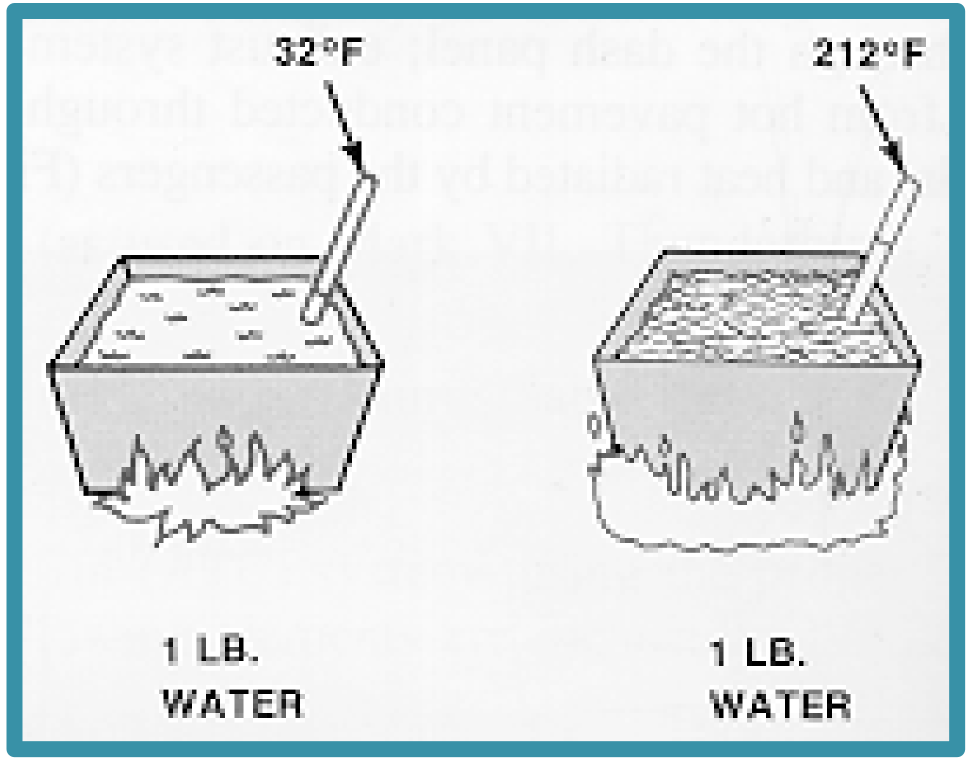
PRINCIPLE ONE

- Heat ALWAYS flows from hot to cold when objects are in contact or connected by a good heat conductor.
- The rate of heat transfer will increase as the difference in temp between the two objects increases



PRINCIPLE TWO

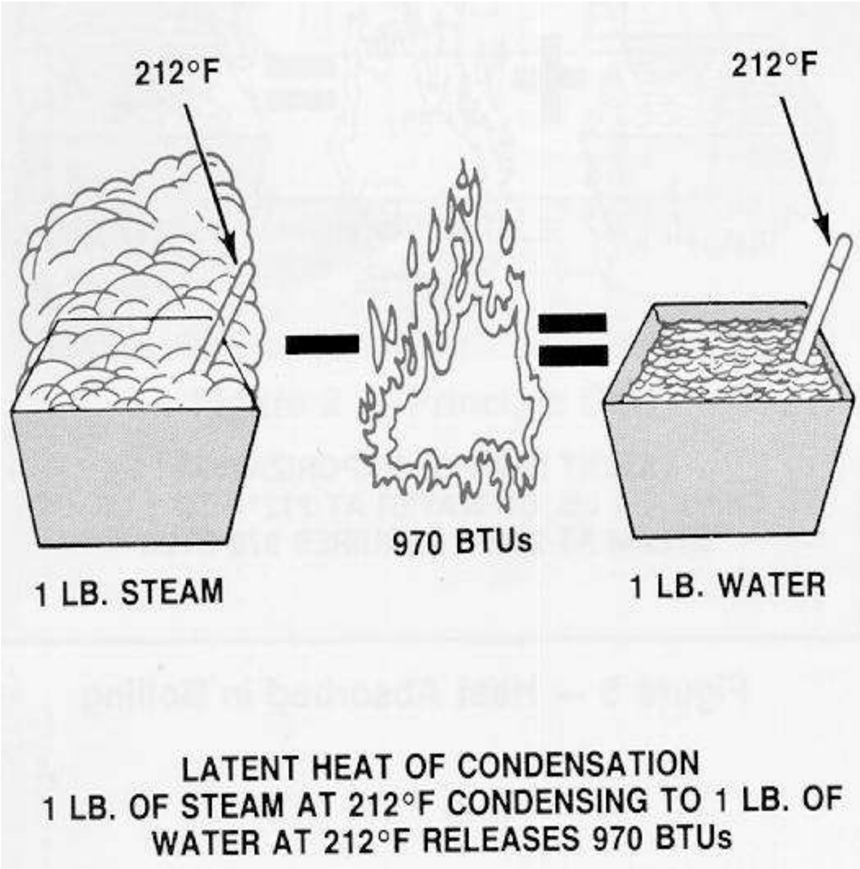
- Cold objects have less internal heat than hot objects of the same mass
- To make an object colder, remove heat; To make is hotter, add heat
- The mass of the object remains the same regardless of the heat content



PRINCIPLE THREE

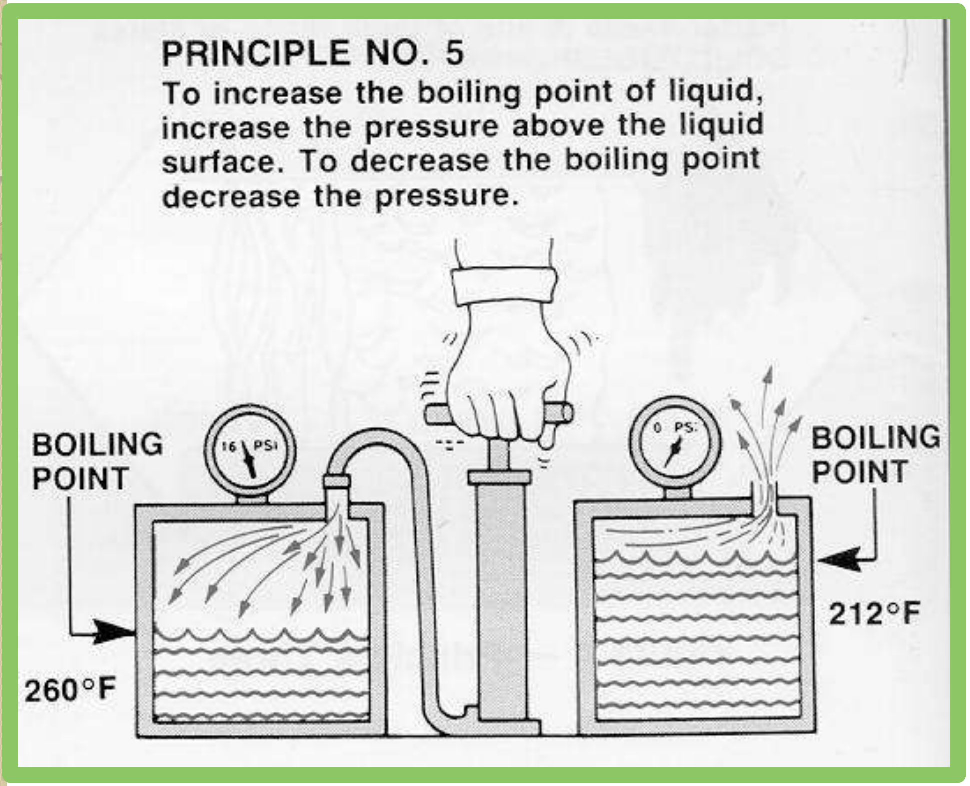
- Everything is composed of matter
- All matter exists in one of three states: solid, liquid or vapor.
- LATENT HEAT OF VAPORIZATION: When matter changes from liquid to vapor or vice versa, it absorbs or releases a relatively large amount of heat <u>without a change in temperature</u>.

PRINCIPLE FOUR



- CONDENSATION When a vapor is cooled below its dew point, it becomes a liquid. (boiling point in reverse)
- When vapor condenses, releases large amount of heat

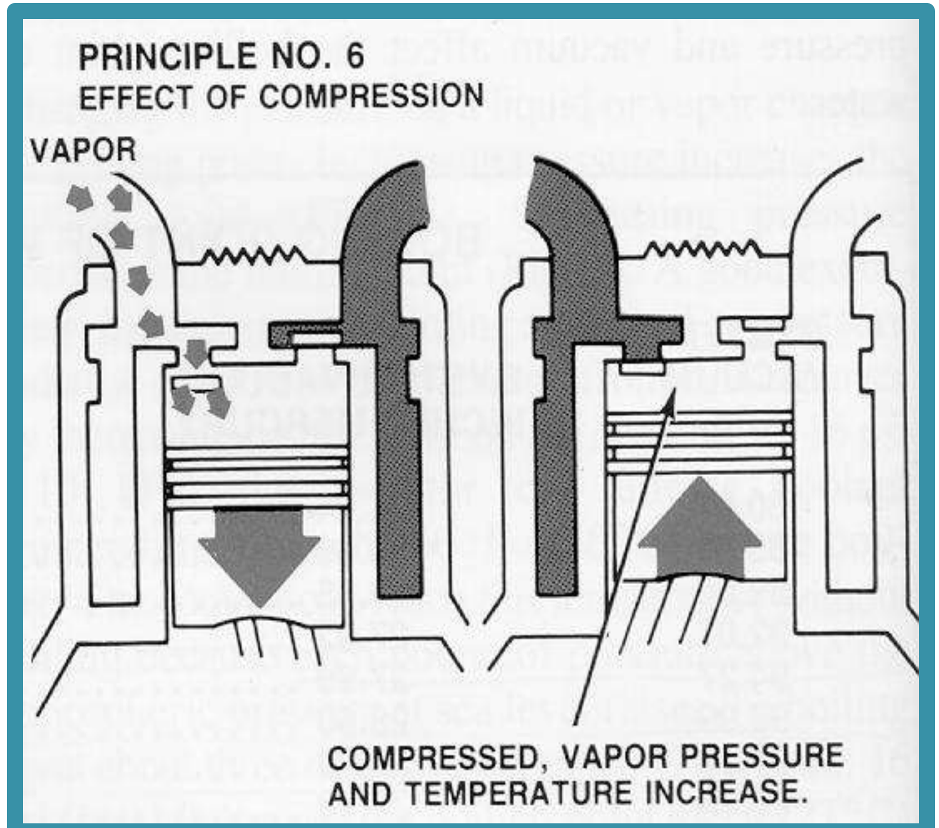
PRINCIPLE FIVE



- Changing the pressure on a liquid or a vapor changes the boiling point.

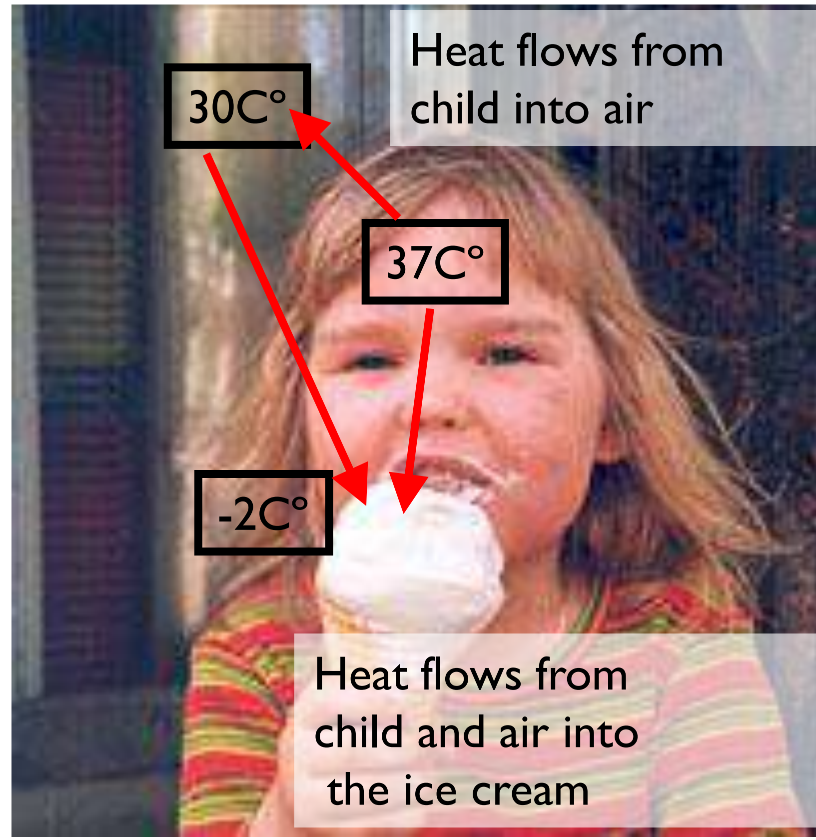
PRINCIPLE SIX

- When a vapor is compressed, its temperature and pressure will increase even though heat has not been added



Heat Transfer

- Heat *always* flows from high temperature objects to low temperature objects.
- Heat flow stops when temperatures equal.
- Various ways by which heat may flow.



Modes of Heat Transfer

- **Conduction:** Flow of heat energy by direct contact & through free electrons e.g. heat flow through solids

- **Convection:** Transfer of heat energy by fluid flowing over a surface e.g. heat transfer from engine surface to surrounding atmospheric air

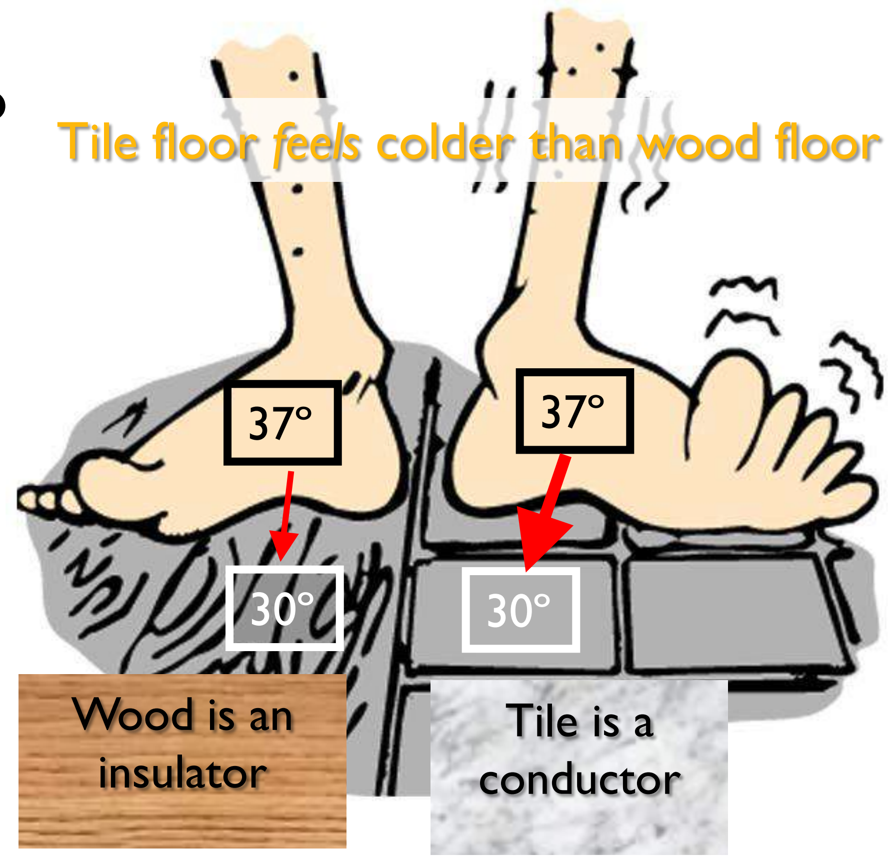
- **Radiation:** Flow of heat energy without any intervening medium e.g. energy of sun reaching the earth.

CONDUCTION

- Heat is transferred through a solid and gets the solid hot. (molecules get hotter, then they in-turn give energy to nearby molecules and they get hotter too)
- Different solids conduct different amounts of heat in a specific time. (copper vs. glass)
- Conduction is the process whereby heat is transferred ectly through a material, ' bulk motion of the terial playing no role in the transfer.
- Those materials that conduct heat well, are called thermal conductors, while those that conduct heat poorly, are known as thermal insulators.
- Most metals are excellent thermal conductors, while wood, glass, and most plastics are common thermal insulators.
- The free electrons in metals are responsible for the excellent thermal conductivity of metals.

Conduction

- Conduction is heat flow by direct contact.
- Some materials are go thermal conductors, others are insulators.



Fourier's Law of Heat Conduction

- Rate of heat transfer by conduction (through a solid) in a given direction is proportional to the area normal to the direction of heat flow and the temp gradient in that direction.

Mathematically ;

$$Q \propto A\frac{\Delta T}{\Delta x} Watt \quad \text{OR} \quad Q = -kA\frac{dT}{dx} Watt\,(J/s)$$

where Q = heat flow rate, Watt (J/s)

A = area normal to heat flow direction, m^2

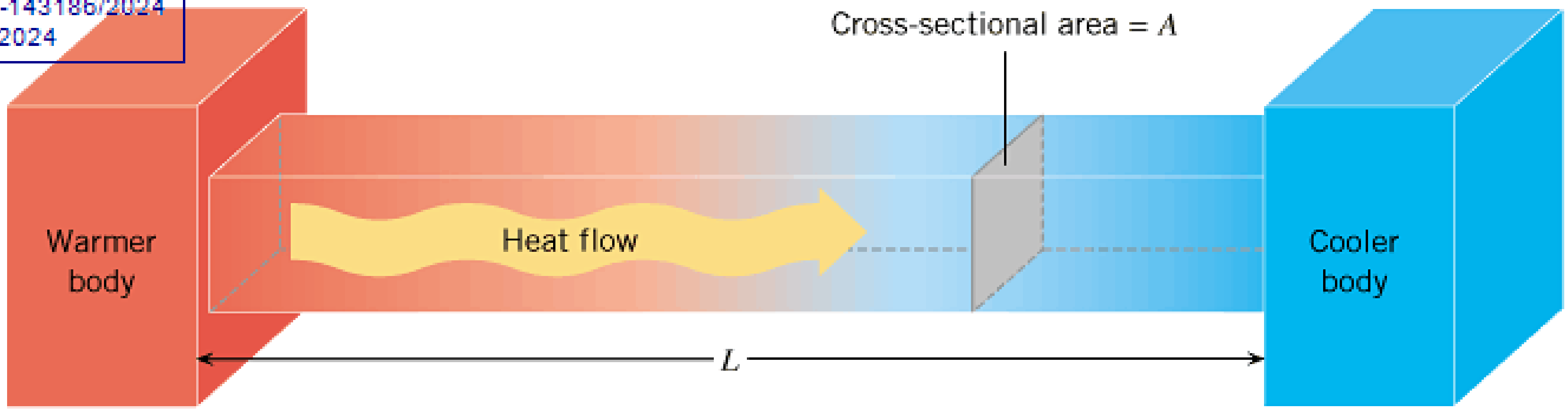
k = conductivity of material (property), W/mK

dT/dx = temp gradient in x direction

ΔT = temp difference across Δx

Δx = thickness of material in heat flow direction

Conduction



Rate of heat transfer by conduction, *Q* through the length, *L* across the cross-sectional area, *A* is given by the following equation, where *k* is the thermal conductivity and *ΔT* is the temperature difference between the two ends.

$$Q = \frac{kA\Delta T}{L}$$

Unit of Thermal Conductivity: J/(s · m · C°)

Assumptions of Fourier's Law

1. Unidirectional heat flow (only one direction)

2. Steady state heat flow

3. Constant temp gradient

4. Constant conductivity, k

5. Both faces isothermal

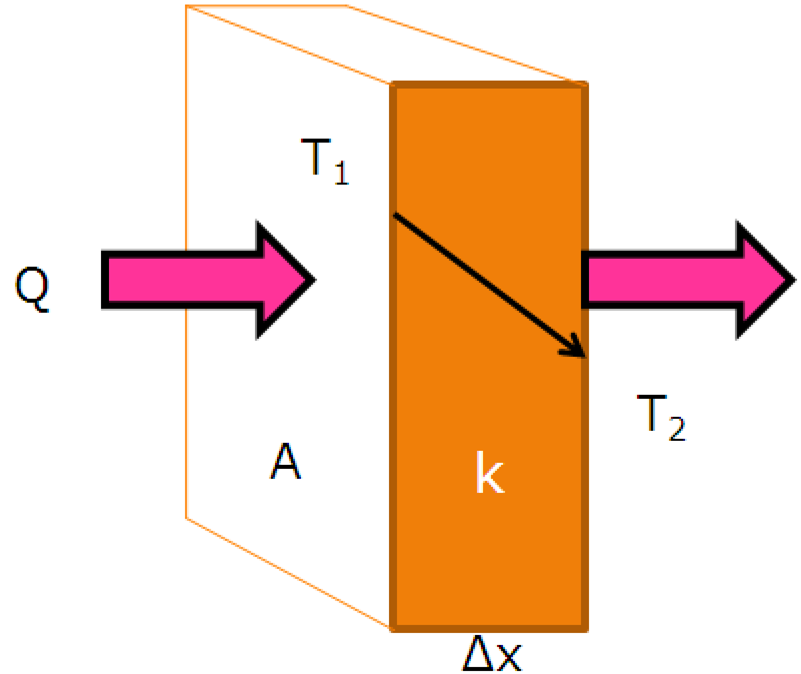
Conduction

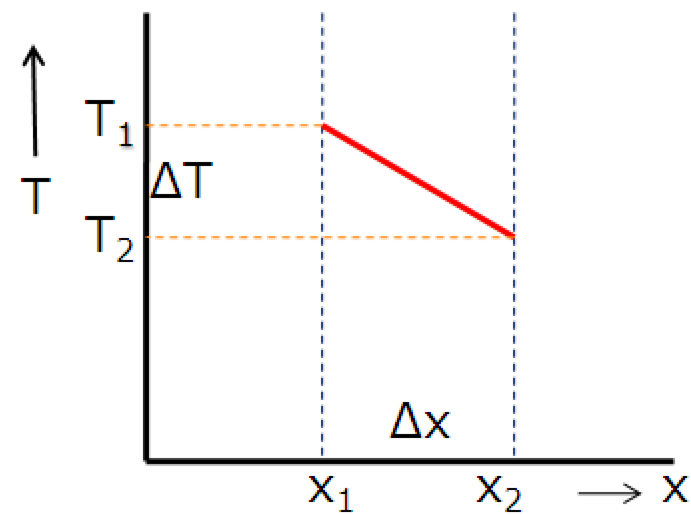
$$Q = -kA\frac{dT}{dx}$$

$$Q = kA\frac{(T_1 - T_2)}{(x_1 - x_2)}$$

$$Q = -kA\frac{(T_1 - T_2)}{(x_2 - x_1)}$$

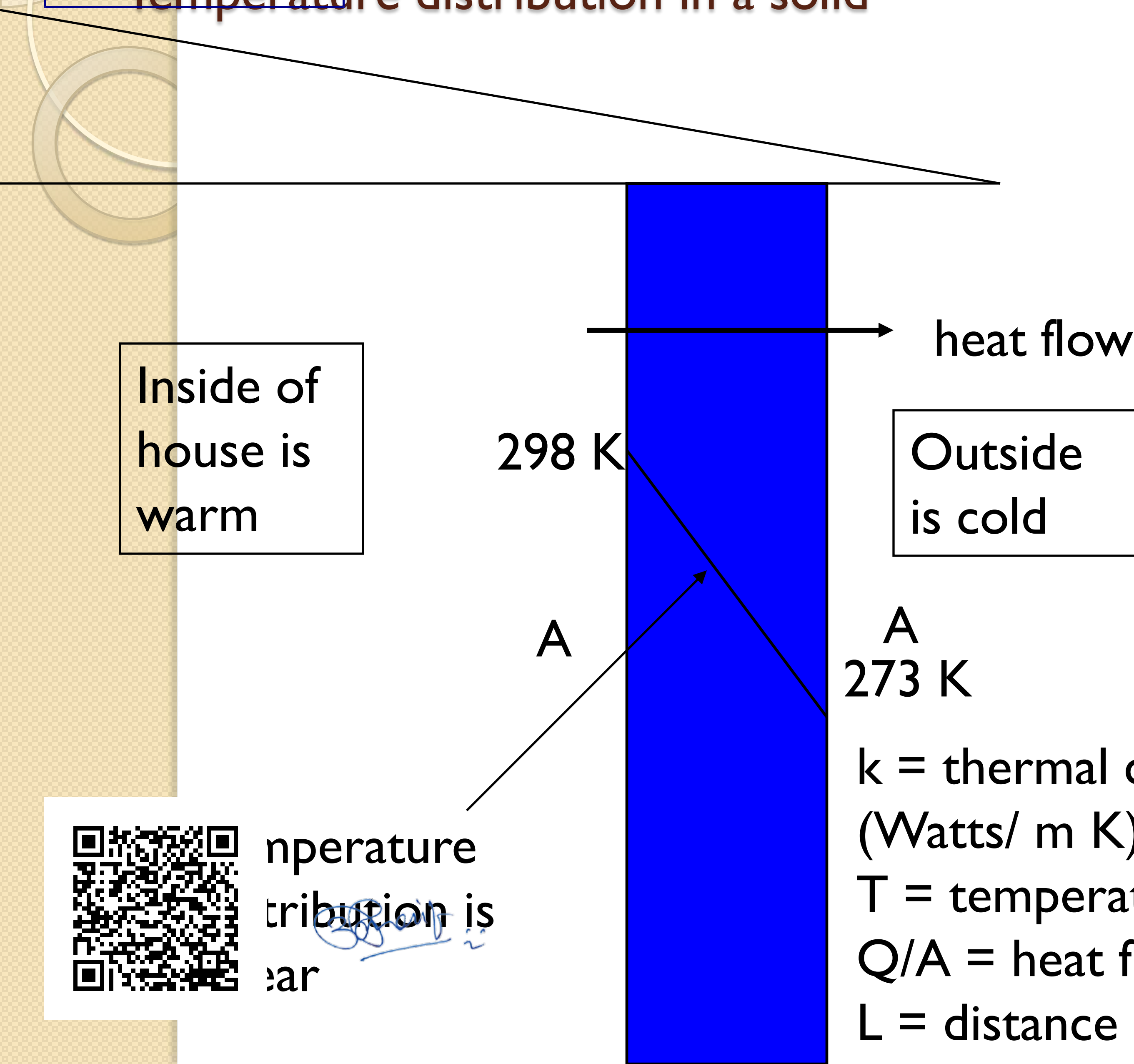
$$Q = kA\frac{\Delta T}{\Delta x}$$





at Flux $q = Q/A, W/m^2$

Temperature distribution in a solid



$$Q = -kA\frac{T_{out} - T_{in}}{L}$$

k = thermal conductivity (Watts/ m K)

T = temperature (K)

Q/A = heat flux (Watts/m^2)

L = distance (m)

Thermal Conductivity

Metals		
Aluminum	240	High conductivity
Brass	110	
Copper	390	High conductivity
Iron	79	
Lead	35	
Silver	420	High conductivity
Steel (stainless)	14	
Gases		
	0.0256	
rogen (H_2)	0.180	
ogen (N_2)	0.0258	

Thermal Conductivities

- Metals have high thermal conductivity, most electrical insulators also have low thermal conductivity.
- Air is a great insulator, except that large air spaces allow heat flow by convection.

Substance	Thermal Conductivity: k W / (m K)
Glass	0.84
Water	0.60
Wood	0.10
Air	0.023

Variation of Thermal Conductivity

1. It is the property of material; defined as ability of material to conduct heat through it.

2. Thermal conductivity in decreasing order :
   Metals » Non-metallic Solids » Liquids » Gases

3. Higher conductivity in metals due to free electrons in their outer orbits

4. k depends on grain structure. When k is different in different directions (k_x , k_y , k_z), material is known as anisotropic. When k is constant in all directions, it is called Isotropic.

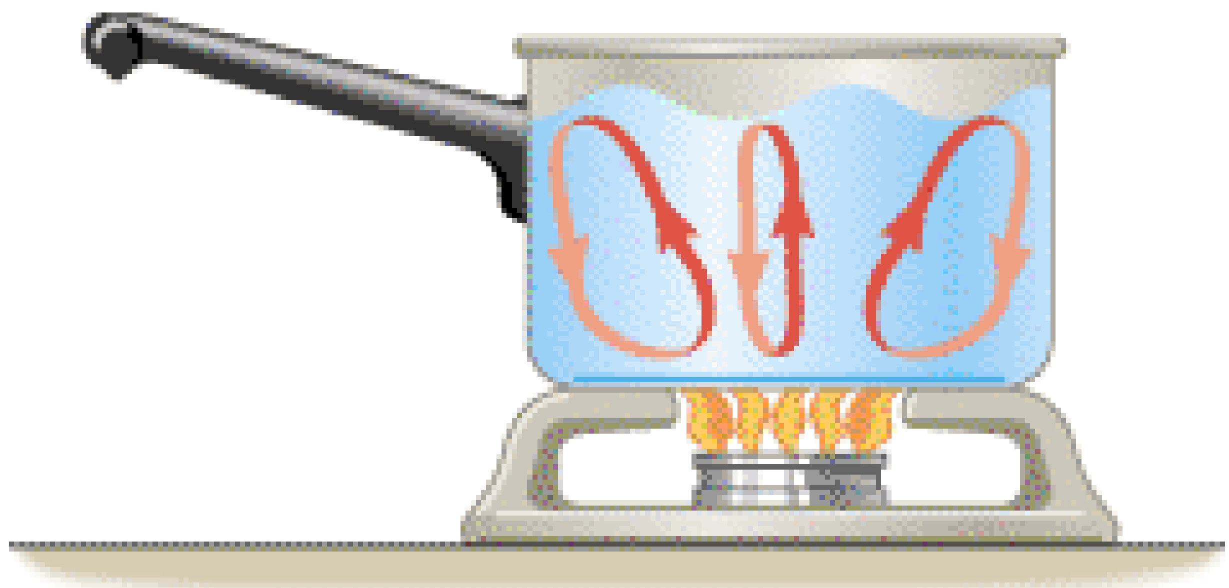
5. k is strongly dependent on temp; $k=k_o(1+\alpha T)$

Isotropic & Anisotropic Materials

- Some materials exhibit same conductivity in all directions. These are called ISOTROPIC materials (kx = ky = kz = k)
- While some materials have different conductivity in different directions(k_x , k_y , k_z), such materials are known as anisotropic.
- Wood exhibits directional conductivity; different along grains

HEAT CONVECTION

- Convection is the process in which heat is carried from place to place by the bulk movement of a fluid.



- Convection currents are set up when a pan of water is heated.

Heat Convection

- When a fluid flows over a solid body or surface and temp of the fluid and solid surface are different, heat transfer between the solid surface and fluid takes place due to motion of fluid relative to the surface.

- If the fluid motion is artificially induced, then heat transfer is said to be by FORCED convection.

- If the fluid motion is set up by buoyancy effects resulting from density difference caused due to temp difference in the fluid, heat transfer is said to be by FREE or NATURAL convection

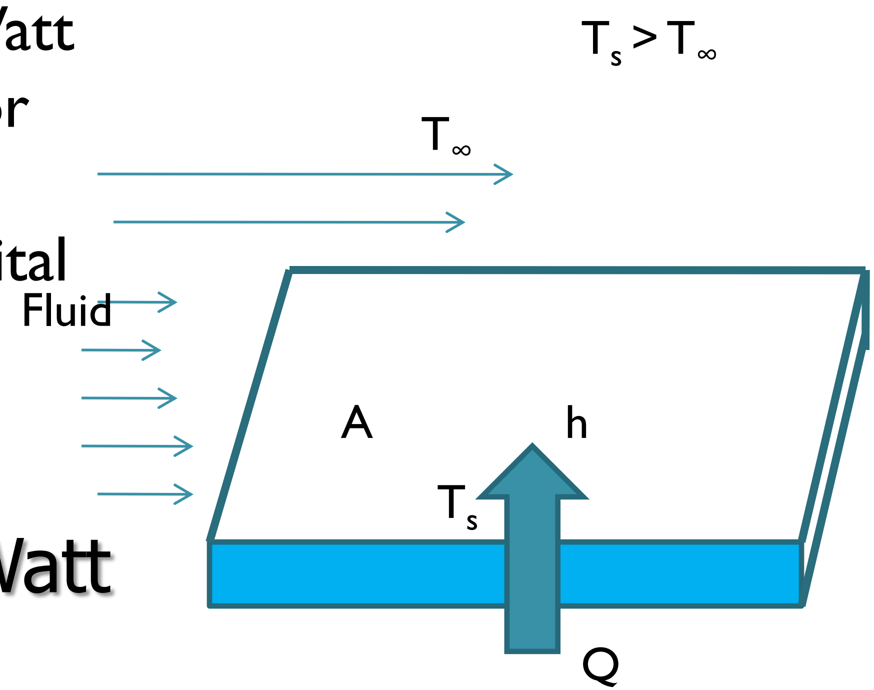
Newton's Law of Cooling

- Rate of heat transfer by convection from a surface to a fluid or vice versa , flowing along it is equal to the product of temp difference between surface and the free stream of the fluid, the area of the surface normal to the direction of heat flow and a quantity h called convective heat transfer coefficient.

Mathematically;

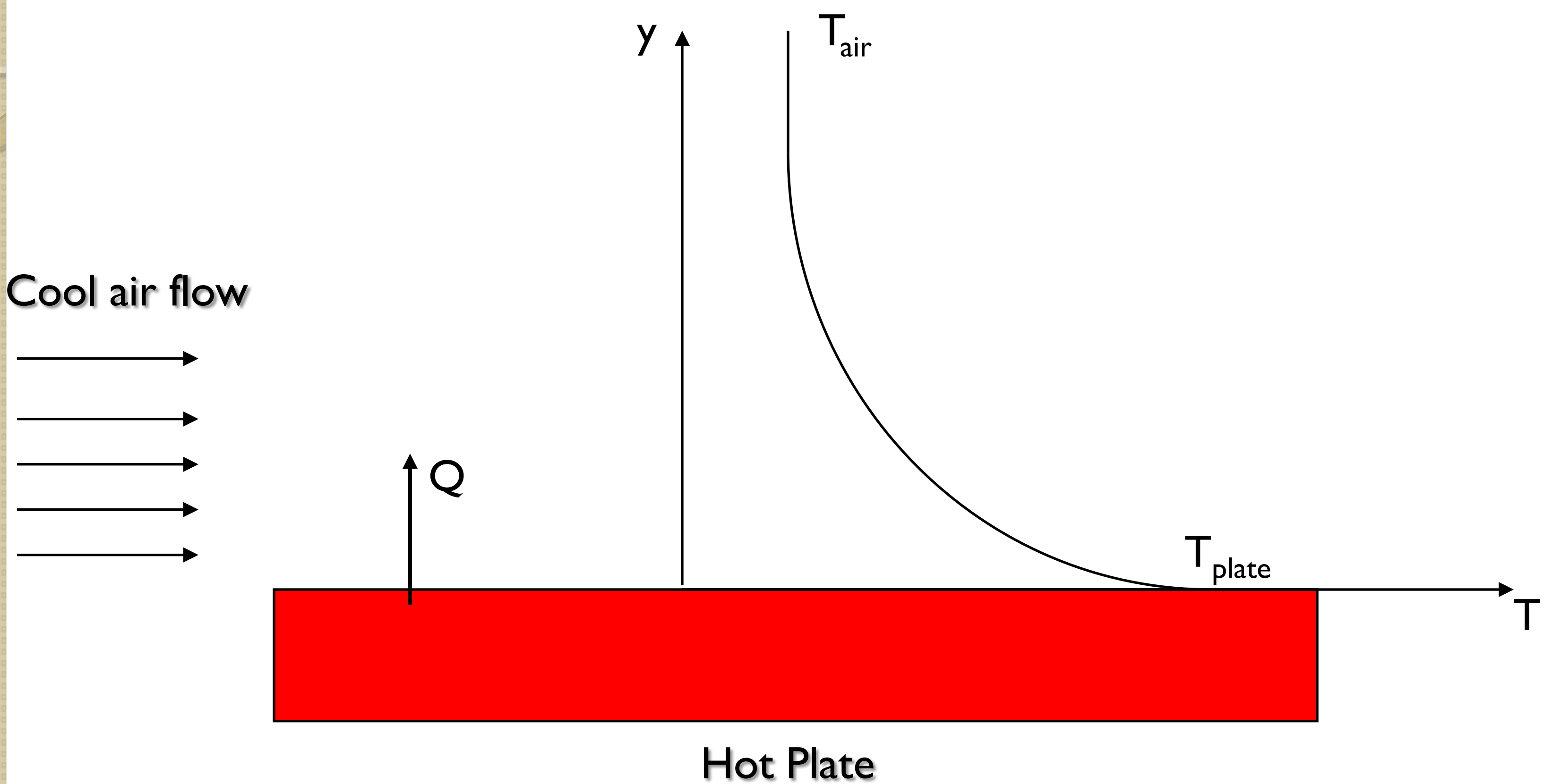
$$Q = hA(T_s - T_\infty); \text{Watt}$$

h is not a property of fluid or surface, but it depends on properties of the fluid and vital dimensions of the surface



$$Q = h\,A\,(T_s - T_\infty);\ \text{Watt}$$

Convection

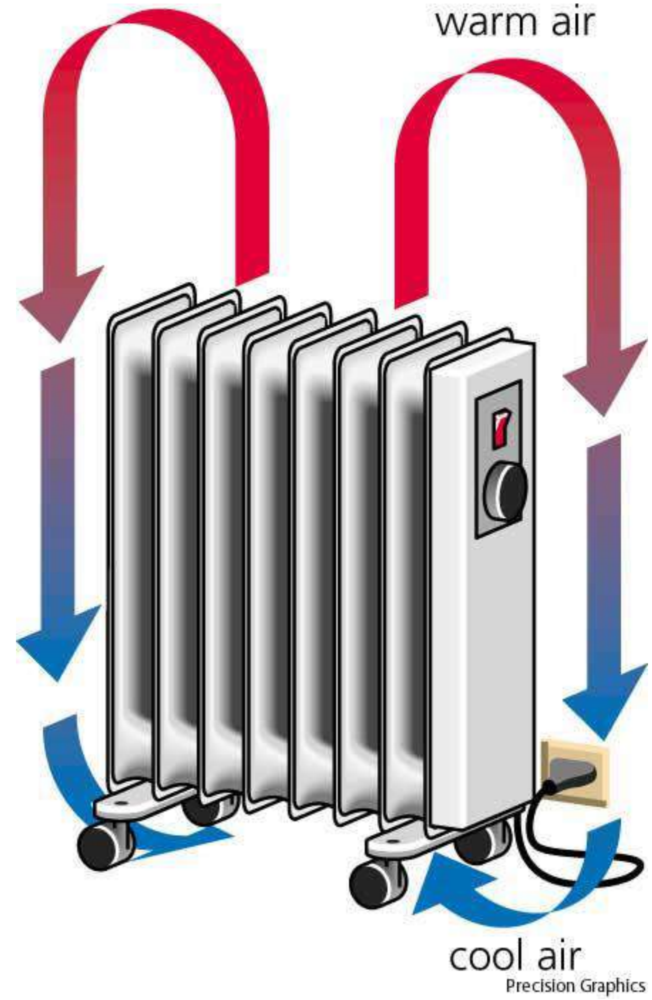


h = convection coefficient Watts/m^2 K

$$= hA(T_{plate} - T_{air})$$

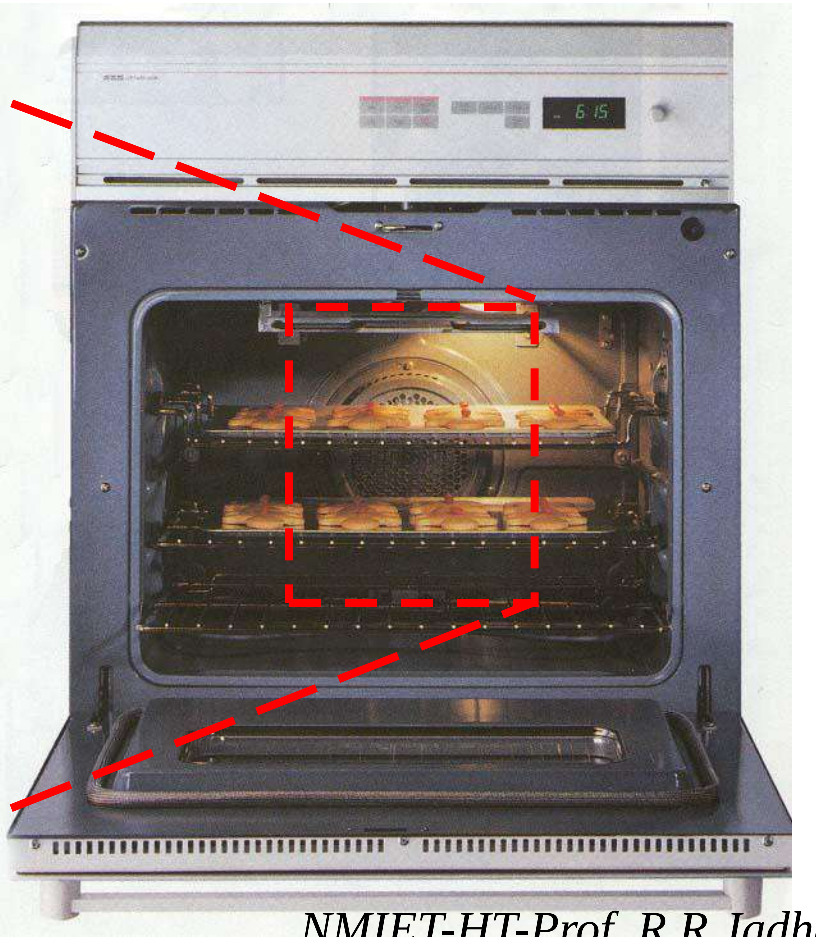
Convection

- Heat transfer in a fluid often occurs mostly by convection.
- Buoyancy causes warm air to rise, which carries thermal energy directly by its motion.



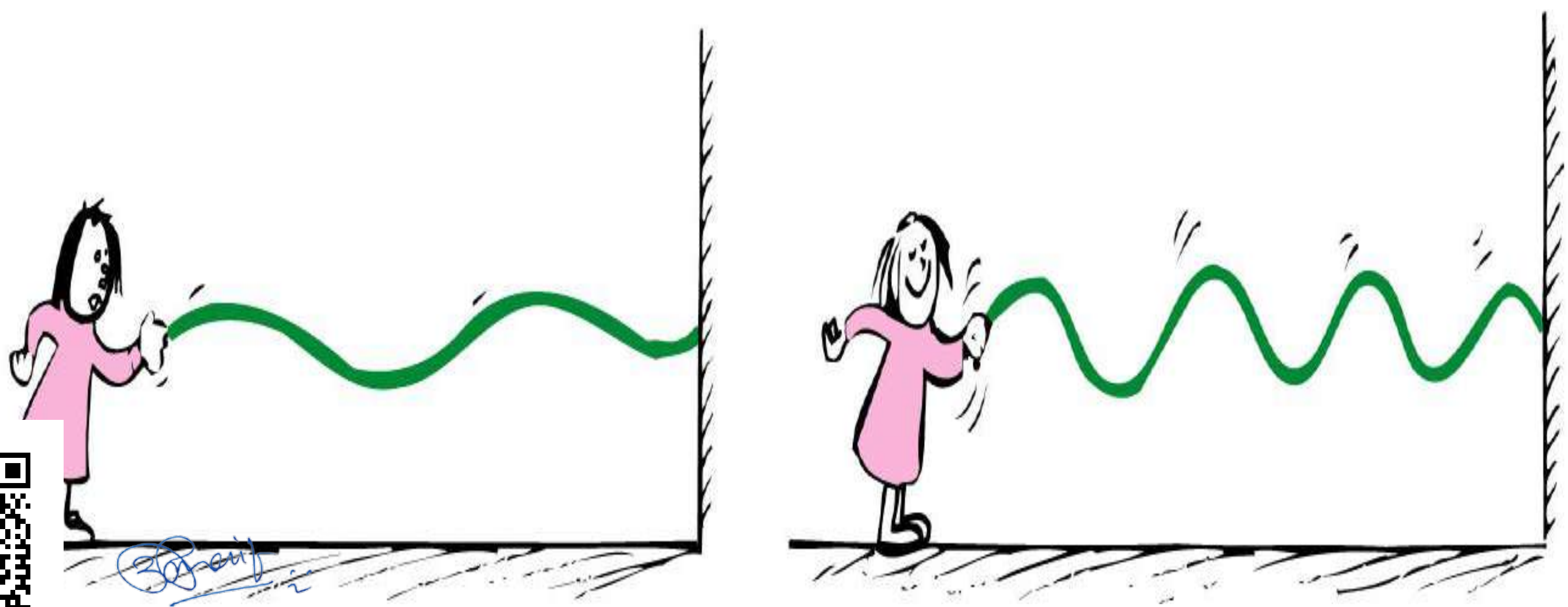
Convection Oven

➢ Convection oven has a fan to enhance the circulation of the air, increasing the transfer of heat.



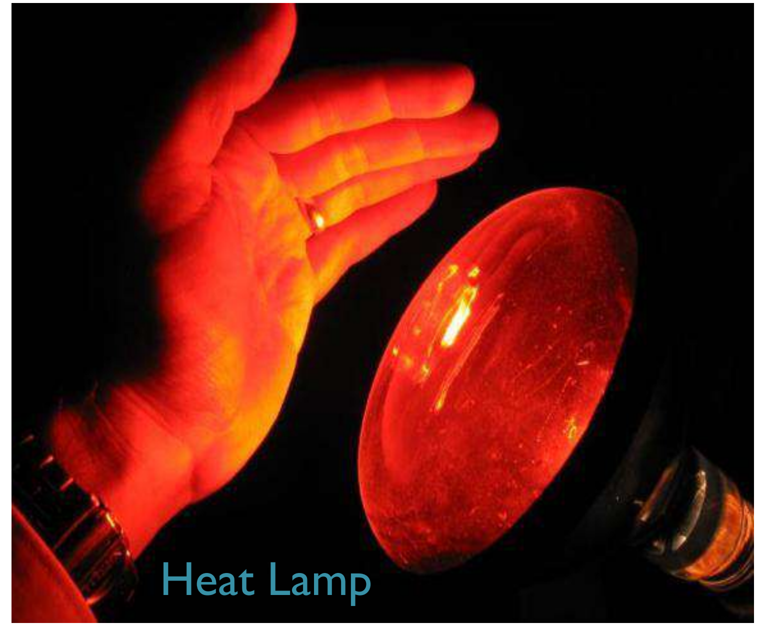
Heat Radiation

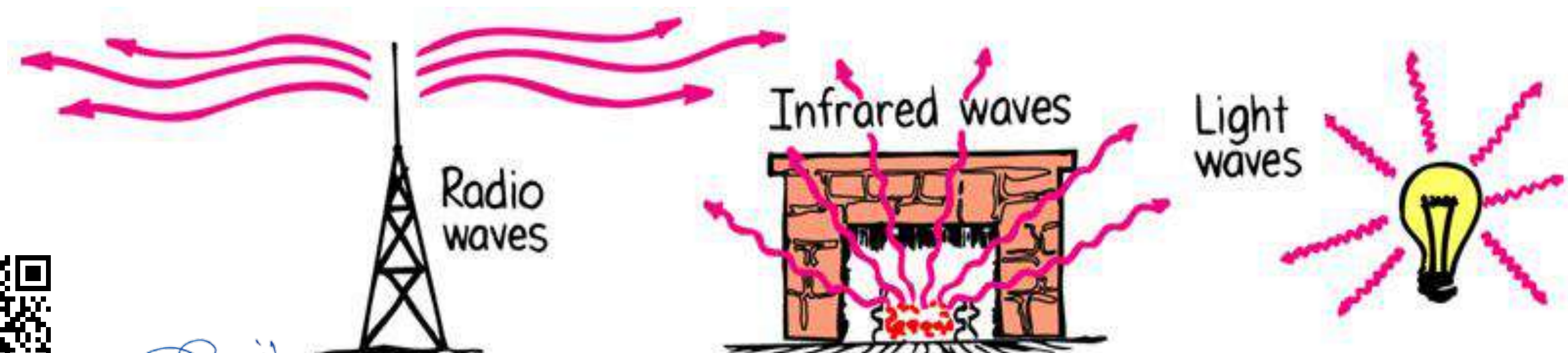
- Energy carried by electromagnetic waves
- Light, microwaves, radio waves, x-rays
- Wavelength is related to vibrational frequency



Radiation

- Light has many different wavelengths, most of which are not visible to the eye.
- All light carries energy, thus transfers heat.





Heat Radiation

- All bodies continuously emit energy if their temp is above zero absolute (0K) and energy thus emitted is called thermal radiation.

- Thermal radiations are electromagnetic waves and do not require any medium for propagation.

- Thermal radiation is a surface phenomenon.

- Theories of Thermal Radiation

1. Wave/Maxwell's Classical Theory : Propagation by electromagnetic waves

   Quantum/ Planck's Theory: Propagation by quanta possessing certain amount of energy

Stefan Boltzmann's Law of Radiation

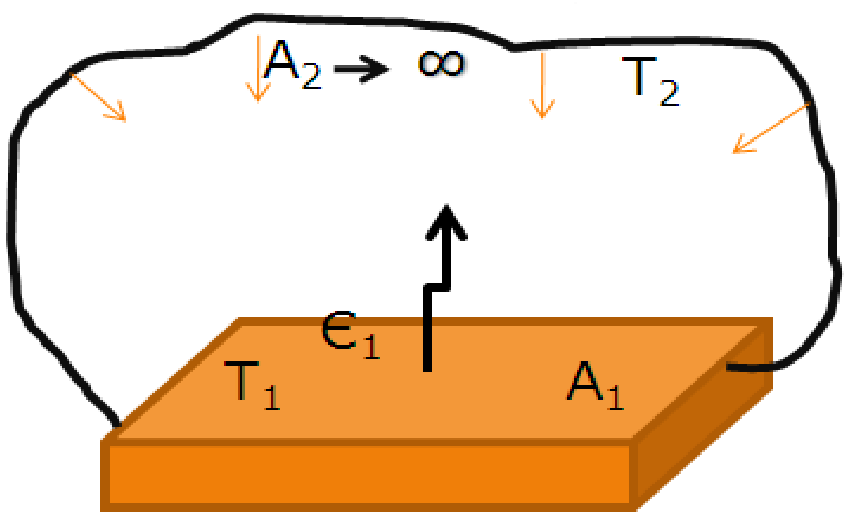
- Thermal radiation emitted by a black body is proportional to the Fourth Power of its absolute temp.

Mathematically;

$$q \propto T^4 \quad W/m^2;$$

$$Q = \sigma A T^4 \ W;$$ where σ is Stefan Boltzmann's constant ($5.67 \times 10^{-8} \ W/m^2K^4$)

$Q = A_1 \epsilon_1 \sigma (T_1^4 - T$



$T_1 > T_2$

The Stefan–Boltzmann's Law Of Radiation

- The rate at which an object emits radiant energy is proportional to the fourth power of its absolute temperature. This is known as Stefan's law and is expressed as follows:

$$Q = \sigma \mathcal{E} A T^4$$

where σ is the Stefan-Boltzmann constant,
$\sigma = 5.67 \times 10^{-8}\ W/m^2\ K^4$.

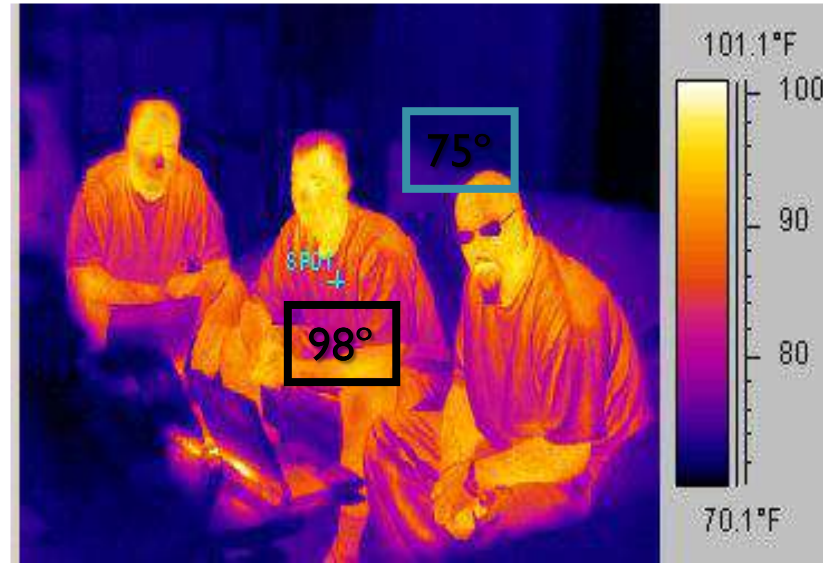
The factor ∈ is called the emissivity, which is a number between 0 and 1.

fect radiators have a value of 1 for ∈.

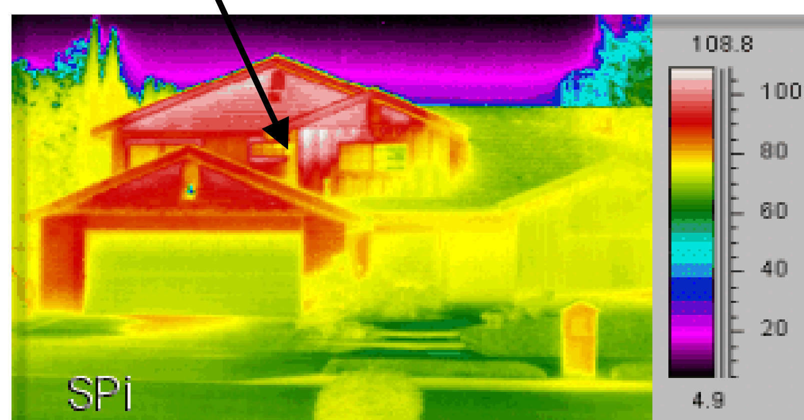
s the surface area and T is the temperature of the radiator in Kelvin.

Emission of Radiant Energy

- All objects radiate light; higher the temperature, higher the frequency.
- At room temperature the radiated light is at frequencies too low for our eyes to see.
- Special cameras are sensitive to this infrared radiation.



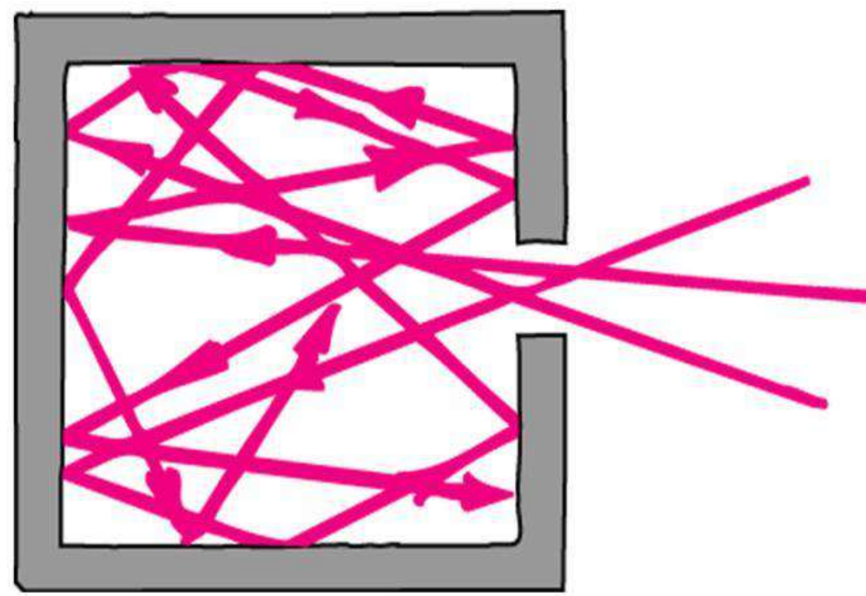
Attics in this house were kept warm for growing marijuana.



Reflection of Radiant Energy

White objects reflect light, black objects don't.

Hole in a box with white interior looks black because almost none of the light entering the hole reflects back out.



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White tubes look black inside.

Numerical Problem

Q1: Air at 20°C blows over a 50cm x 75cm hot plate at 250°C. The film heat transfer coefficient is 25 W/m²K. 300 W is lost from the plate surface by radiation. Calculate heat transfer rate and other side plate temp. Thermal conductivity of the plate material is 43 W/mK. The plate is 2cm thick.

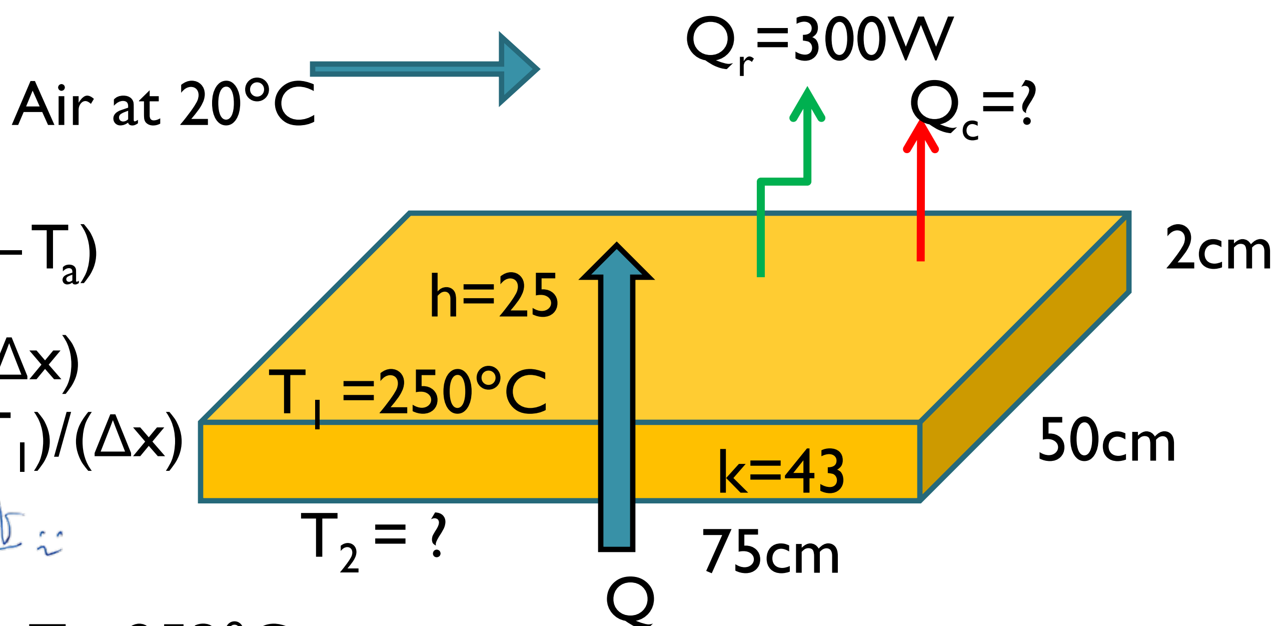
Q=?

$Q = Q_c + Q_r$

$Q_c = hA(T_1 - T_a)$

$Q = kA(\Delta T/\Delta x)$

$kA(T_2 - T_1)/(\Delta x)$



$Q = 2456W$; $T_2 = 253°C$

Electrical Analogy

	Electrical Energy	Heat Energy
What flows?	Electrons	Heat energy through electrons
Driving Potential	Voltage Diff, ΔV	Temp Diff, ΔT
Flow	Current, I	Heat Transfer Rate, Q
Resistance to flow	ρ, A, L of conductor	R, Thermal Resistance

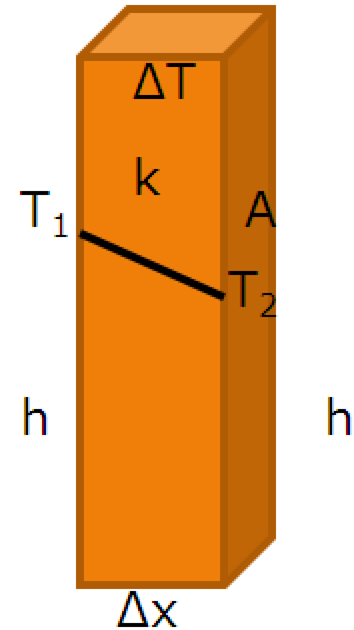
Electrical Analogy

As per Ohm's Law, I = ΔV/R

Similarly, Heat Flow Rate, Q = ΔT/R = C.ΔT;
where R is thermal resistance & 1/R=C conductance

Conductive Resistance:

$$Q = kA\frac{\Delta T}{\Delta x} = \frac{\Delta T}{\frac{\Delta x}{kA}} = \frac{\Delta T}{R}$$



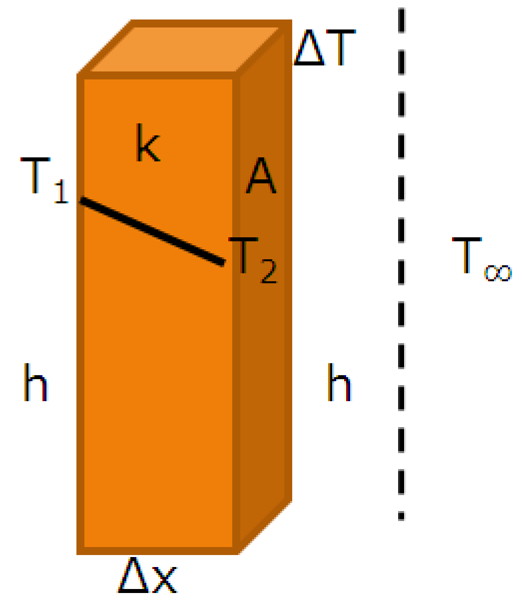
$$\text{ence}, R_{conductive} = \frac{\Delta x}{kA}$$

Electrical Analogy

Convective Resistance:

$$Q = hA(T_2 - T_\infty) = \frac{\Delta T}{\frac{1}{hA}} = \frac{\Delta T}{R}$$

$$Hence, R_{convective} = \frac{1}{hA}$$



Heat Transfer In Composite Structures

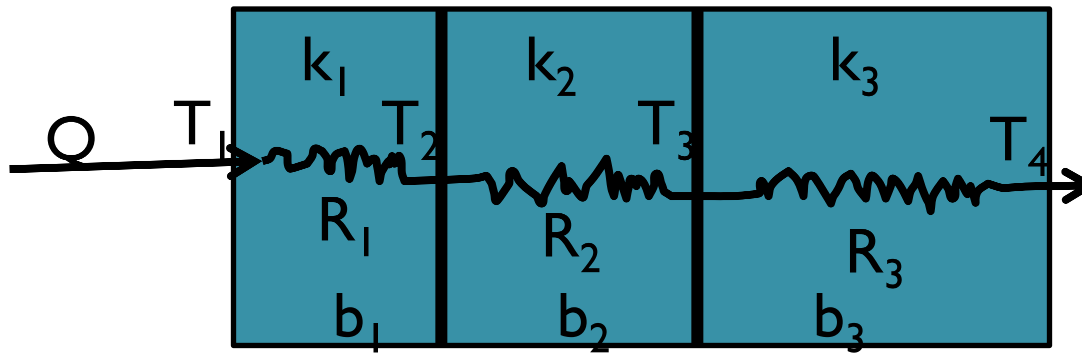
Resistance In Series

$Q=\Delta T/R$

$=(T_1 - T_2)/R_1$

$=(T_2 - T_3)/R_2$

$=(T_3 - T_4)/R_3$



On adding up;

$T_1 - T_4=Q\ (R_1+R_2+R_3)$ or $Q=(T_1-T_4)/(R_1+R_2+R_3)$

$Q=\Delta T/R$;

nce $R=R_1+R_2+R_3$

$=b_1/k_1A$; $R_2=b_2/k_2A$; $R_3=b_3/k_3A$

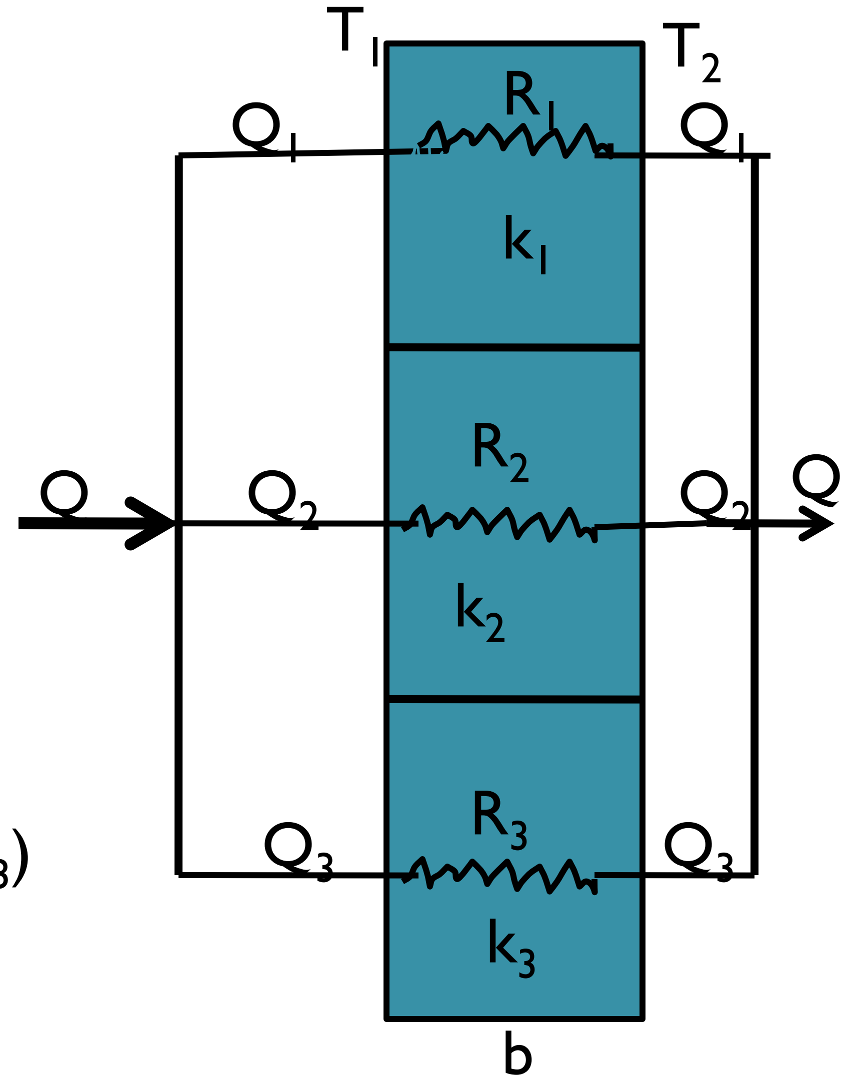
Heat Transfer In Composite Structures

Resistance In Parallel

$Q_1 = (T_1 - T_2)/R_1$

$Q_2 = (T_1 - T_2)/R_2$

$Q_3 = (T_1 - T_2)/R_3$



On adding;

$Q = Q_1 + Q_2 + Q_3$

$= (T_1 - T_2) * (1/R_1 + 1/R_2 + 1/R_3)$

$: \Delta T * 1/R;$

nce $1/R = 1/R_1 + 1/R_2 + 1/R_3$

Examples of Composite Structures

- Walls of buildings
- Walls of home refrigerators
- Insulated pipe carrying steam
- Walls of a furnace
- Walls of a cold storage
- Hot case for food

Unsteady State Heat Transfer

- Whenever a heat transfer system is switched on/ started, it takes some time to attain steady value of heat transfer rate. Heat transfer rate under these conditions keeps varying with passage of time. This heat transfer system is said to be transferring heat under unsteady state / transient conditions. Here, temperature also keeps varying at various locations in the system with time. Hence, temp is a function of both location as well as time.

- Similar situation occurs when a heat transfer system is switched off / shut off, but in reverse direction

- Examples are starting/firing of a furnace, heating of a body, switching on a heater, starting of an engine, etc

Steady State Heat Transfer

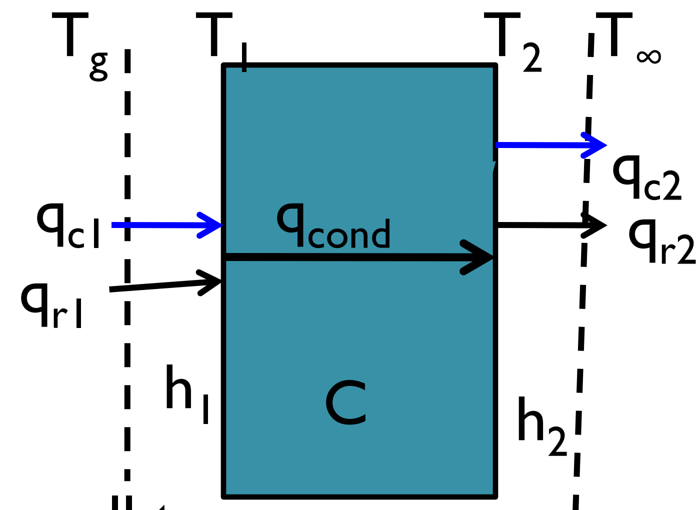
- Whenever a heat transfer system is switched on/ started, it takes some time to stabilize the heat transfer rate when it becomes constant and does not change with time. This heat transfer system is said to be transferring heat under steady state conditions. Here, temperatures attain constant values at various locations in the system and do not vary with time. Hence, temp is a function of only location and not of time.

- Heat transfer rate is directly proportional to temp difference. Since temps attain constant values, temp difference also become constant hence heat transfer rate attains steady value.

- This implies that whatever amount of heat energy is being eceived by the system, at same rate it is transferring out.

- is means that under steady state, system transfers / eives constant amount of heat energy per unit time

Q2. In a furnace, temp of hot gases is 2100°C. Ambient temp is 40°C. Heat flow by radiation from hot gases to inner surface of the wall is 23kW/m². Convective heat transfer coeff. between hot gases and the inner surface of the wall is 12W/m²K. Thermal conductance of the wall is 58W/m²K. Heat flow by radiation from external surface of the wall to surroundings is 10kW/m². Temp of inside surface of the wall is 900°C. For the external surface of the wall, find surface temp and convective heat transfer coefficient.



(Ans. T_2=255.2°C; h_2=127.3W/m²K

h_2 = ?

Solution:

$$Q_{C1} = h_1 A(T_g - T_1) = 12x1(2100 - 900) = 14.4 kW/m^2$$

$$Q = Q_{C1} + Q_{r1} = 14.4 + 23 = 37.4 kW/m^2$$

This is the heat conducted through slab.

$$Hence\ Q = C(T_1 - T_2) = \frac{(T_1 - T_2)}{1/C}$$

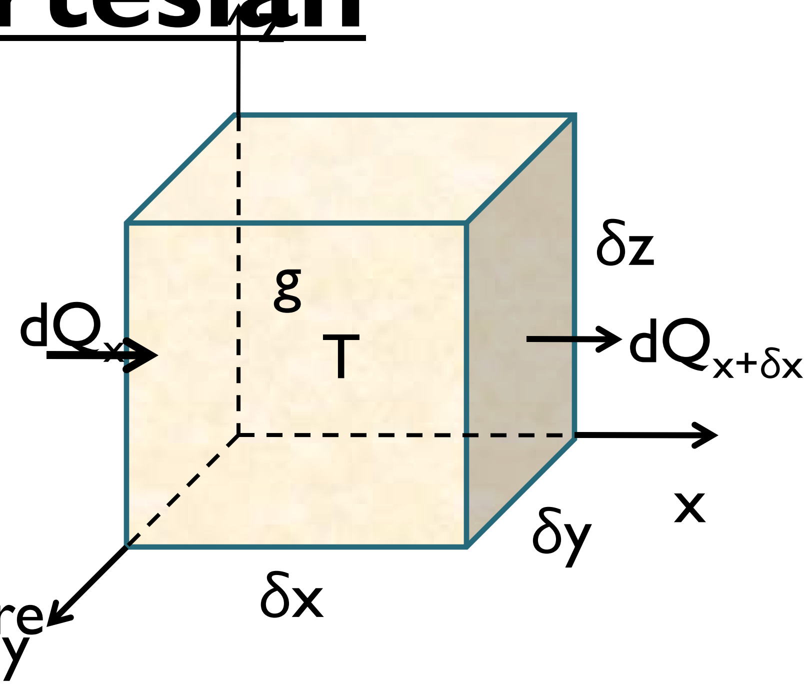
$$= \frac{(900 - T_2)}{1/58} \Rightarrow T_2 = 255.2°C$$

$$Q = Q_{C2} + Q_{r2} \Rightarrow 37400 = Q_{C2} - 10000$$

$$ence\ Q_{C2} = 27400 = h_2 A(T_2 - T_\infty)$$

$$h_2 = 127.3 W/m^2 K$$

General Heat Conduction Equation In Cartesian Coordinates



- Consider a small rectangular volume of sides δx, δy & δz parallel to the three axes in medium, in which temp is varying with loc & time.
- Let T denote the temp at centre of this elemental volume.
- Also, let there be internal heat generation at the rate of g Watt per unit volume (W/m^3) due to heat source
- …t the material be anisotropic implying that thermal …ductivities have values k_x, k_y & k_z in x, y & z directions …pectively

General Heat Conduction Equation

- Consider heat entering and leaving this volume through its six faces.

- Let heat entering the elemental volume per unit time normal to the area/face $\delta y \delta z$ at x be dQ_x and heat leaving the volume in the direction normal to the area $\delta y \delta z$ at $x+\delta x$ be $dQ_{x+\delta x}$.

- As per Fourier's Law, heat entering $dQ_x = -k_x(\delta y \delta z)\partial T/\partial x$

- Similarly, heat leaving, $dQ_{x+\delta x} = dQ_x + \partial/\partial x(dQ_x)\delta x$

General Heat Conduction Equation

So, net heat flow into the element in x-direction/time;

$$dQ_x - dQ_{x+dx} = -\frac{\partial}{\partial x}\left(dQ_x\right)\delta x$$

$$= -\frac{\partial}{\partial x}\left(-k_x \delta y \delta z . \frac{\partial T}{\partial x}\right)\delta x$$

$$= \frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right)\delta x \delta y \delta z$$

General Heat Conduction Equation

Similarly, net heat flow into the element per unit time in y & z directions respectively are;

$$dQ_y - dQ_{y+\delta y} = \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y} \right)\delta x \delta y \delta z \quad and$$

$$dQ_z - dQ_{z+\delta z} = \frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z} \right)\delta x \delta y \delta z$$

Thus, net heat flow in to the element from all directions by conduction in certain time δt will be:

$$\left[\left(k_x \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z} \right) \right] \delta x \delta y \delta z \delta t$$

General Heat Conduction Equation

Now, internal heat generation in time $\delta t = g.\delta x \delta y \delta z \delta t$

Heat gain by the element from above, will result in energy storage and will increase its temp.

Let δT be the rise in temp in time δt, the net heat storage in the element in time δt ;

$$(mC_p \Delta T) = \rho V C_p \, \delta T$$

$$= \rho C_p \, \delta T \, \delta x \delta y \delta z$$

General Heat Conduction Equation

<u>Energy Balance Equation:</u>

Net heat conducted in to the element from all Directions +Heat generated within the element = Energy stored in the element

$$\left[\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right)\right]\delta x\delta y\delta z\delta t$$

$$+\, g.\delta x\delta y\delta z\delta t \quad = \quad \rho C_p \delta T.\delta x\delta y\delta z$$

Dividing the Equation by $\delta x\delta y\delta z\delta t$, we get;

General Heat Conduction Equation

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right)+g=\rho C_p \frac{\partial T}{\partial t}$$

This is three dimensional heat conduction equation in Cartesian Coordinates for anisotropic material for Unsteady state conditions.

For isotropic material, $k_x = k_y = k_z = k$ constant

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{g}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$Where\ \alpha\ is\ thermal\ diffusivity=\frac{k}{\rho C_p}\ m^2/s$$

General Heat Conduction Equation

Fourier's Equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Poisson's Equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{g}{k} = 0$$

Laplace Equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

Steady State, One Dimensional Equation w/o g:

$$\frac{d^2 T}{dx^2} = o$$

Types of Problems In Heat Transfer

1. Plate/Slab/Wall

2. Tube/Pipe/Cylinder

3. Sphere

- To increase Heat Transfer Rate

- To decrease Heat Transfer Rate

General Heat Conduction Equation in Cylindrical Coordinates

By substituting x=r.cosθ; y=r.sinθ and z=z, we get General Heat Conduction Equation in Polar/ Cylindrical Coordinates:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} + \frac{g}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\cdots For\ isotropic\ material\ with\ \ k = constt$$

Poisson's Equation:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{g}{k} = 0$$

lial heat conduction w/o g:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

General Heat Conduction Equation In Spherical Coordinates

Similarly, by substituting x=r.sinθ.cosΦ; y= r.sinθsinΦ and z=r.sinθ, we get heat conduction equation in Spherical Coordinates:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2}+\frac{g}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

For isotropic material with $k = \text{const}$*t*

Poisson's Equation:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)+\frac{g}{k}=0$$

ial heat conduction w/o g:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=0$$

Thermal Diffusivity

- Thermal Diffusivity is the ratio of thermal conductivity to heat storage capacity of the material.

$$Denoted\ by\ \alpha, it\ is\ defined\ as: \quad \alpha = \frac{k}{\rho C_p} \quad m^2/s$$

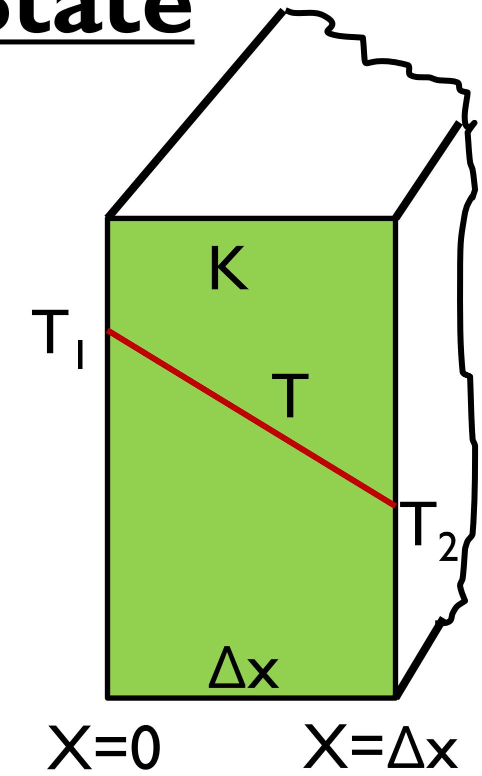
- Larger the value of α, faster shall be the heat diffusion through the material.

Steady state heat conduction does not contain α, hence temp distribution through material is determined by k only, where as in unsteady state heat conduction, temp distribution is determined by α. (Both by k & ρC_p)

ample: Cooking steel utensils having copper bottom

One Dimensional Steady State Heat Conduction through Slab/Plane Wall

Consider a plane wall of thickness Δx of material having conductivity k with its faces maintained at temp T_1 & T_2



Steady state, one dimensional Heat conduction eqn will be:

$$\frac{d^2T}{dx^2} = 0$$

$$Integrating\ \ this\ \ equation\ \ twice;$$

$$We\ \ have\quad \frac{dT}{dx} = C_1 \text{..........}(1)\ Slope\ of\ Temp\ Profile$$

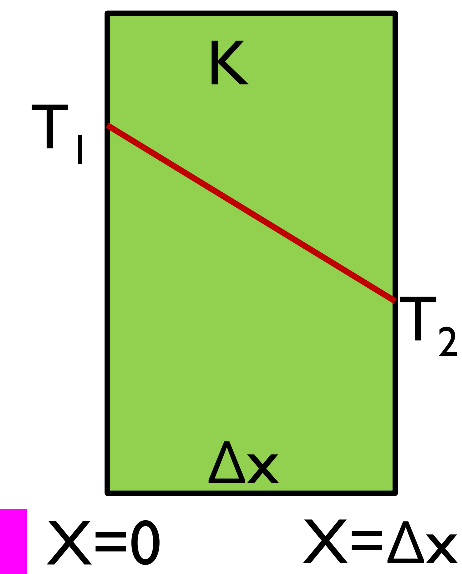
$$and\quad T = C_1 x + C_2 \text{.........}(2)\ Temp\ \ Profile$$

Heat Conduction through Slab/Plane Wall

Boundary Conditions:

$(T=C_1.x+C_2)$....(2)

1) At x=0; $T=T_1$
2) At $x=\Delta x$; $T=T_2$



Applying BC 1), we get $T_1=C_1.0+C_2$

Hence $C_2=T_1$

Applying BC 2), we get

$T_2=C_1.\Delta x+C_2$

Or $T_2=C_1.\Delta x+T_1$

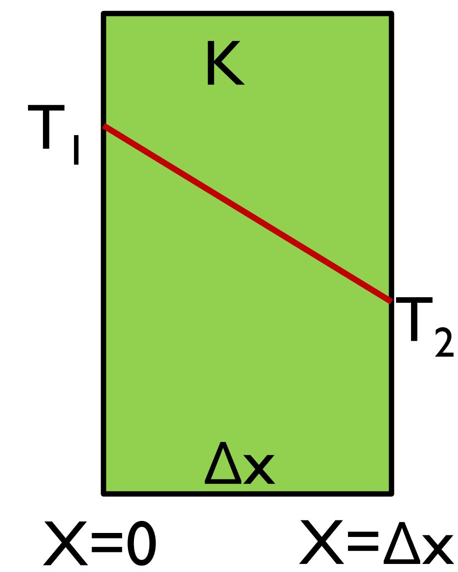
$$\Rightarrow C_1 = \frac{T_2 - T_1}{\Delta x}$$

Substituting C_1 and C_2 in Eqn..(2)

$$get \; T = \frac{T_2 - T_1}{\Delta x}.x + T_1 ....Temp\ Distribution$$

Heat Conduction through Slab/Plane Wall

$$Heat\ Flow\ Rate \quad Q = -kA\frac{dT}{dx}$$

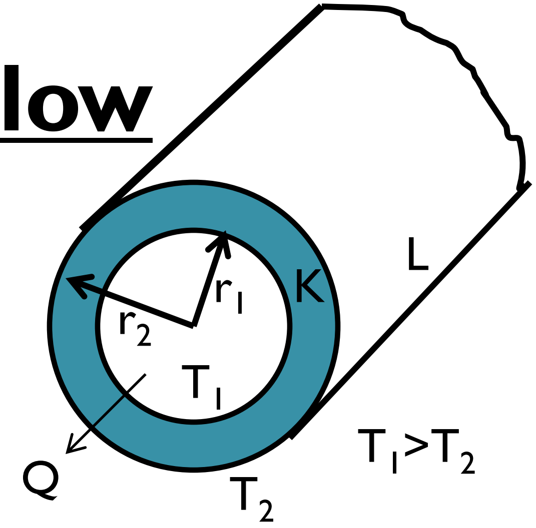


$$From\ Eqn..(1);\ \frac{dT}{dx} = C_1 = \frac{T_2 - T_1}{\Delta x}$$

$$Hence\ Q = kA\frac{(T_1 - T_2)}{\Delta x} = \frac{\Delta T}{\frac{\Delta x}{kA}} = \frac{\Delta T}{R}$$

$$nce \quad R_{cond} = \frac{\Delta x}{kA} ..... for\ Slab$$

One Dimensional (Radial) Steady State Heat Conduction through Hollow Cylinder



Consider a hollow cylinder of inner radius r_1 and outer r_2 of length L of a material having conductivity k.

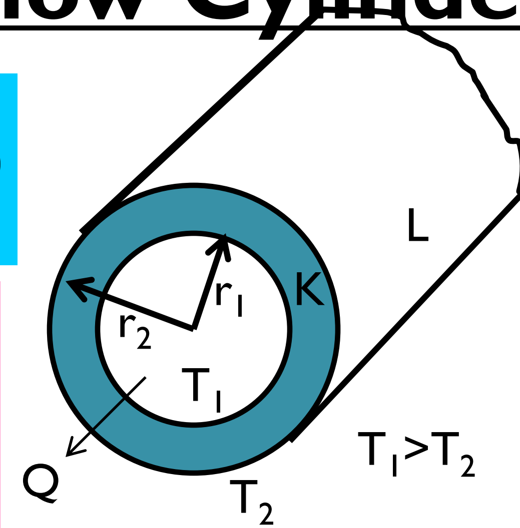
Inner surface of cylinder is at temp T_1 and outer at T_2

Conduction Equation for one dimensional (radial) Heat flow (without g) will be:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)=0$$

One Dimensional Steady State Heat Conduction through Hollow Cylinder

Integrating Equation: $\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)=0$



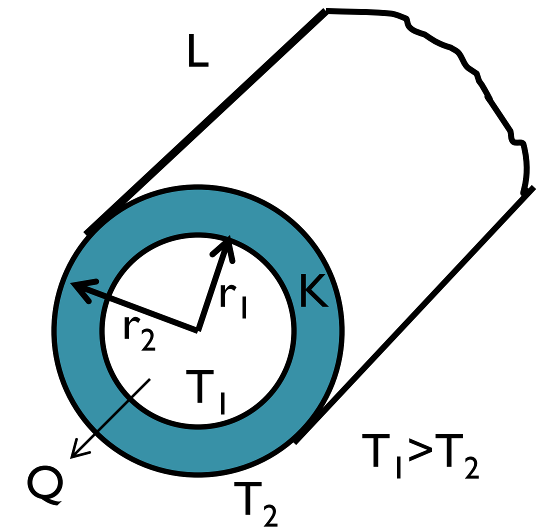
$$We\ have\quad \int \frac{d}{dr}\left(r\frac{dT}{dr}\right)=0$$

$$\Rightarrow r\frac{dT}{dr}=C_1 \quad or \quad \frac{dT}{dr}=\frac{C_1}{r}\,.......(1)$$

On further Integration;

$$We\ have\quad T=C_1\ln r+C_2\,.......(2)$$

Heat Conduction through Hollow Cylinder



Boundary Conditions:

Eqn (2) $T=C_1.\ln r+C_2$

1) At $r=r_1; T=T_1$
2) At $r=r_2; T=T_2$

Substituting in Eqn ….(2); We have

$T_1=C_1.\ln r_1+C_2$ …..(3)

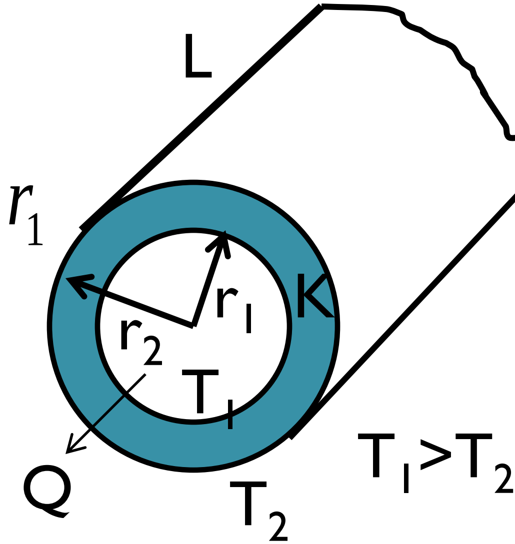
$T_2=C_1.\ln r_2+C_2$ ….(4)

Subtracting eqn (4) from (3) and further substitution;

$$= \frac{T_2 - T_1}{\ln \frac{r_2}{r_1}} \quad and\ C_2 = T_1 - \frac{T_2 - T_1}{\ln \frac{r_2}{r_1}} . \ln r_1$$

Heat Conduction through Hollow Cylinder

$T = C_1 . \ln r + C_2$ …….(2)

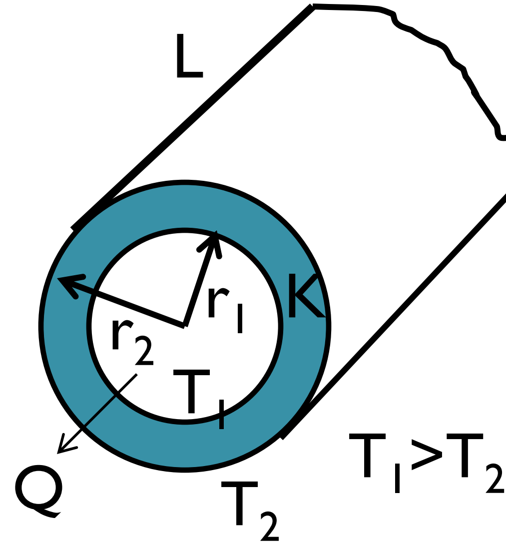


$$C_1 = \frac{T_2 - T_1}{\ln \frac{r_2}{r_1}} \quad and \quad C_2 = T_1 - \frac{T_2 - T_1}{\ln \frac{r_2}{r_1}} . \ln r_1$$

Substituting values of C_1 & C_2 in Eqn ….(2); We have

$$T = \frac{\ln \frac{r}{r_1}}{\ln \frac{r_2}{r_1}} \left(T_2 - T_1 \right) + T_1 \quad OR \quad \frac{T - T_1}{T_2 - T_1} = \frac{\ln \frac{r}{r_1}}{\ln \frac{r_2}{r_1}}$$

Heat Conduction through Hollow Cylinder



Heat Flow Rate:

$$C_1 = \frac{T_2 - T_1}{\ln \frac{r_2}{r_1}}$$

$$Q = -kA\frac{dT}{dr}$$

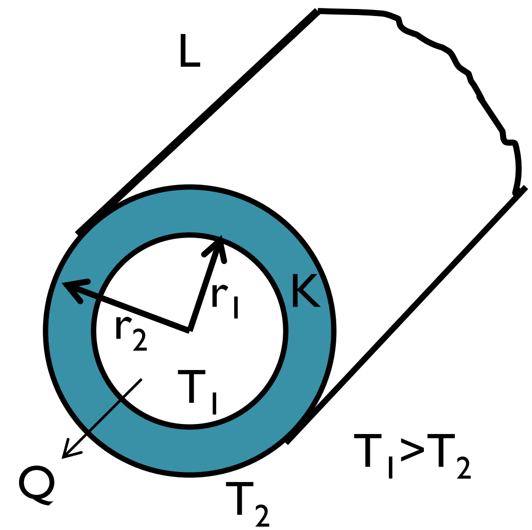
$$\frac{dT}{dr} = \frac{C_1}{r} \text{.... from Eqn....(1)}$$

$$Therefore,\ Q = -k.2\pi rL.\frac{C_1}{r} = -2\pi kLC_1$$

$$Substituting\ C_1;$$

$$= -2\pi kL.\frac{(T_2 - T_1)}{\ln \frac{r_2}{r_1}} = 2\pi kL.\frac{(T_1 - T_2)}{\ln \frac{r_2}{r_1}}$$

Heat Conduction through Hollow Cylinder



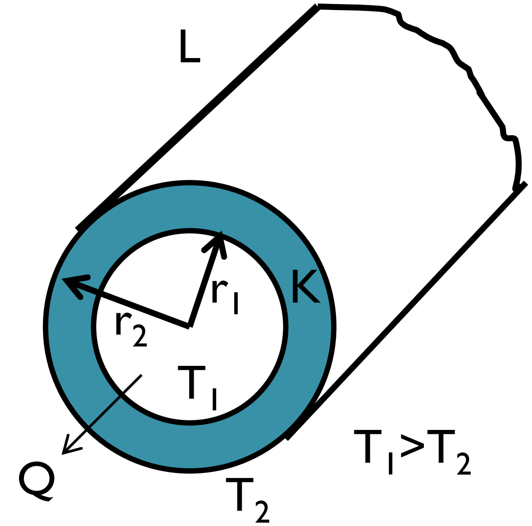
Heat Flow Rate:

$$Q = -2\pi k L . \frac{(T_2 - T_1)}{\ln \frac{r_2}{r_1}}$$

$$Q = \frac{(T_1 - T_2)}{\frac{\ln \frac{r_2}{r_1}}{2\pi k L}} = \frac{\Delta T}{R}$$

$$\text{ence} \quad R_{Cond} = \frac{\ln \frac{r_2}{r_1}}{2\pi k L} \quad \text{for Cylinder}$$

Heat Conduction through Hollow Cylinder



In case of cylinder, in Q expression,
Q= -k A (dT/dr);
area transferring heat A=2 л r L
changes with r, unlike in case of slab.
Therefore, it is convenient to work out
Mean Area A_m for use in analogous
formula
for slab Q=k A (ΔT/Δx).

Logarithmic Mean Area (LMA)

$$If\ we\ write\ Q = \frac{2\pi kL.\Delta T}{\ln \frac{r_2}{r_1}} = k.A_m . \frac{\Delta T}{r_2 - r_1}$$

en A_m is mean area which can be utilized
formula for slab

Heat Conduction through Hollow Cylinder

Logarithmic Mean Area (LMA)

$$To\,obtain\,value\,of\ LMA\,i,e.\ A_m;$$

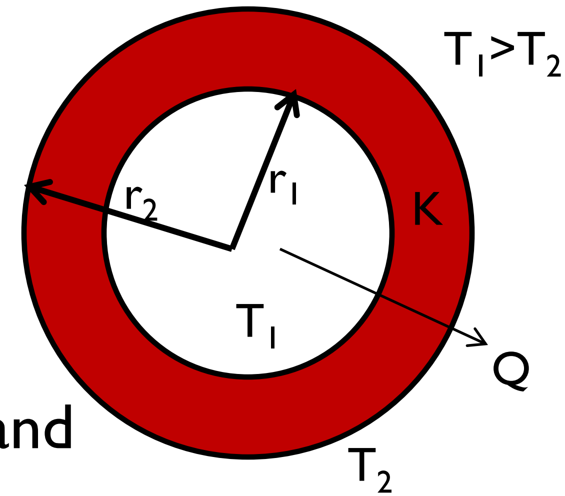
$$We\,multiply\ \&\ divide\ Q\,expression\,by\,(r_2 - r_1)\,as;$$

$$Q = \frac{2\pi kL.\Delta T}{\ln\frac{r_2}{r_1}}.\frac{(r_2 - r_1)}{(r_2 - r_1)} = \frac{k.2\pi L(r_2 - r_1).}{\ln\frac{r_2}{r_1}}\frac{\Delta T}{(r_2 - r_1)}$$

$$Comparing\ with\ Q = k.A_m.\frac{\Delta T}{r_2 - r_1};$$

$$e\,have\quad A_m = \frac{2\pi L(r_2 - r_1)}{\ln\frac{r_2}{r_1}} = \frac{A_o - A_i}{\ln\frac{A_o}{A_i}}$$

One Dimensional (Radial) Steady State Heat Conduction through Hollow Sphere

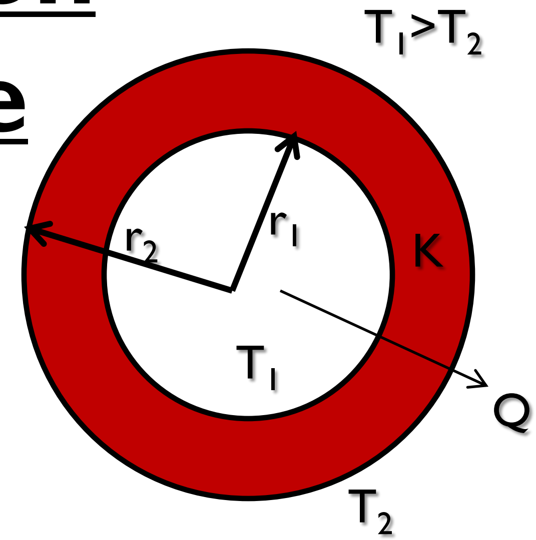


- Consider a hollow sphere of inner radius r_1 and outer r_2 of a material having conductivity k.
- Inner surface of sphere is at temp T_1 and outer at T_2
- Conduction Equation for one dimensional (radial)

Heat flow (without g) will be:

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=0 \Rightarrow \frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=0$$

One Dimensional (Radial) Steady State Heat Conduction through Hollow Sphere



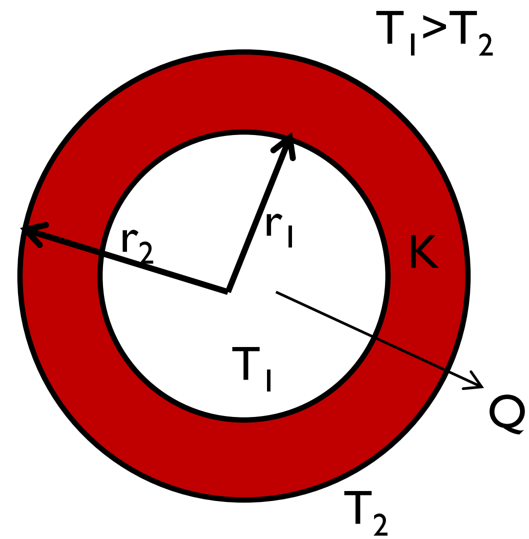
$$Integrating\ Eqn\ldots\int \frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = 0$$

$$We\ have\ r^2 \frac{dT}{dr} = C_1 \ or\ \frac{dT}{dr} = \frac{C_1}{r^2}\ldots(1)$$

$$On\ further\ Integration, we\ have$$

$$T = -\frac{C_1}{r} + C_2\ldots\ldots\ldots\ldots(2)$$

One Dimensional (Radial) Steady State Heat Conduction through Hollow Sphere



Boundary Conditions:

1) At $r=r_1$; $T=T_1$
2) At $r=r_2$; $T=T_2$

Substituting in Eqn $T=-\frac{C_1}{r}+C_2 \ldots(2)$

$$We\,have \quad C_1=\frac{(T_1-T_2).r_1 r_2}{r_1-r_2}$$

$$nd \quad C_2=T_1+\frac{(T_1-T_2).r_2}{r_1-r_2}$$

One Dimensional (Radial) Steady State Heat Conduction through Hollow Sphere

$$Substituting\ C_1\ \&\ C_2\ in\ Eqn\ T = -\frac{C_1}{r} + C_2$$

$$T = \frac{r_1}{r}.\frac{r_2 - r}{r_2 - r_1}.T_1 + \frac{r_2}{r}.\frac{r - r_1}{r_2 - r_1}.T_2$$

s is the Temp Profile across the thickness
phere

One Dimensional (Radial) Steady State Heat Conduction through Hollow Sphere

$$\text{Heat Flow Rate } Q = -kA\frac{dT}{dr} = -k.4\pi r^2.\frac{dT}{dr}$$

$$\text{Substituting } \frac{dT}{dr} = \frac{C_1}{r} \Rightarrow Q = -k.4\pi r^2.\frac{C_1}{r^2} = -4\pi k C_1$$

$$\text{Substituting } C_1;$$

$$Q = 4\pi k.r_2 r_1.\frac{T_1 - T_2}{r_2 - r_1} = \frac{T_1 - T_2}{\dfrac{r_2 - r_1}{4\pi k.r_2 r_1}}$$

$$\text{erefore } \therefore R_{cond} = \frac{r_2 - r_1}{4\pi k r_2 r_1} \text{ and } A_m = 4\pi r_1 r_2$$

Conductive Resistances

For Slab:

$$R = \frac{\Delta x}{kA}$$

For Hollow Cylinder:

$$R = \frac{\ln \frac{r_2}{r_1}}{2\pi k L}$$

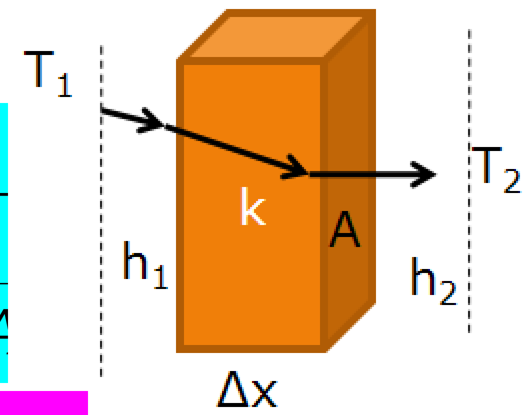
For Sphere:

$$R = \frac{r_2 - r_1}{4\pi k r_2 r_1}$$

Overall Heat Transfer Coefficient

Heat Flow Rate can also be given as Q=UAΔT;
where U is called as overall heat transfer coefficient

For plane wall:



$$Q = UA\Delta T = \frac{\Delta T}{\frac{1}{UA}} = \frac{\Delta T}{\frac{1}{h_1 A} + \frac{\Delta x}{kA} + \frac{1}{h_2 A}}$$

$$hence \quad \frac{1}{UA} = \frac{1}{h_1 A} + \frac{\Delta x}{kA} + \frac{1}{h_2 A}$$

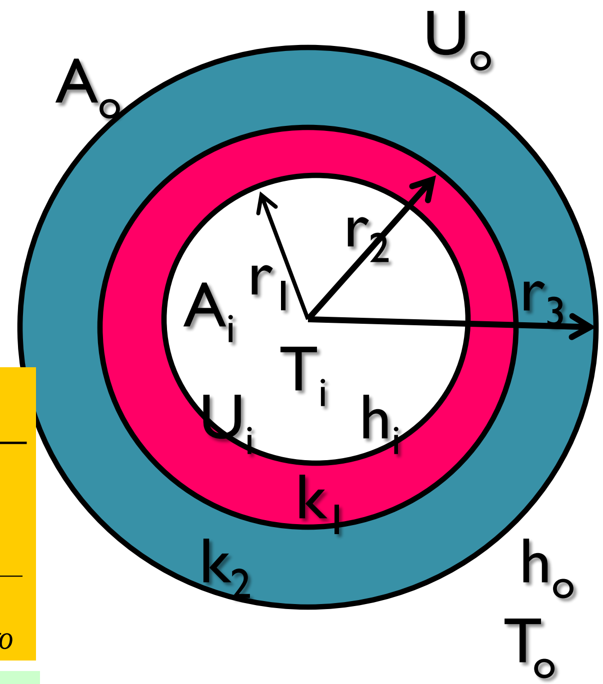
erefore, $$\frac{1}{U} = \frac{1}{h_1} + \frac{\Delta x}{k} + \frac{1}{h_2}$$

where U is Overall Heat Transfer Coeff

Overall Heat Transfer Coefficient

For Cylinder:

$$Q = U_i A_i (T_i - T_o) = U_o A_o (T_i - T_o)$$



$$= \frac{\Delta T}{\frac{1}{U_i A_i}} = \frac{\Delta T}{\frac{1}{U_o A_o}} = \frac{\Delta T}{\frac{1}{h_i A_i} + \frac{\ln {r_2}/{r_1}}{2\pi k_1 L} + \frac{\ln {r_3}/{r_2}}{2\pi k_2 L} + \frac{1}{h_o A_o}}$$

$$\frac{1}{U_i A_i} = \frac{1}{U_o A_o} = \frac{1}{h_i A_i} + \frac{\ln {r_2}/{r_1}}{2\pi k_1 L} + \frac{\ln {r_3}/{r_2}}{2\pi k_2 L} + \frac{1}{h_o A_o}$$

re U_i is Overall heat transfer coeff based on inner surface area A_i

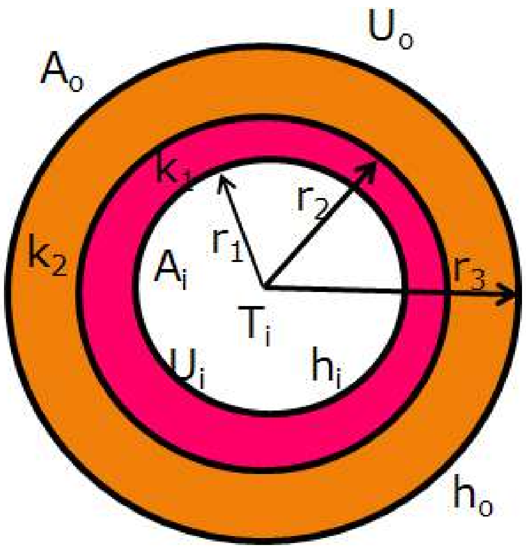
U_o is Overall heat transfer coeff based on outer surface area A_o

$= 2\pi r_1 L$ and $A_o = 2\pi r_3 L$

Overall Heat Transfer Coefficient

For Sphere:

$$Q = U_i A_i \left(T_i - T_o\right) = U_o A_o \left(T_i - T_o\right)$$



$$= \frac{\Delta T}{\frac{1}{U_i A_i}} = \frac{\Delta T}{\frac{1}{U_o A_o}} = \frac{\Delta T}{\frac{1}{h_i A_i} + \frac{r_2 - r_1}{4\pi k_1 r_2 r_1} + \frac{r_3 - r_2}{4\pi k_2 r_3 r_2} + \frac{1}{h_o A_o}}$$

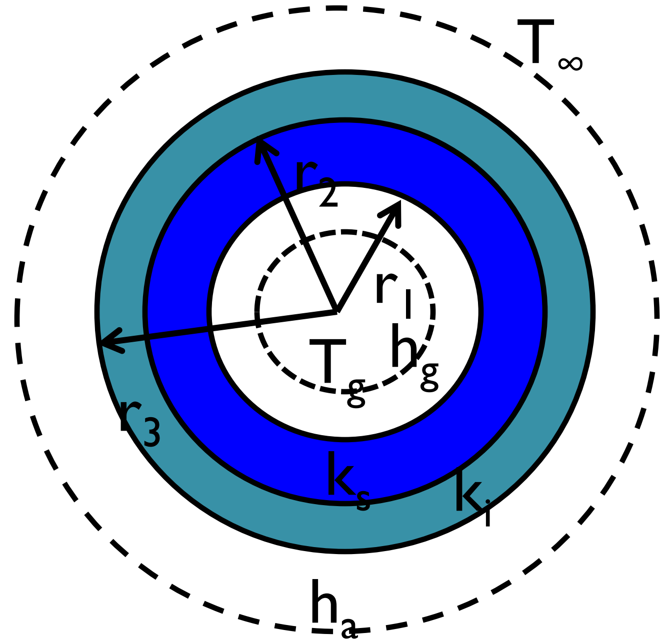
$$\frac{1}{U_i A_i} = \frac{1}{U_o A_o} = \frac{1}{h_i A_i} + \frac{r_2 - r_1}{4\pi k_1 r_2 r_1} + \frac{r_3 - r_2}{4\pi k_2 r_3 r_2} + \frac{1}{h_o A_o}$$

re U_i is Overall heat transfer coeff based on inner surface area A_i

U_o is Overall heat transfer coeff based on outer surface area A_o

$A_i = 4\pi r_1^{\,2}$ and $A_o = 4\pi r_3^{\,2}$

Q3. A steel tube with 8cm OD, 6cm ID and k=15W/mK, is covered with an insulation covering of thickness 2cm and k=0.2W/mK. A hot gas at 300°C with h_g=400W/m^{2}K flows inside the tube. The outer surface of insulation is exposed to cool air at 30°C with h_a=50W/m^{2}K. Calculate over all heat transfer coeff. U_o based on outer surface of insulation and heat loss from the tube for its 25m length.



U_o=?

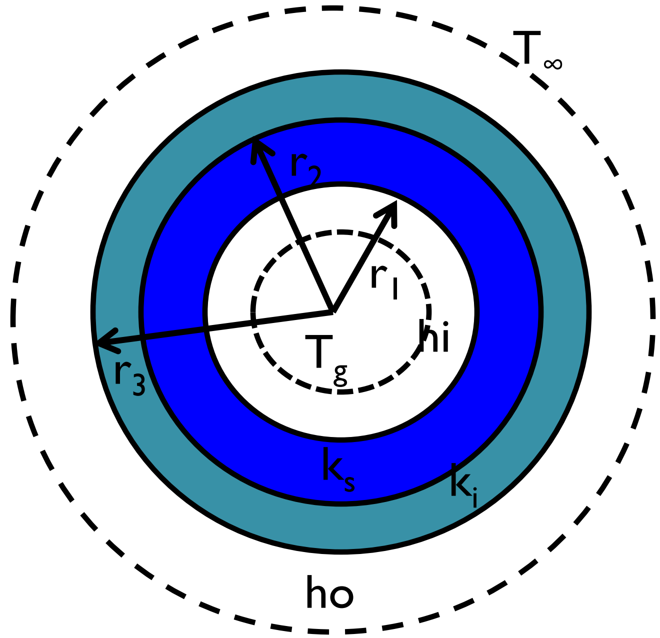
6.76W/m^{2}K

Q=?

19.19kW

Solution:

$$Q = U_o A_o \Delta T = U_i A_i \Delta T$$



$$\frac{1}{U_o A_o} = \frac{1}{h_o A_o} + \frac{\ln\frac{r_3}{r_2}}{2\pi K_i L} + \frac{\ln\frac{r_2}{r_1}}{2\pi k_s L} + \frac{1}{h_i A_i}$$

$$A_o = 2\pi r_3 L = 9.42 m^2;\ A_i = 2\pi r_1 L$$

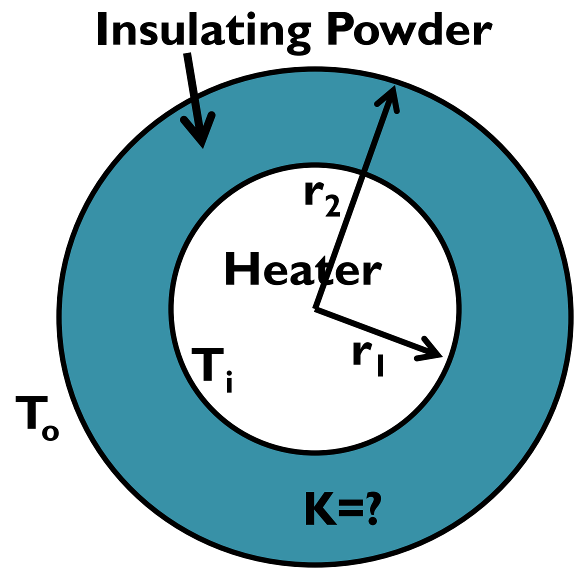
$$\Rightarrow U_o A_o = 63.67 \Rightarrow U_o = 6.759 W/m^2 K$$

$$Q = UoAo\Delta T$$

$$= 63.67(300 - 30)$$

$$= 17.19\, kW$$

Q1. An insulating powder is densely packed in the annular space between two concentric spheres with radii 75mm and 50mm. The inner sphere is uniformly heated with electric power input of 30 W. Steady state temp attained by the inner sphere is 120°C and that by outer surface is 30°C.



Neglecting the thermal resistance offered by the spheres:

a) Draw analogous electrical cct diagram

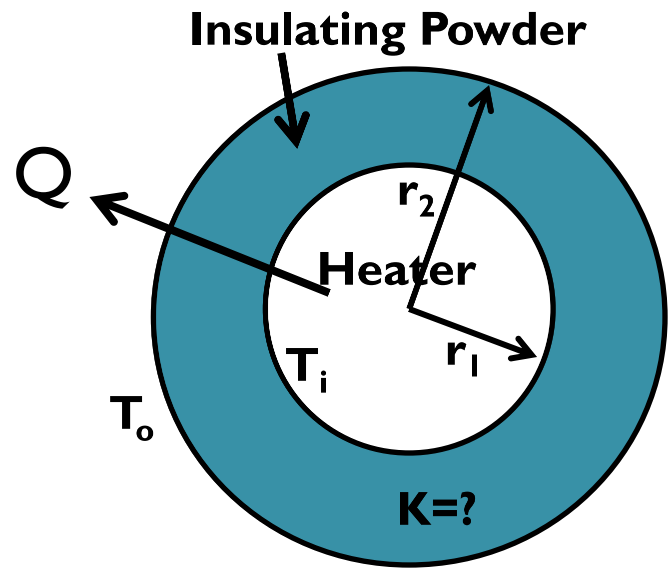
Calculate thermal conductivity of the powder

k=?=0.177W/mK

Analogous cct ?

Solution:

Analogous cct ?

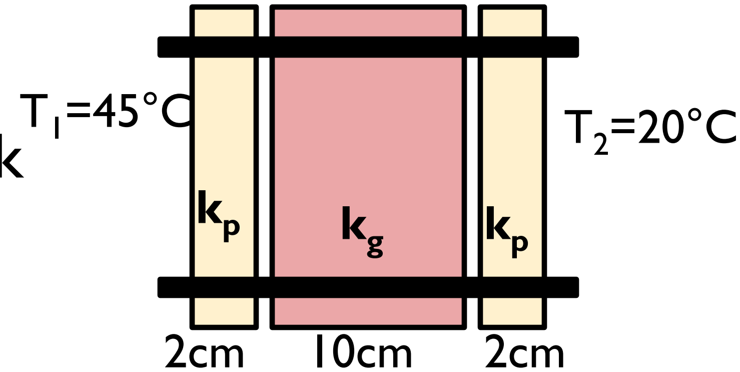


k=?

$$Q = \frac{T_i - T_o}{\dfrac{r_2 - r_1}{4\pi k_{ins} r_1 r_2}} = 30\,W$$

$$\frac{120 - 30}{\dfrac{0.075 - 0.05}{\pi\, k x 0.075 x 0.05}} = 30 \quad \Rightarrow k_{ins} = 0.1768 W / mK$$

Q5. The insulation boards for air-conditioning purposes are made of three layers, the middle being of packed grass 10cm thick (k_g=0.02 W/mK) and the sides are made of plywood , each of 2cm thickness (k_p=0.12 W/mK). They are glued with each other.

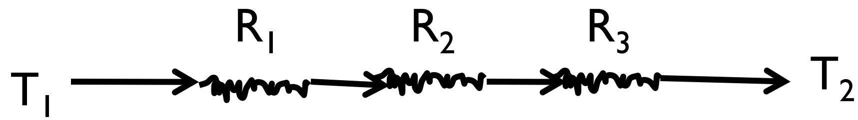


Q_a=? **Q_b=?**

a) Determine heat flow rate if one surface is at 45°C and the other at 20°C. Neglect resistance of the glue.

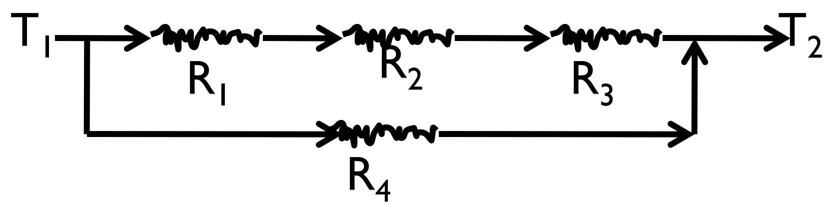
b) Instead of glue, if these three boards are bolted by 4 steel bolts (k_s=40 W/mK) of 1cm dia each at the corners per m^2 area of the board, then find whether heat flow rate is going to increase or decrease and what percent ?

Solution:



$$Q = \frac{T_1 - T_2}{\frac{\Delta X_p}{k_p.A} + \frac{\Delta X_g}{k_g.A} + \frac{\Delta X_p}{k_p.A}} = \frac{45-20}{\frac{0.02}{0.12x1} + \frac{0.1}{0.02x1} + \frac{0.02}{0.12x1}} = 4.69\,\mathrm{W/m}^2$$

With Bolts:



$$A_b = \frac{\pi}{4} x0.01^2 x4 = 3.14x10^{-4} m^2 ; Q_{new} = \frac{45-20}{3.61} = 6.9 W/m^2$$

$$R = \frac{\Delta X}{k_s A} = \frac{(2+10+2)10^{-2}}{40x3.14x10^{-4}}$$

$$= 3.61$$

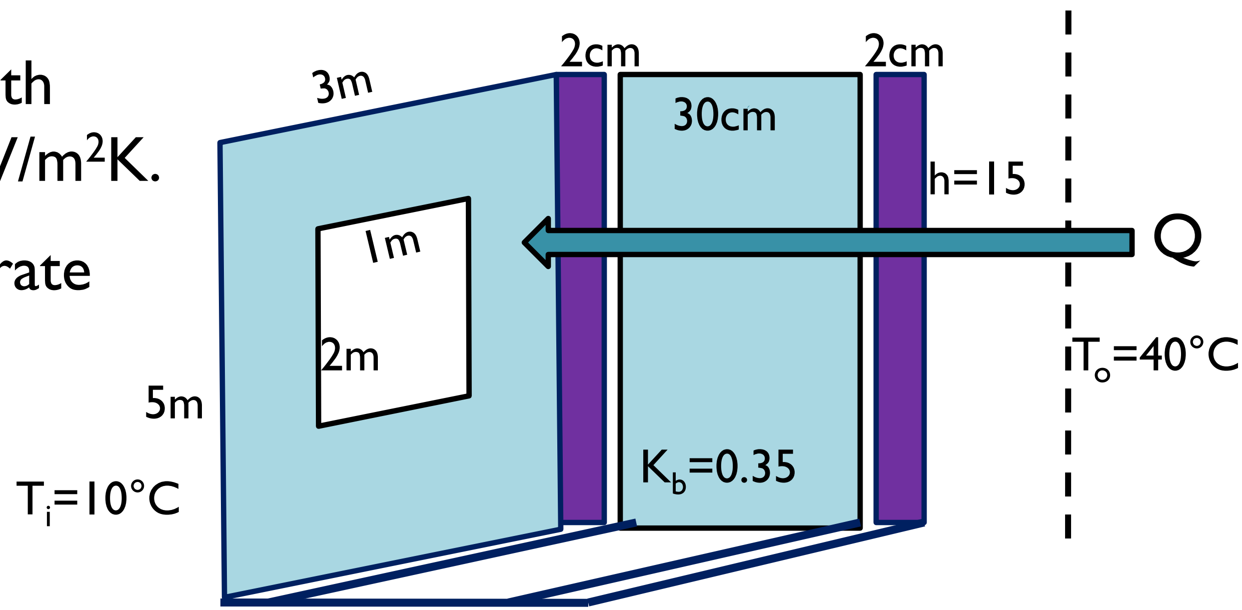
$$Increase = \frac{6.9 - 4.69}{4.69} x100$$

$$= 47.7\,\%$$

Q6. A wall 30cm thick of size 5mX3m made of red brick (k_b=0.35W/mK). It is covered on both sides by layers of plaster 2cm thick (k_p=0.6W/mK). The wall has a window of size 1mX2m. The door of window made up of 12mm thick glass has a conductivity k_g=1.2 W/mK.

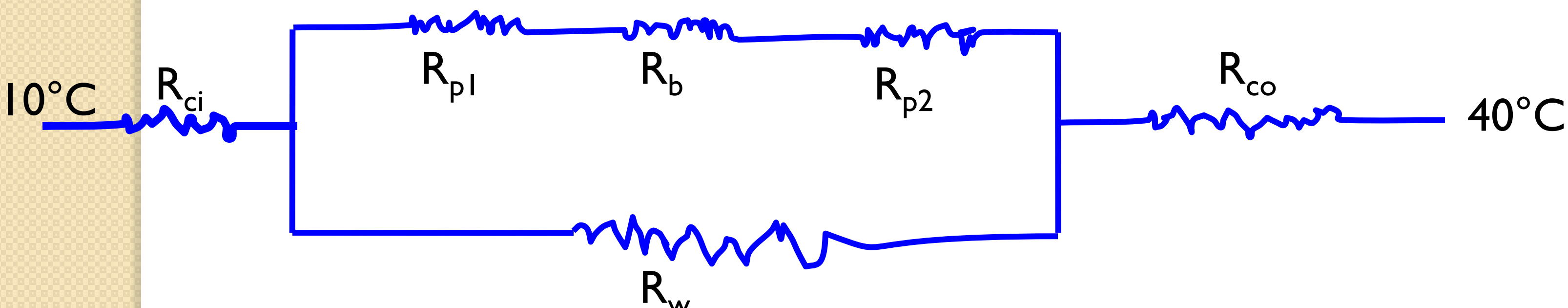
Inner and outer air temp are 10 & 40°C. Take h on both sides as 15 W/m²K.

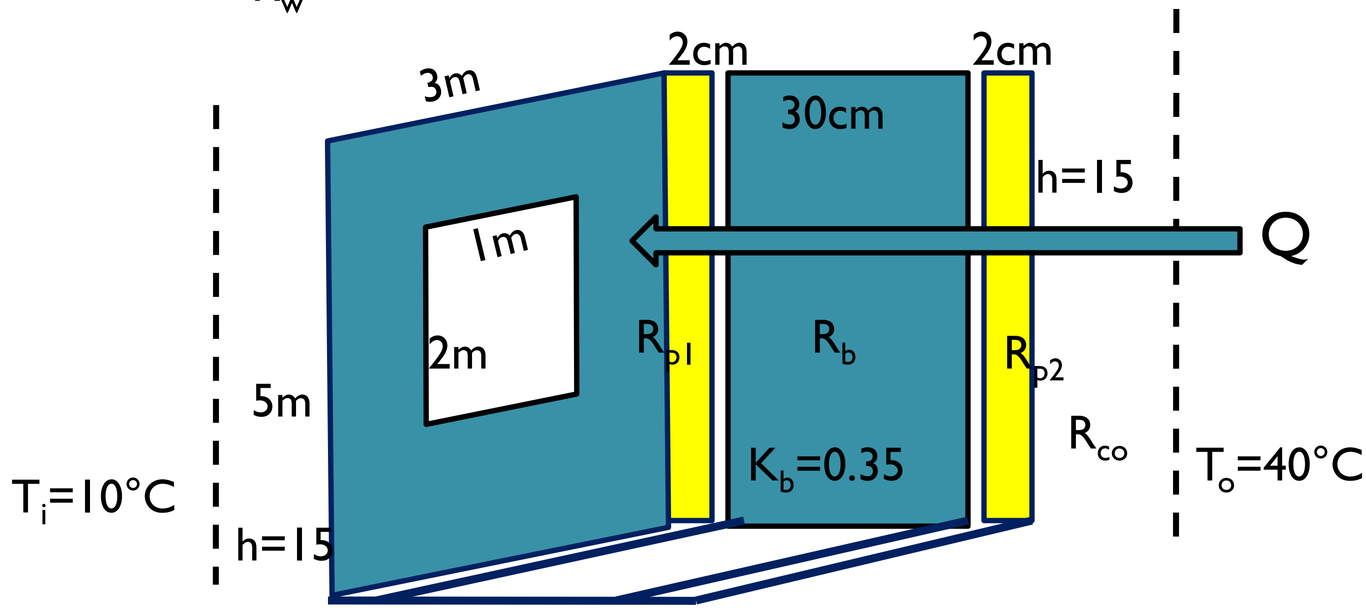
Estimate the rate of heat flow through the wall.



Solution:

$$Q = \frac{To - Ti}{R} = \frac{40 - 10}{R}$$





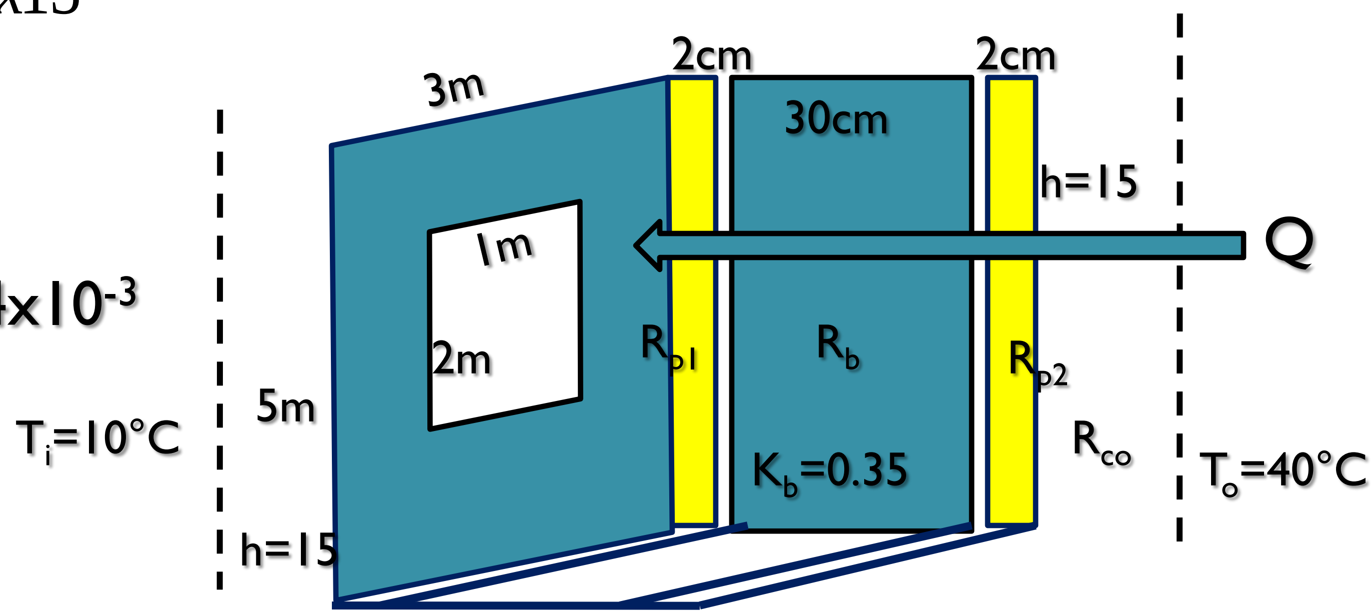
Solution:

$$R_{ci} = \frac{1}{hA} = \frac{1}{15x5x3} = 4.44x10^{-3} = R_{co}$$

$$R_{p1} = \frac{\Delta X}{k_p A} = \frac{0.02}{0.6x13} = 2.56x10^{-3} = R_{p2}$$

$$R_b = \frac{0.3}{0.35x13} = 65.9x10^{-3} \qquad R_w = \frac{0.012}{1.2x2x1} = 5x10^{-3}$$

$R_t = 13.54x10^{-3}$



$$= \frac{40-10}{13.54x10^{-3}} = 2215.66W \quad Ans.$$

Insulating Materials

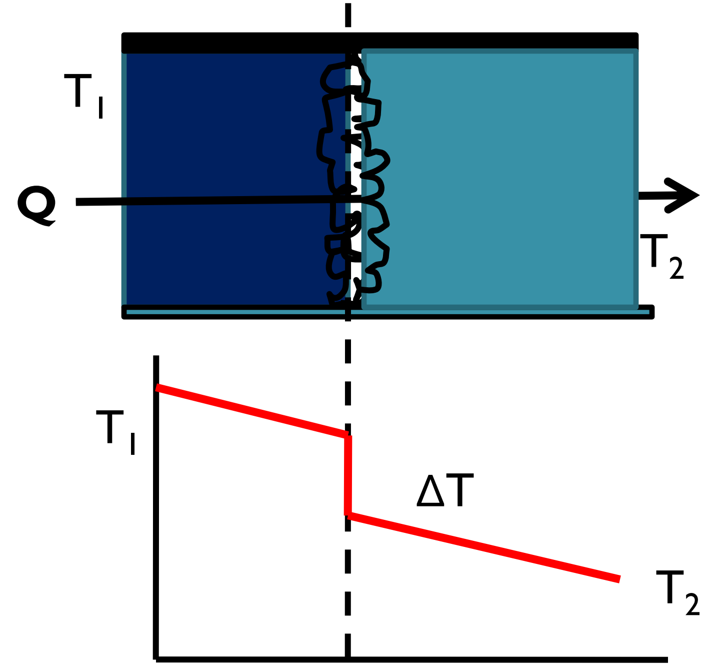
- Materials which are used to reduce the heat transfer rate from / to the system, are known as INSULATOTRS
- Examples are glass wool, plastics, wood, brick, cement, asbestos, rubber, grass, saw dust, cork, glass, clay, etc
- Insulators have low conductivity (generally k <2 W/mK)
- Insulating materials should be cheaper, able to withstand higher temp and humidity, should remain in applied shape and have long life, odorless, non-reactive,
- Practical applications are in refrigerators & air conditioners, buildings, conduits carrying high temp fluids like steam/ emicals, plastic handles of kitchen utensils, furnaces, cold orages, offices etc

Conductivities of some Insulating Materials

Materials	Conductivity k (W/mK)
Wood	1.2 – 0.8
Brick	0.9 – 1.3
Concrete	0.8 – 0.9
Glass	0.7 – 0.8
Asbestos	0.2 – 0.4
Glass fiber	0.04
Cork	0.03
Plastics	0.9 – 0.04
Air	0.02
Clay	1.02
Gypsum	0.3
Saw Dust	0.07

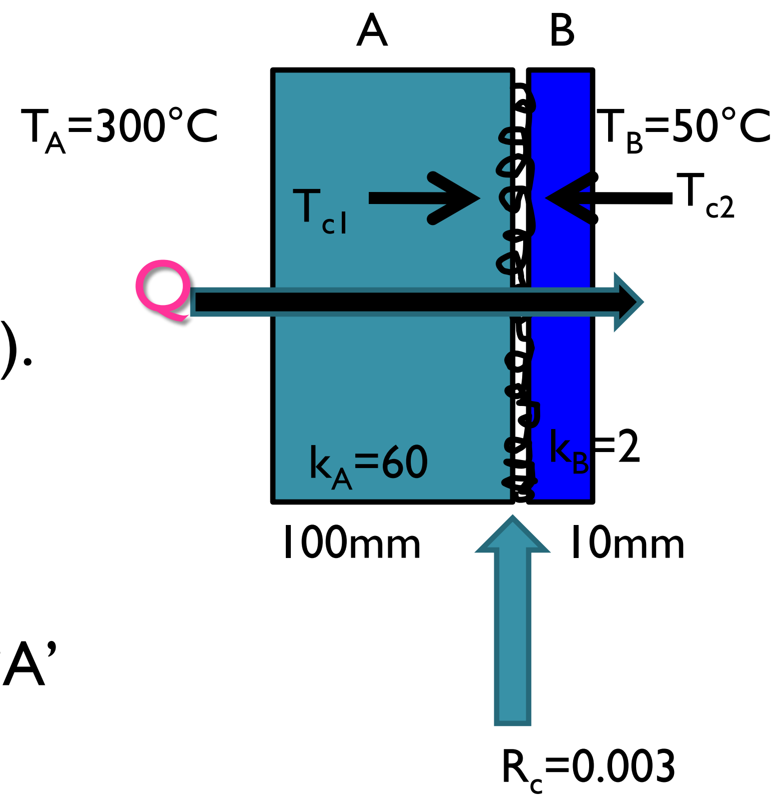
Thermal Contact Resistance

When heat flows through two solids in contact, temp profile experiences a sudden drop across the interface of the solids. This temp drop occurs due to thermal contact resistance.



Due to rough surfaces at contact, direct contact is made at few points only and voids get filled with air or surrounding fluid, whose conductivity is much lower than solids in contact. Therefore, interface acts as resistance to heat flow causing sudden temp drop. s resistance is called **Thermal Contact Resistance.**

Q2. A plane composite slab with unit cross sectional area is made up of material 'A' (thickness=100mm, kA=60W/mK) and material 'B' (thickness=10mm, kB=2W/mK). Thermal contact resistance at their interface is $0.003 m^2K/W$.



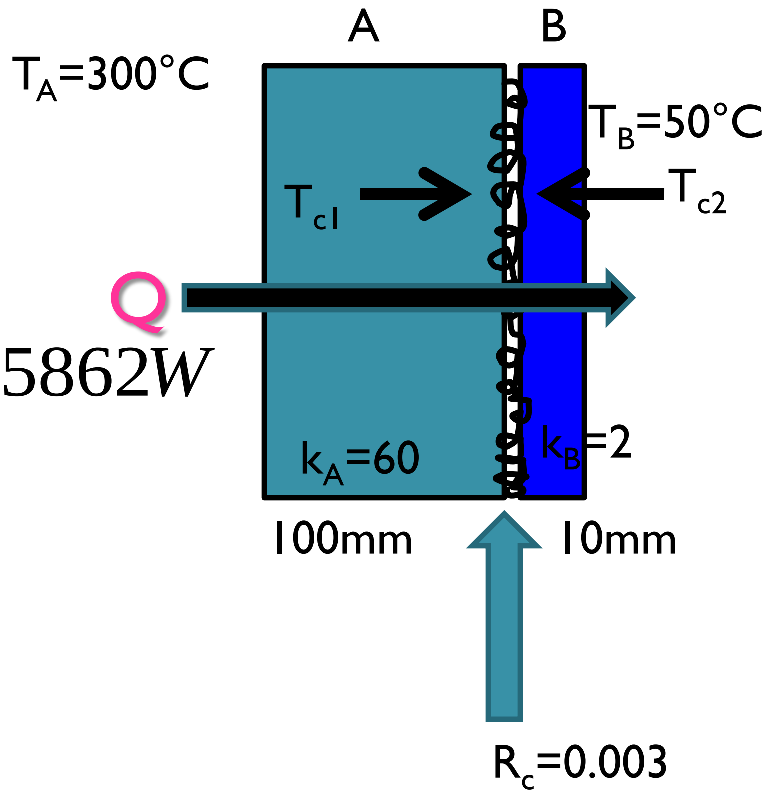
The temp of open side of slab 'A' is 300°C and that of open side of slab 'B' is 50°C.

Calculate:

a) The rate of heat flow through the slab (Q) (25862W)

emp on both sides of the interface (Tc1 &Tc2) (257;179C)

Solution:



$$Q = \frac{T_A - T_B}{R_A + Rc + R_B}$$

$$= \frac{300 - 50}{\frac{0.1}{60x1} + 0.003 + \frac{0.01}{2x1}} = 25862W$$

$$Q = \frac{T_A - T_{C1}}{R_A} = \frac{T_{C2} - T_B}{R_B}$$

$$Hence \frac{300 - T_{C1}}{1.667x10^{-3}} = \frac{T_{C2} - 50}{50x10^{-4}} = 25862$$

$$T_{C1} = 256.89°C \; \& \; T_{C2} = 179.3°C$$

Variable Thermal Conductivity

Thermal conductivity of materials is strongly dependent on temperature.

Metals: k is inversely proportional to temp

In general, $k = k_o (1 + aT + bT^2 + \ldots\ldots)$

$= k_o (1 \pm aT)$; where –ve for metals

& +ve for non-metals

Non-metallic Solids: k is directly proportional to temp

Liquids: k is inversely proportional to temp (except Water)

ases: k is directly proportional to temp. $k = f(T, Pr,$

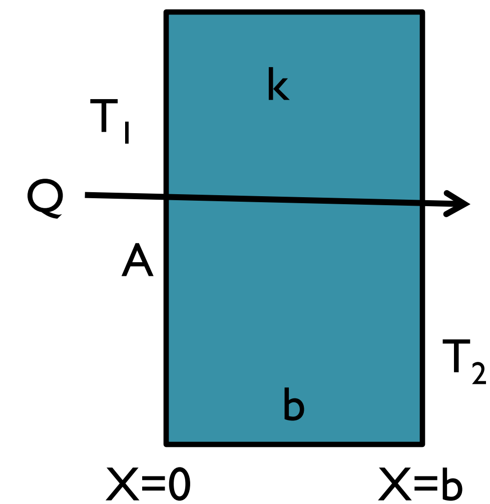
umidity)

Variable Thermal Conductivity

Q3: A plane wall of isothermal faces of temps T_1 at x=0 and T_2 at x=b has a thermal conductivity $k=k_0(1+aT)$. A is the area of faces. Show that heat conducted through wall is given by $Q=k_0.A/b[1+a/2(T_1+T_2)](T_1-T_2)$

From Fourier's Law

$$Q = -kA\frac{dT}{dx}$$



<u>BCs:</u> 1) At x=0; $T=T_1$

2) At x=b; $T=T_2$

Separating Variables

e have $$\frac{Q}{A}.dx = -kdT$$

Variable Thermal Conductivity

Substituting value of $k = k_0(1 + aT)$ *and Integrating*

$$We\ have \quad \frac{Q}{A}.\int_0^b dx = -k_0 \int_{T_1}^{T_2} (1 + aT)dT$$

$$or \quad \frac{Q}{A}.[x]_0^b = -k_0 \left[T + \frac{a}{2}.T^2 \right]_{T_1}^{T_2}$$

$$Or \quad \frac{Q}{A}.b = -k_0 \left[(T_2 - T_1) + \frac{a}{2}.(T_2^{\ 2} - T_1^{\ 2}) \right]$$

$$Q = k_0.\frac{A}{b}.\left[(T_1 - T_2)\left\{ 1 + \frac{a}{2}.(T_1 + T_2) \right\} \right]$$

Hence proved

Plotting Temp Distribution Across Thickness of Slab with Variable Conductivity

$$Q = -kA\frac{dT}{dx}$$

Under steady state conditions, Heat Flow Rate Q remains const. Since k changes with temp T, for the LHS of the eqn, i,e. Q to remain const, something on the RHS must change accordingly in opposite direction. As area A is const, it is dT/dx, which should change.

1. For a=0;

$k=k_0$ from expression $k=k_0(1+aT)$

Therefore, for Q to remain const, since k is not ...nging, dT/dx=const; hence const slope of temp ...file across thickness of the wall

Plotting Temp Distribution Across Thickness of Slab with Variable Conductivity

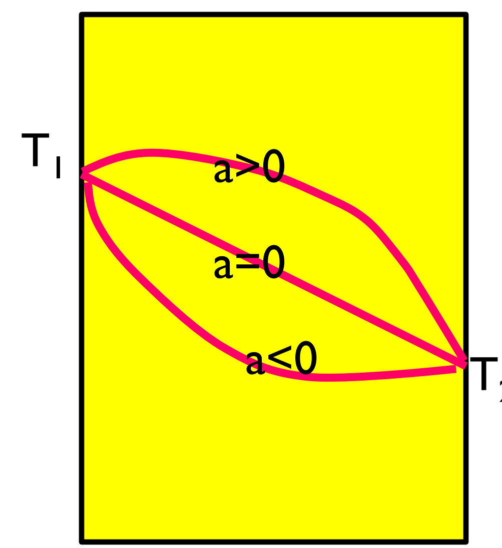
2. For a>0:

We have k proportional to T

Since temp decreases in +ve x-dir; k also decreases;
So dT/dx must increase to keep RHS const

3. For a<0:

We have k inversely proportional to T.
As T decreases in +ve x-dir; k will increase. Therefore, to keep Q const, dx must decrease



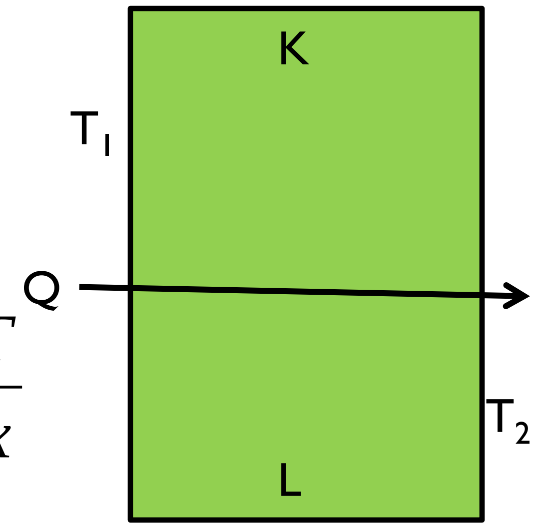
Q4. Variation of thermal conductivity of a wall material is given by:

$$k = k_0\left(1 + \alpha T + \beta T^2\right)$$

If the thickness of the wall is L and its two surfaces are maintained at temp T_1 and T_2, find the expression for heat flow through the wall.

<u>Solution:</u>

$$As\ per\ Fourier's\ Law\ Q = -kA\frac{dT}{dx}$$



$$Substituting\ Q = -k_0(1 + \alpha T + \beta T^2)\frac{dT}{dx}$$

parating Variables, we have

$$dx = -k_0\left(1 + \alpha T + \beta T^2\right)dT$$

A

Example: Variable Conductivity (Contd)

Integrating both sides;

$$\frac{Q}{A}.\int_0^L dx = -k_0 \int_{T_1}^{T_2} \left(1 + \alpha T + \beta T^2\right).dT$$

$$\Rightarrow \frac{Q}{A}.L = -k_0.\left[T + \alpha.\frac{T^2}{2} + \beta.\frac{T^3}{3}\right]_{T_1}^{T_2}$$

On substitution & simplification, we get;

$$Q = k_0.\frac{A}{L}.\left[1 + \frac{\alpha}{2}.(T_1 + T_2) + \frac{\beta}{3}\left(T_1^2 + T_2^2 + T_1T_2\right)\right].(T_1 - T_2)$$

End of Unit - I

Heat Conduction with Internal Heat Generation

Internal heat generation is the one, where heat is uniformly generated throughout the material at a constant rate (expressed as W/m^3).

Examples : 1. Heat generated due to passage of current through the metals like electrical conductor.

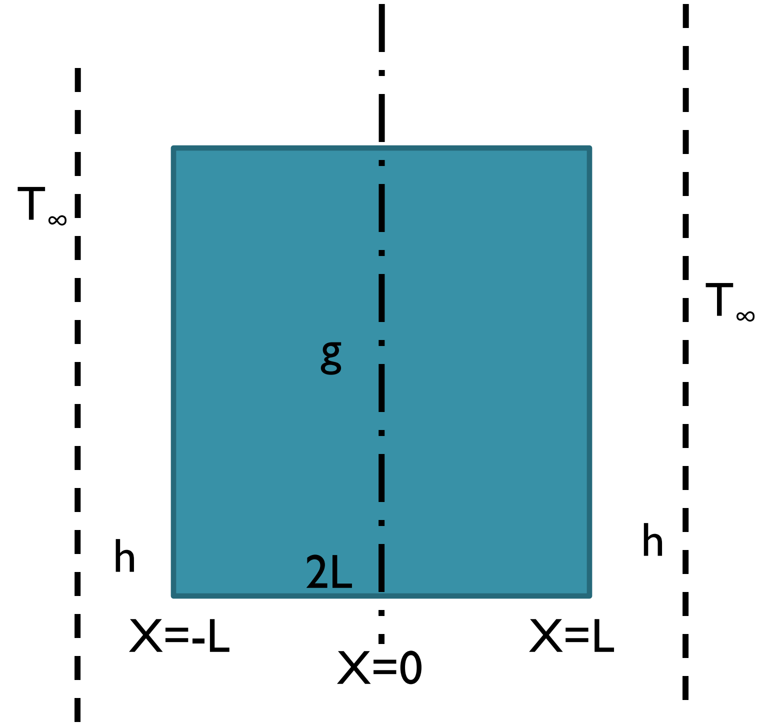
2. Heat generated due to fission or fusion reaction in Nuclear fuel.

3. Setting of concrete slab by releasing heat uniformly.

Combustion of fuel in IC Engines

Heat Conduction with Internal Heat Generation Through A Slab (Symmetrical BCs)

Consider an infinite slab of thickness 2L. Let g (W/m^3) be internal heat generation at const rate and same surrounding fluid both sides be at temp T_∞ .



For convenience, x=0 has been aligned with centre line of thickness of slab; so that left face of the slab is at x=-L and right face at x=L

...sson's Eqn applicable ...he present case is:

$$\frac{d^2T}{dx^2} + \frac{g}{k} = 0$$

Heat Conduction with Internal Heat Generation through A Slab

Aim is to find out Temp Distr $T_{(x)}$ through Slab and Heat Flow Rate Q

$$Applicable\ \ equation \qquad \frac{d^2T}{dx^2} + \frac{g}{k} = 0$$

$$OR \qquad \frac{d^2T}{dx^2} = -\frac{g}{k}$$

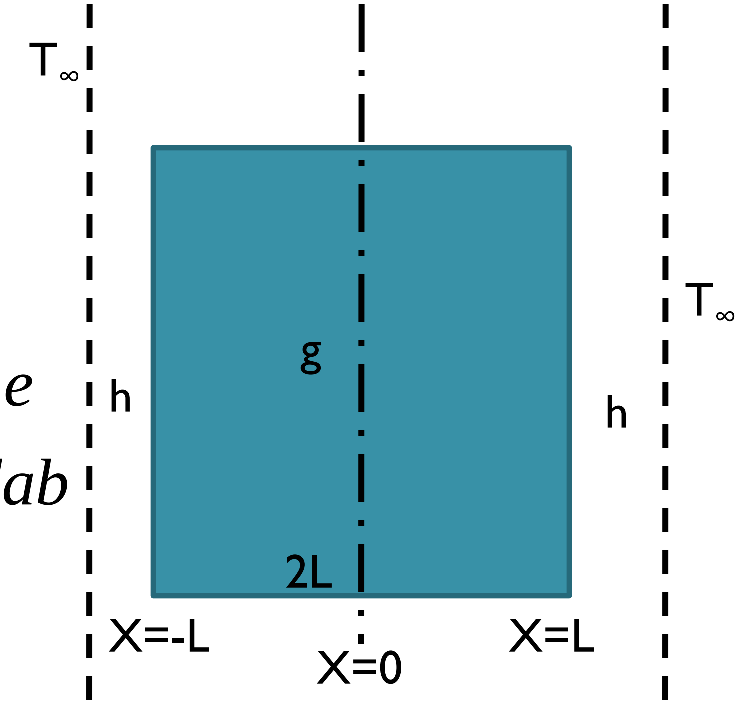
$$Integrating\ twice, We\ have;$$

$$\frac{dT}{dx} = -\frac{g}{k}.x + C_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$And \quad T = -\frac{g}{2k}.x^2 + C_1.x + C_2 \ldots\ldots(2)$$

Heat Conduction with Internal Heat Generation Through A Slab

Boundary Conditions:



1. $\frac{dT}{dx} = 0 \quad at\ x = 0;\ as\ Temp$ $will\ be\ \max\ at\ the\ centre\ as\ same$ $conditions\ exist\ on\ both\ sides\ of\ slab$

2. $Heat\ Conducted\ upto\ face = Heat\ Convected\ from\ the\ face$

$$\left[-kA\frac{dT}{dx}\right]_{x=L} = hA\left[T_L - T_\infty\right]_{x=L}$$

Heat Conduction with Internal Heat Generation Slab

$$From\ Eqn...(1), \frac{dT}{dx} = 0\ for\ x = 0 \Rightarrow C_1 = 0$$

$$Hence Eqn...(2)\ becomes\ T = -\frac{g}{2k}.x^2 + C_2$$

$$Applying\ \ BC...(2), We\ have;$$

$$-kA\left(\frac{-gL}{k}\right) = hA\left(\frac{-gL^2}{2k} + C_2 - T_\infty\right)$$

$$\Rightarrow C_2 = \frac{gL^2}{2k} + \frac{gL}{h} + T_\infty$$

Heat Conduction with Internal Heat Gen. in Slab

$$Substituting\ C_2 = \frac{gL^2}{2k} + \frac{gL}{h} + T_\infty\ in\ \text{modified}\ eqn...(2)$$

$$We\ have: T = -\frac{gx^2}{2k} + \frac{gL^2}{2k} + \frac{gL}{h} + T_\infty$$

$$T = \frac{g}{2k}\left(L^2 - x^2\right) + \frac{gL}{h} + T_\infty ...Temp\ Distribution$$

$$Max\ Temp\ will\ occur\ at\ Centre\ (At\ x = 0);$$

$$Hence\quad T_{\max} = \frac{gL^2}{2k} + \frac{gL}{h} + T_\infty$$

$$For\ Surface\ Temp,\ putting\ x = L,$$

$$We\ have: T_s = \frac{gL}{h} + T_\infty$$

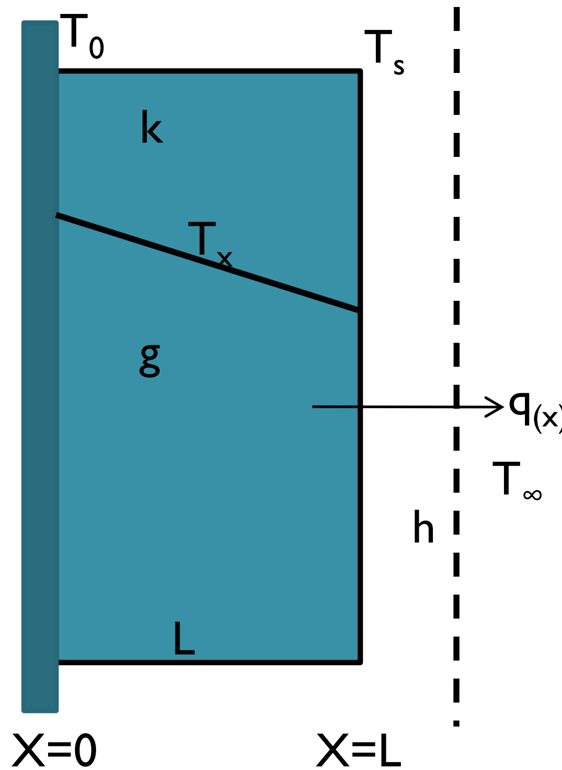
Heat Flux

$q_{(x)} = g.x$

Heat Conduction with Internal Heat Generation Through A Slab (Unsymmetrical BCs)

Q5:Consider a slab of thickness L and k conductivity, in which energy is generated at a const rate of g W/m^3 . Surface at x=0 is insulated and surface at x=L dissipates heat by convection with h to a fluid at T_∞

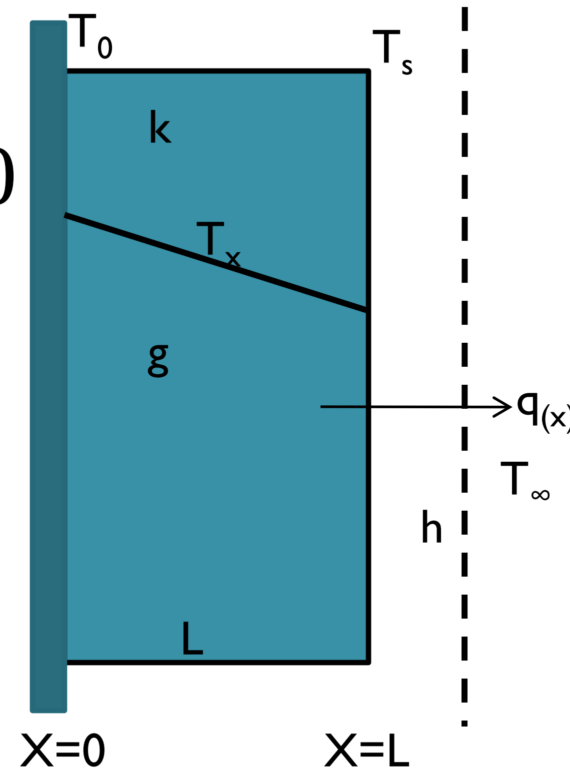
We have to find out Temp Distr $T_{(x)}$ through slab & heat flux $q_{(x)}$



...o, We have to calculate temp T_0 at x=0 and ...t x=L for L=1cm, k=20 W/mK, g=8x10^7 W/m^3, ...000 W/m^{2}K & T_∞=100°C

Heat Conduction with Internal Heat Generation Through A Slab (Unsymmetrical BCs)

$$Conduction\ Eqn: \frac{d^2T}{dx^2} + \frac{g}{k} = 0$$



Integrating twice, we have,

$$\frac{dT}{dx} = -\frac{g}{k}.x + C_1 ....(1) \quad and$$

$$= -\frac{gx^2}{2k} + C_1 x + C_2 ....(2)$$

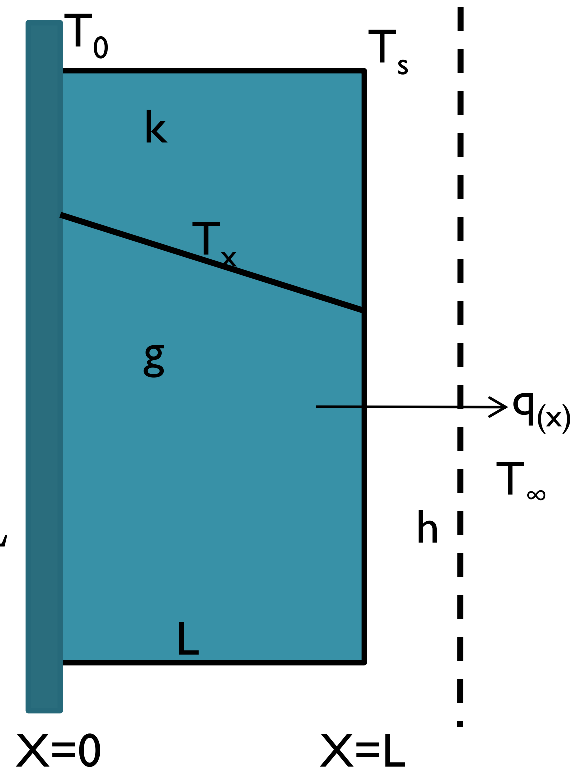
Heat Conduction with g (Unsymmetrical BCs)

$BC\;\; 1)\; Surface\; at\;\; x = 0\; is\;\; insulated\;,$

$hence\; NO\; heat\; transfer\; will\; take$

$place\;\; from\; this\; surface; Temp\; will$

$$be\;\; \max; \; Therfore\, \frac{dT}{dx} = 0\, at\;\; x = 0$$



$BC\;\; 2)\; Heat\; Conducted\; upto\; surface\; at\; x = L$

$= Heat\; convected\;\; from\; the\; surface\; at\; x = L$

$$-k\left(\frac{-g.2x}{2k}\right) = h\left(\frac{-gx^2}{2k} + C_2 - T_\infty\right)_{at\, x=L}$$

$$C_2 = \frac{gL^2}{2k} + \frac{gL}{h} + T_\infty$$

Heat Conduction with Internal Heat Generation Through A Slab (Unsymmetrical BCs)

$$Substituting\ C_1\ \&\ C_2\ in\ Equation..(2), We\ have$$

$$Temp\ \text{Profile}\quad T = \frac{g}{2k}\left(L^2 - x^2\right) + \frac{gL}{h} + T_\infty$$

$$q_{(x)} = -k\frac{dT}{dx} = -k\left(\frac{-gx}{k}\right) = g.x$$

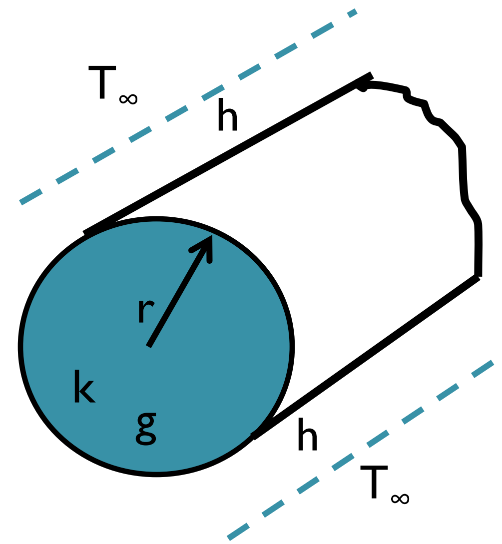
$$Temp\ at\quad x = 0;\quad T = \frac{gL^2}{2k} + \frac{gL}{h} + T_\infty$$

$$T_0 = \frac{8X10^7.(0.01)^2}{2X20} + \frac{8X10^7.(0.01)}{4000} + 100 = 500°C$$

$$= \frac{gL}{h} + T_\infty = \frac{8X10^7.(0.01)}{4000} + 100 = 300°C$$

Heat Conduction with Internal Heat Generation Through A Long Solid Cylinder (Symmetrical BCs)

Consider a solid cylinder of radius r of conductivity k, in which internal heat is generated at a const rate of g W/m^3 .



This cylinder is exposed to a fluid with heat transfer coefficient h at temp T_∞ , to which it is dissipating heat by convection

Example: An electric conductor carrying current exposed to atmospheric air

Heat Conduction with Internal Heat Generation Through A Cylinder (Symmetrical BCs)

$$\textit{Poisson's Equation}: \frac{1}{r}.\frac{d}{dr}\left(r.\frac{dT}{dr}\right)+\frac{g}{k}=0$$

$$\text{or} \quad \frac{d}{dr}\left(r.\frac{dT}{dr}\right)=-\frac{gr}{k}$$

Integrating above Eqn Twice;

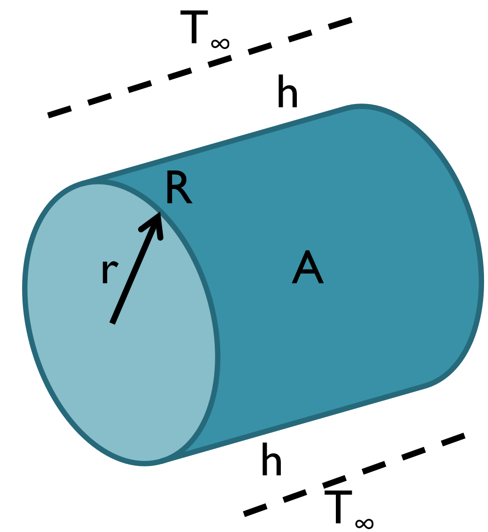
$$r.\frac{dT}{dr}=-\frac{gr^2}{2k}+C_1 \quad \text{or}$$

$$\frac{dT}{dr}=-\frac{gr}{2k}+\frac{C_1}{r}\text{...........}(1) \quad \text{and}$$

$$= -\frac{gr^2}{4k}+C_1\ln r+C_2\text{.........}(2)$$

Heat Conduction with g through a Cylinder

Boundary Conditions



$$1.\ \frac{dT}{dr} = 0\ at\ r = 0;\ because\ Temp\ will\ be$$

$$\max\ at\ the\ centre\ as\ same\ conditions$$

$$exist\ on\ all\ sides\ of\ cylinder\ \Rightarrow C_1 = 0$$

$$2.\ Heat\ Conducted\ upto\ surface\ of\ the\ cylinder$$

$$= Heat\ Convected\ from\ the\ surface\ of\ the\ cylinder$$

$$\left[-kA\frac{dT}{dr}\right]_{r=R} = hA\left[T_R - T_\infty\right]_{r=R}$$

$$Substituting\ \frac{dT}{dr}\ \&\ T_r$$

$r = R$ from eqns.(1) & (2), we have $\Rightarrow C_2 = \dfrac{gR^2}{4k} + \dfrac{gR}{2h} + T_\infty$

Heat Conduction with Internal Heat Generation Through A Cylinder (Symmetrical BCs)

$$Substituting\ C_1\ \&\ C_2\ in\ Equation..(2);$$

$$We\ have: T = \frac{g}{4k}\left(R^2 - r^2\right) + \frac{gR}{2h} + T_\infty$$

$$Max\ Temp\,(at\ Centre) \qquad T_{\max(r=0)} = \frac{gR^2}{4k} + \frac{gR}{2h} + T_\infty$$

$$Temp\ at\ surface\,(r = R) \qquad T_s = \frac{gR}{2h} + T_\infty$$

$$Heat\ Flux \quad q_{(r)} = \left[-k\frac{dT}{dr}\right]_{r=R}$$

$$k.\left(\frac{-gR}{2k}\right) = \frac{g.R}{2} \qquad W/m^2$$

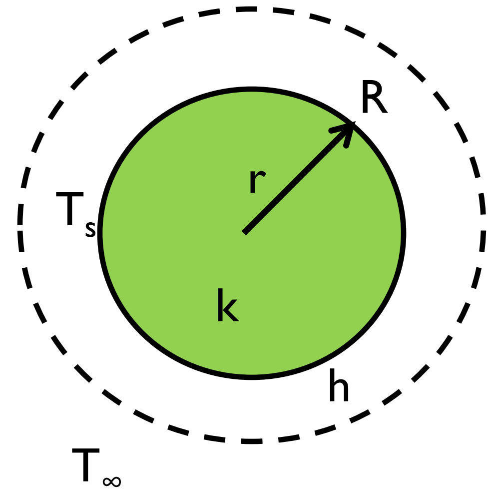
$$Q = -k.A.\left[\frac{dT}{dr}\right]_{r=R} = -k.2\pi R\left(\frac{-gR}{2k}\right)$$

$$= g.\pi R^2(1m) = g.Volume.....W/m$$

Heat Conduction with Internal Heat Generation Through A Solid Sphere (Symmetrical BCs)

Poison's Equation for sphere:

$$\frac{1}{r^2}\cdot\frac{d}{dr}\cdot\left(r^2\frac{dT}{dr}\right)+\frac{g}{k}=0$$



$$OR\ \frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=-\frac{gr^2}{k}$$

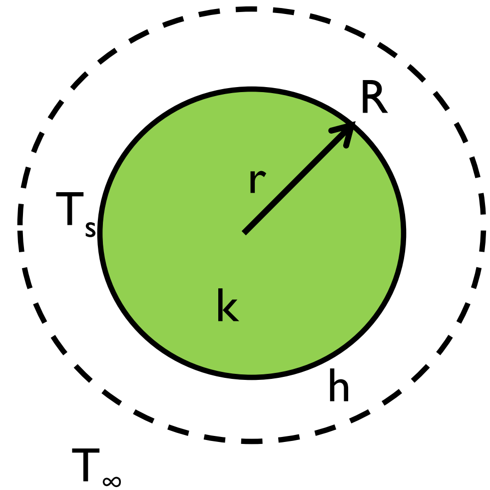
$$On\ Integrating\ twice;\quad r^2\frac{dT}{dr}=-\frac{gr^3}{3k}+C_1$$

$$or\ \frac{dT}{dr}=-\frac{gr}{3k}+\frac{C_1}{r^2}\,.........(1)$$

$$and\ T=-\frac{gr^2}{6k}-\frac{C_1}{r}+C_2\,..........(2)$$

Heat Conduction with Internal Heat Generation Through A Solid Sphere (Symmetrical BCs)

Boundary Conditions:



$$1.\quad \frac{dT}{dr} = 0 \; at \quad r = 0;$$

$$\Rightarrow C_1 = 0 \; from \; Eqn...(1)$$

$$2.\quad Heat\ Conducted\ upto\ Surface$$

$$= Heat\ Convected\ out\ from\ the\ Surface$$

ence $\quad -k.A.\left[\frac{dT}{dr}\right]_{r=R} = h.A.\left[T_s - T_\infty\right]_{r=R}$

Heat Conduction with Internal Heat Generation Through A Solid Sphere (Symmetrical BCs)

$$\Rightarrow C_2 = \frac{gR^2}{6k} + \frac{gR}{3h} + T_\infty$$

Substituting values of C_1 & C_2 *in Eqn...(2)*

$$\textit{We have;} \quad T = -\frac{gr^2}{6k} + \frac{gR^2}{6k} + \frac{gR}{3h} + T_\infty$$

$$\textit{or} \qquad T = \frac{g}{6k}\left(R^2 - r^2\right) + \frac{gR}{3h} + T_\infty$$

$$r) = -k.\left(\frac{dT}{dr}\right) = -k.\left(\frac{-gr}{3k}\right) = \frac{gr}{3} \quad W/m^2$$

Heat Conduction with Internal Heat Generation

Slab:

$$T = \frac{g}{2k}.\left(L^2 - x^2\right) + \frac{gL}{h} + T_\infty ; \quad q_{(x)} = g.x \quad W/m^2$$

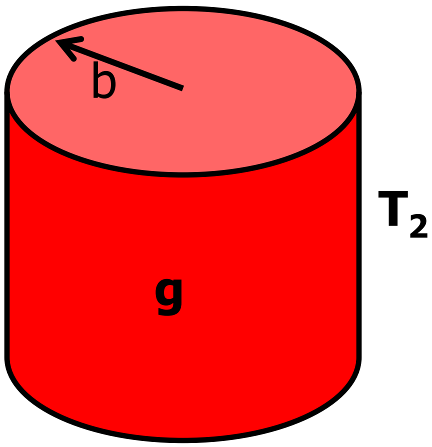
Cylinder:

$$T = \frac{g}{4k}\left(R^2 - r^2\right) + \frac{gR}{2h} + T_\infty ; q_{(r)} = \frac{gr}{2} \quad W/m^2$$

S

$$T = \frac{g}{6k}\left(R^2 - r^2\right) + \frac{gR}{3h} + T_\infty ; \quad q_{(r)} = \frac{gr}{3} \quad W/m^2$$

Q6: Consider a solid cylinder of radius r=b, in which energy is generated at a const rate of g W/m^3 while boundary surface at r=b is maintained at temp T_2.



Develop an expression for one dimensional(radial), steady state temp distr $T_{(r)}$ and heat flux $q_{(r)}$.

Calculate centre temp and heat flux at the boundary surface r=b for b=1cm, $g=2x10^8$ W/m^3, k=20W/mK, & T_2=100°C

Solution:

$$\text{Relevant Equation}: \frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{g}{k}=0$$

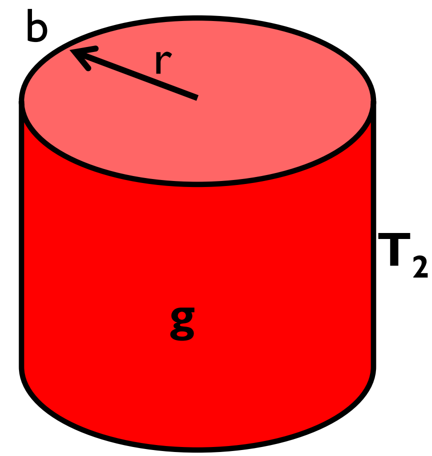
$$or \quad \frac{d}{dr}\left(r\frac{dT}{dr}\right)=-\frac{gr}{k}$$

$$\text{Integrating, we have}: r\frac{dT}{dr}=-\frac{gr^2}{2k}+C_1$$

$$or \ \frac{dT}{dr}=-\frac{gr}{2k}+\frac{C_1}{r}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$=-\frac{gr^2}{4k}+C_1\ln r+C_2\ldots\ldots\ldots\ldots(2)$$

Boundary Conditions:



1) dT/dr=0 at r=0

$$r\frac{dT}{dr} = -\frac{gr^2}{2k} + C_1$$

2) $T=T_2$ at r=b

Applying BC 1) to eqn (1); $C_1=0$

Hence eqn (2) becomes: $$T = -\frac{gr^2}{4k} + C_2 \ldots\ldots\ldots(3)$$

Applying BC 2) to eqn (3); $$T_2 = -\frac{gb^2}{4k} + C_2$$

$$\therefore C_2 = \frac{gb^2}{4k} + T_2$$

Substituting in eqn (3)

$$= -\frac{gr^2}{4k} + \frac{gb^2}{4k} + T_2 = \frac{g}{4k}\left(b^2 - r^2\right) + T_2$$

Solution (Contd):

$$q_{(r)} = -k\frac{dT}{dr} = -k\left(\frac{-gr}{2k}\right) = \frac{gr}{2}W/m^2$$

$$For\ \ flux\ at\ surface,\ \ putting\ \ r = b$$

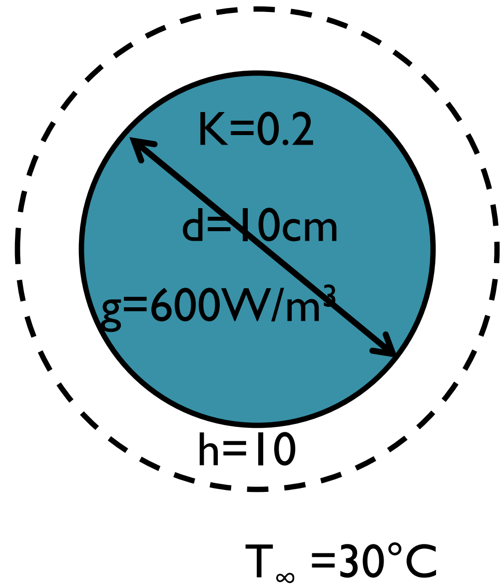
$$q_{(b)} = \frac{qb}{2} = \frac{2x10^8\,x0.01}{2} = 10^6\,W/m^2$$

$$For\ \ CentreTemp,\ putting\ \ r = 0\ in\ Temp\ Eqn$$

$$T_{(r)} = \frac{g}{4k}\left(b^2 - r^2\right) + T_2\,,\quad We\ have;$$

$$) = \frac{gb^2}{4k} + T_2 = \frac{2x10^8\,x(0.01)^2}{4x20} + 100 = 350°C$$

Q7: Heat is generated in a solid sphere of 10 cm dia. at the rate of 600 W/m^3. Surface heat transfer coeff. is 10 W/m^{2}K and surrounding air temp 30°C. k of material is 0.2W/mK .



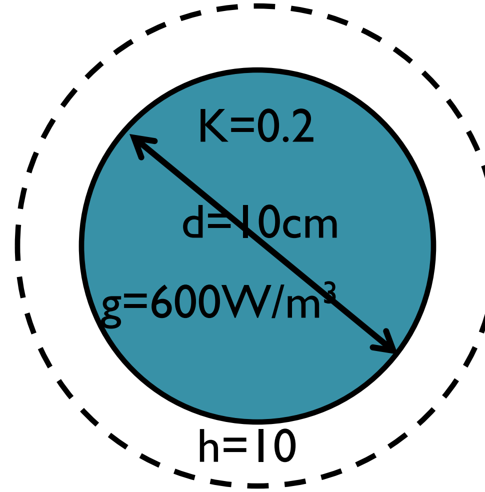
Find:

a) Max temp in the sphere

b) Surface temp of sphere

Solution:

$$For\ sphere;\ T_{(r)} = \frac{g}{6k}\left(R^2 - r^2\right) + \frac{gR}{3h} + T_\infty$$



$T_\infty = 30°C$

Max temp will be at r=0

i,e. at the centre of sphere.

Therefore, putting r=0 in above eqn,

We have:

$$T_{max} = \frac{gR^2}{6k} + \frac{gR}{3h} + T_\infty$$

$$T_{max} = \frac{600x0.05^2}{6x0.2} + \frac{600x0.05}{3x10} + 30 = 32.5°C$$

$$For\ \ Surface\ Temp,\ putting\ \ r = R\ in\ Temp\ Eqn$$

$$= \frac{gR}{3h} + T_\infty = \frac{600x0.05}{3x10} + 30 = 31°C \quad Ans$$

Extended Surfaces (Fins)

- There are large no of engg equipment, where unutilized heat energy is required to be discarded to atm, otherwise systems will fail due to overheating

- Examples are cooling of IC engs, heat removal from nuclear reactors, ICs, electrical Tx, compressors, motors, refrigeration & air-conditioning systems, various types of heat exchangers, etc

- In most of the above cases, heat is rejected from solid surfaces by convection to atm (low h values)

Our aim would be to increase heat transfer rate so that temp of solid surface is maintained within desired limits to avoid failure of the system

Extended Surfaces (Fins)

- We know that heat flow by convection :

$$Q = hA(T - T_\infty);$$

  - where h is almost constant (5 to 12 W/m^2K), whenever heat is convected to atm & can not be controlled
  - $(T - T_\infty)$ is temp diff between solid surface & atm, which also can not be controlled

- So, the only way to increase heat transfer rate Q is by increasing the surface area A

urface area of solid is increased by providing xtended surfaces called FINS

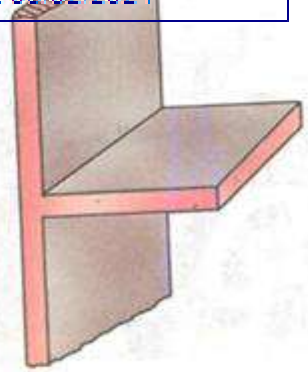
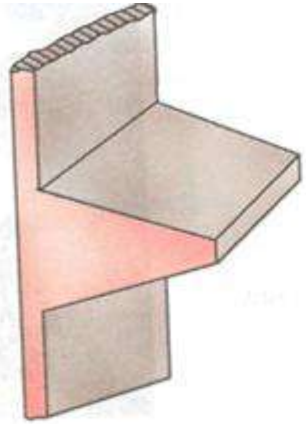
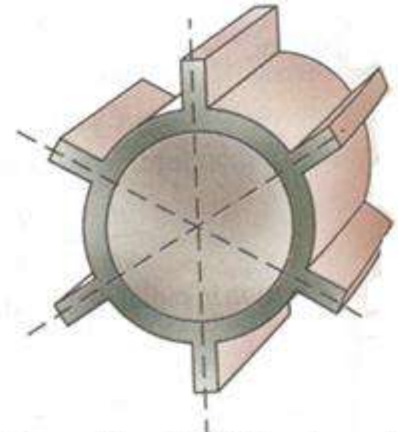


Plate Fin

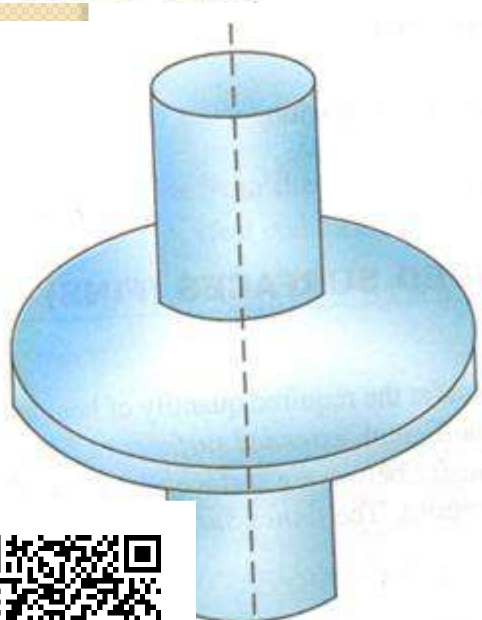


Tapered Fin

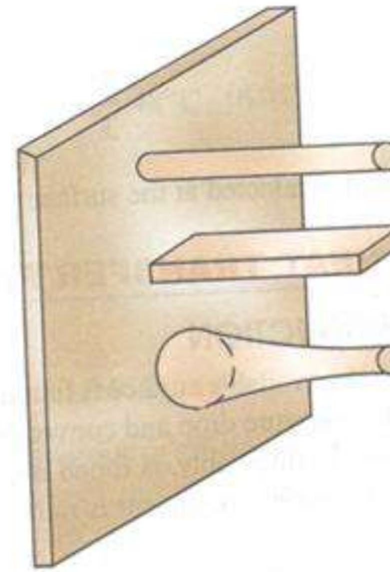


Radial Plate Fins

Different Types Of Fins

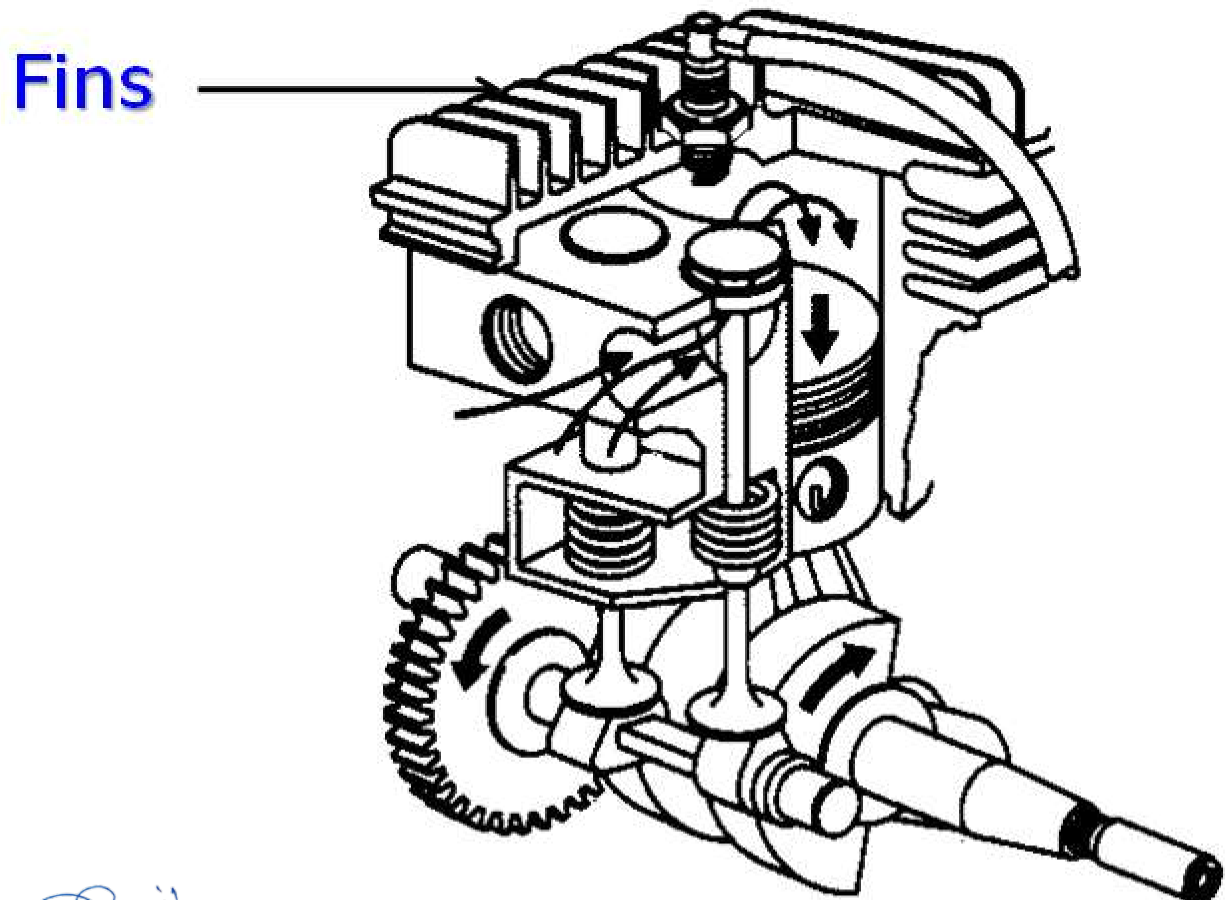


sk Fin

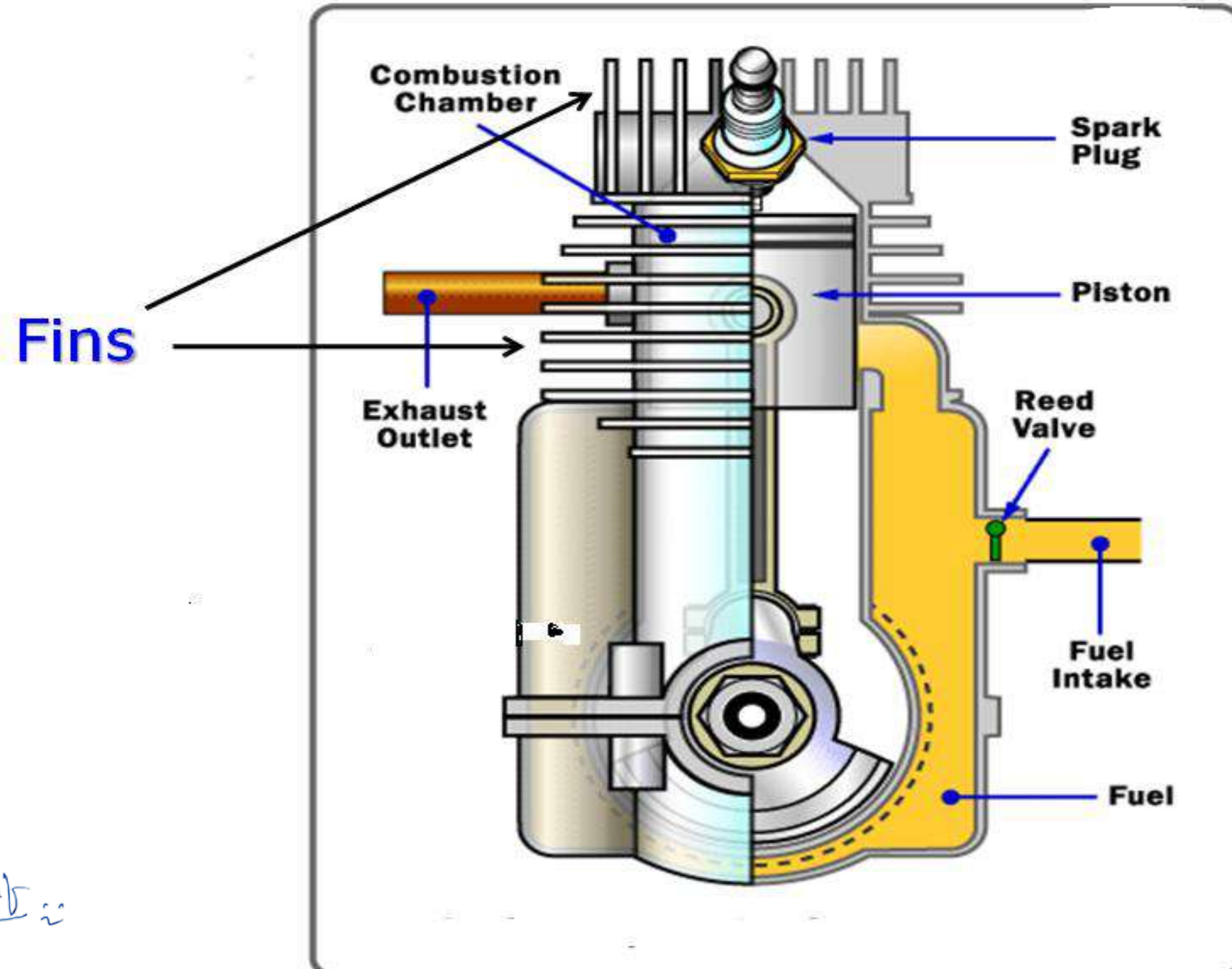


Pin Fins

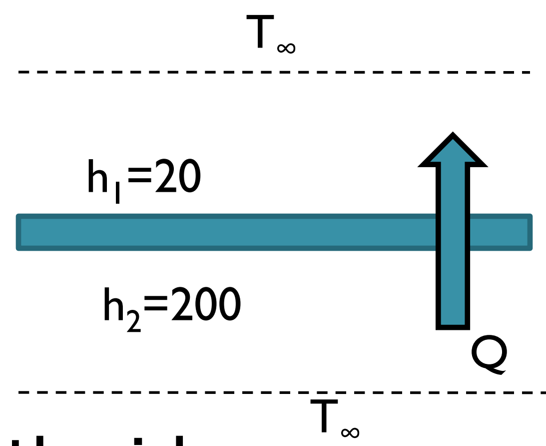
Plate Fins



Fins


Combustion Chamber
Spark Plug
Fins
Piston
Exhaust Outlet
Reed Valve
Fuel Intake
Fuel

Which side of the two Fluids, Fins should Be Provided?



Whichever side resistance to heat flow is more.

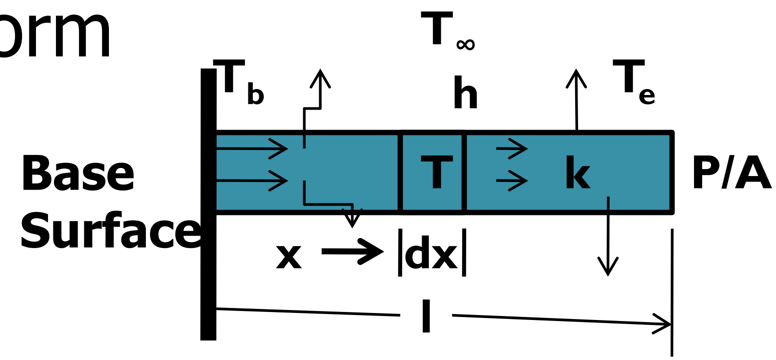
Resistance to heat flow = 1/hA

Assuming area A to be same on both sides,
Convective resistance will be higher on lower h side
Therefore, FINS should be provided on lower h side

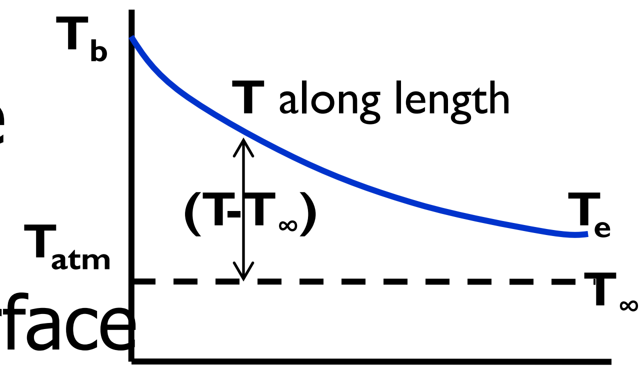
all above examples, lower h is on air/atm side;
ence fins are provided on air/atm side

Analysis of PIN FIN/THIN ROD

- Consider a pin fin of uniform Cross sectional area of A (perimeter P)



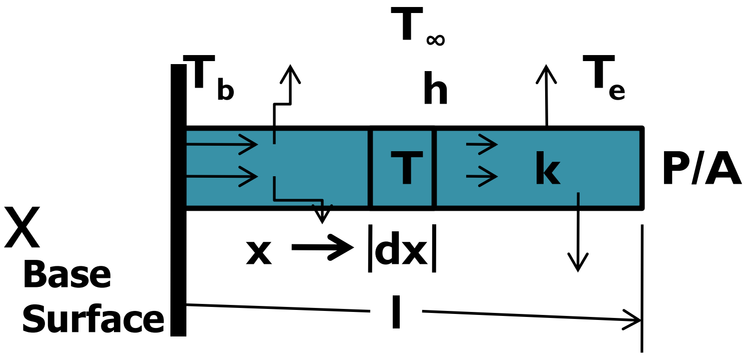
- Fin is connected to base surface,which is at temp T_b and from where heat is to be removed



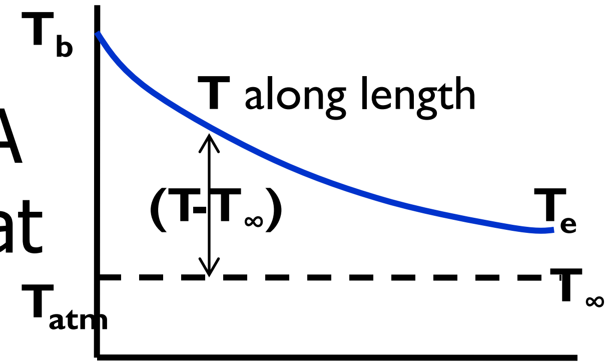
- Heat transfer from base surface to fin and through the fin is by conduction and to atm/surrounding fluid from fin surface by
nvection
emp variation along the length of fin is shown in
g; It reduces along the length away from base

Analysis of PIN FIN/THIN ROD

- Consider heat flow to and from an elemental section of length dx at a distance x from the base at temp T

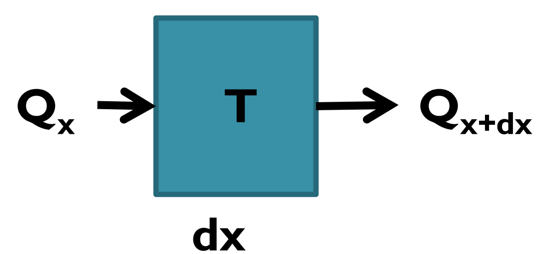


- Let heat entering elemental section dx at face x of area A be Q_x which shall be the heat conducted into the element



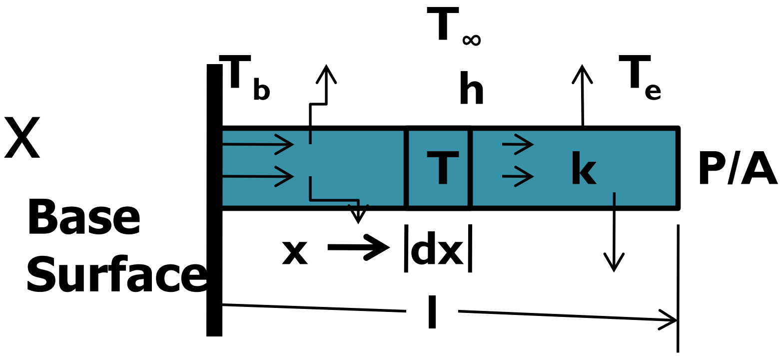
- Hence Q_x can be given as:

$$= -kA\frac{dT}{dx} \quad \textit{from Fourier's Law}$$



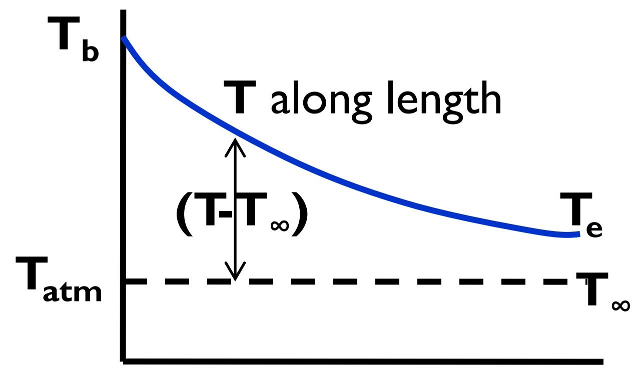
Analysis of PIN FIN/THIN ROD

- And heat conducted out from element at face x+dx will be Q_{x+dx}

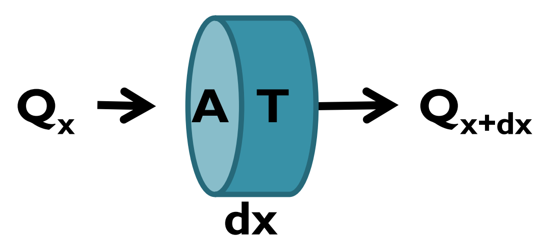


- We know that:

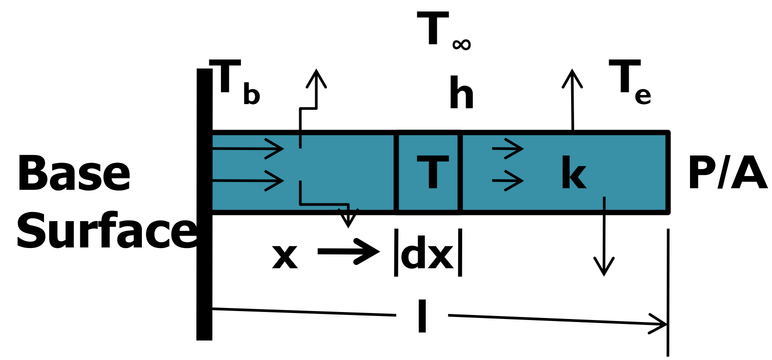
$$\frac{d}{dx}(Q_x) = \frac{Q_{x+dx} - Q_x}{dx};$$



nce $\quad Q_{x+dx} = Q_x + \frac{d}{dx}(Q_x)dx$

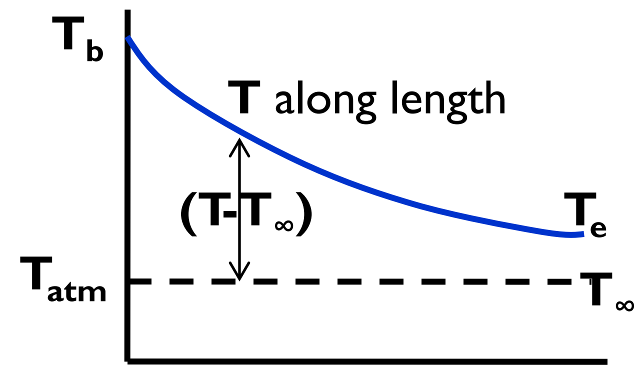


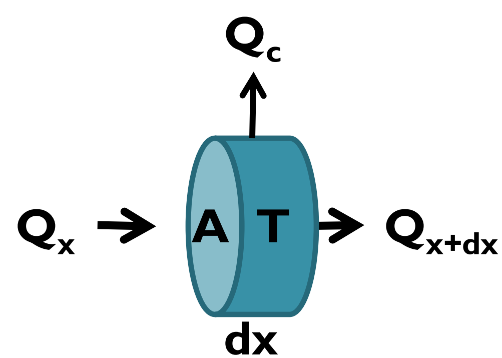
Analysis of PIN FIN/THIN ROD



- And heat convected out from surface of element dx to surroundings will be:

$$Q_c = h.P.dx\,(T - T_\infty)$$





Analysis of PIN FIN/THIN ROD

- Writing energy balance eqn for element dx :

Heat conducted in to the element = Heat conducted out + Heat convected out of element

$$Q_x = Q_x + \frac{d}{dx}(Q_x)dx + hPdx(T - T_\infty) \quad OR$$

$$0 = \frac{d}{dx}(Q_x)dx + hPdx(T - T_\infty) \quad OR$$

$$= \frac{d}{dx}\left(-kA\frac{dT}{dx}\right)dx + hPdx(T - T_\infty) \quad OR$$

Analysis of PIN FIN/THIN ROD

$$kA\frac{d^2T}{dx^2}dx = hPdx(T - T_\infty) \quad OR$$

$$\frac{d^2T}{dx^2} = \frac{hP}{kA}(T - T_\infty)$$

- Putting $(T - T_\infty) = \theta$; the excess temp and let $hP/kA = m^2$; We have :

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

Analysis of PIN FIN/THIN ROD

We have :

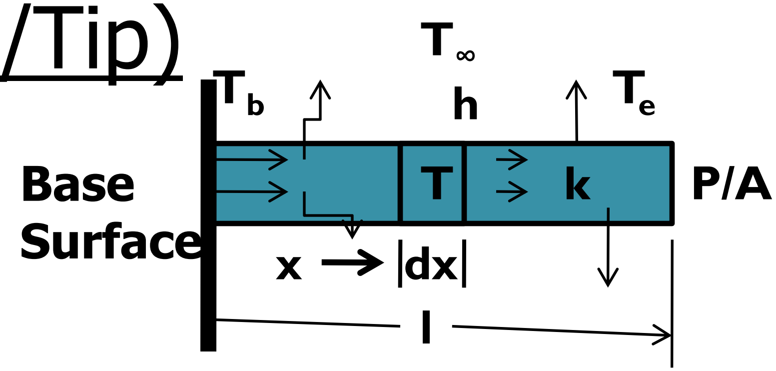
$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

- This is the second order differential equation for temp distribution along the fin, whose general solution is of the form:

$$\theta = C_1 e^{mx} + C_2 e^{-mx}$$

$$nd\ the\, Slope\ \ \frac{d\theta}{dx} = mC_1 e^{mx} - mC_2 e^{-mx}$$

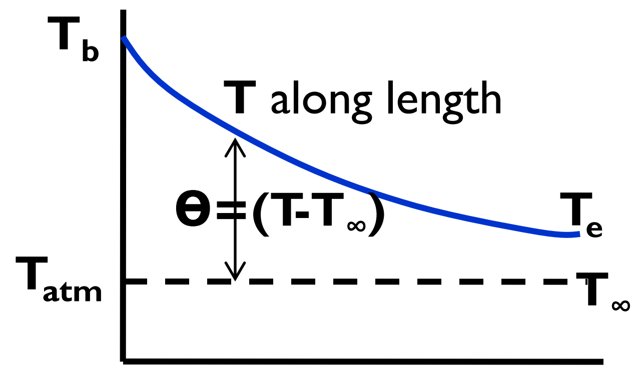
Case – I : Adequately Long FIN (Fin with Insulated End/Tip)



Boundary Conditions:

1. At x=0; $T=T_b$;$\theta=\theta_b$
2. At x=l; Q=0; $d\theta/dx=0$

(As no heat flow by convection from Tip, so assumed insulated Actually, there is no insulation)



- Applying BC 1) we get:

$$_1 + C_2 = \theta_b \ldots\ldots\ldots\ldots\ldots(1)$$

Case – I : Fin with Insulated End/Tip

$$Applying \;\; BC \;\; 2), We \; have:$$

$$\frac{d\theta}{dx} = mC_1 e^{mx} - mC_2 e^{-mx}$$

$$\left[\frac{d\theta}{dx}\right]_{x=l} = mC_1 e^{ml} - mC_2 e^{-ml} = 0$$

$$\Rightarrow C_1 e^{ml} - C_2 e^{-ml} = 0 \ldots\ldots\ldots\ldots(2)$$

$$From \; Equations \;\; (1) \; \& \; (2); we \; have:$$

$$= \frac{\theta_b e^{-ml}}{e^{ml} + e^{-ml}} \;\; and \;\; C_2 = \frac{\theta_b e^{ml}}{e^{ml} + e^{-ml}}$$

Case – I : Fin with Insulated End/Tip

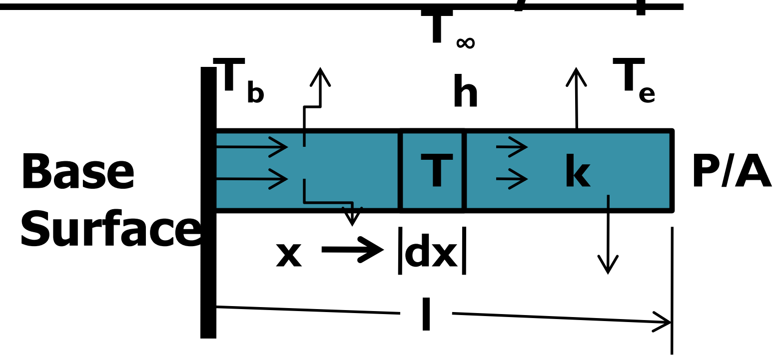
$$Substituting\ C_1\ \&\ C_2\ in\ Eqn\ \theta = C_1 e^{mx} + C_2 e^{-mx}$$

$$\frac{\theta}{\theta_b} = \frac{e^{-ml}.e^{mx} + e^{ml}.e^{-mx}}{e^{ml} + e^{-ml}}$$

$$\frac{\theta}{\theta_b} = \frac{e^{m(l-x)} + e^{-m(l-x)}}{e^{ml} + e^{-ml}} = \frac{Cosh\ m(l-x)}{Cosh\ ml}$$

$$Also,\ \theta\ at\ the\ tip\ will\ be\ \frac{\theta_e}{\theta_b} = \frac{1}{Cosh\, ml}$$

Case – I : Fin with Insulated End/Tip



Heat Flow Rate from Fin:

Heat conducted to Fin at the base shall be the heat Flow rate from Fin

$$Hence\ Q = -kA\frac{d\theta}{dx}\ at\ \ x = 0$$

$$Substituting\ \ \frac{d\theta}{dx}\ \ at\ \ x = 0;\ We\ have$$

$$Q = kAm\theta_b \tanh ml$$

OR $$Q = \theta_b\sqrt{hPkA}\,\tanh ml$$

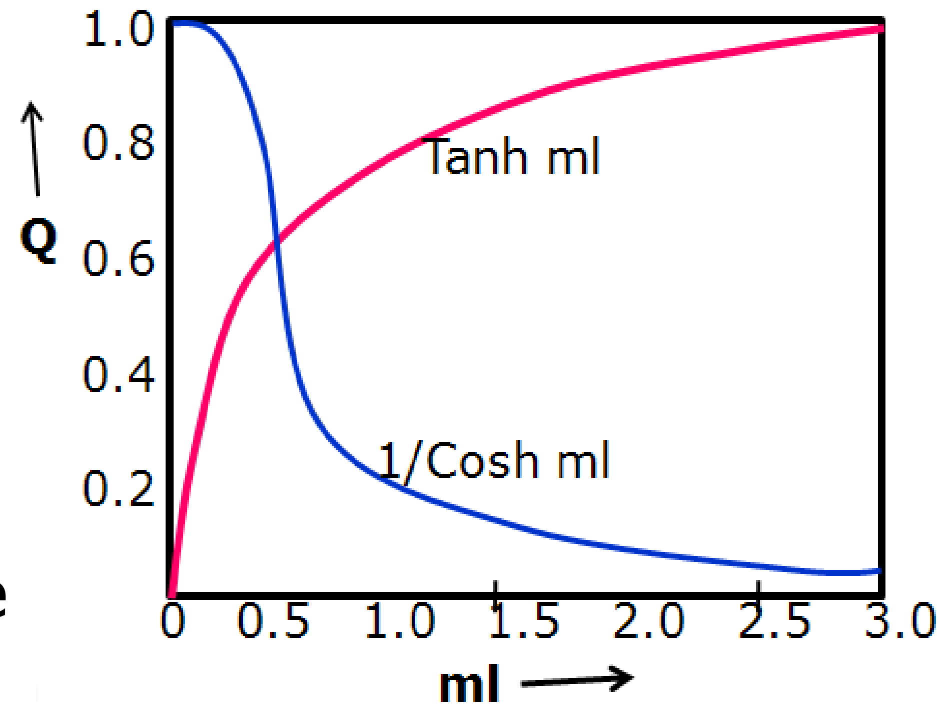
Case – I : Adequately Long FIN

(Fin with Insulated End/Tip)

What is adequately long Fin?

Let us look at Q v/s ml & θ v/s ml plots

As ml or l increases, tanh ml first increases rapidly and then become asymptotic at ml ≈3.

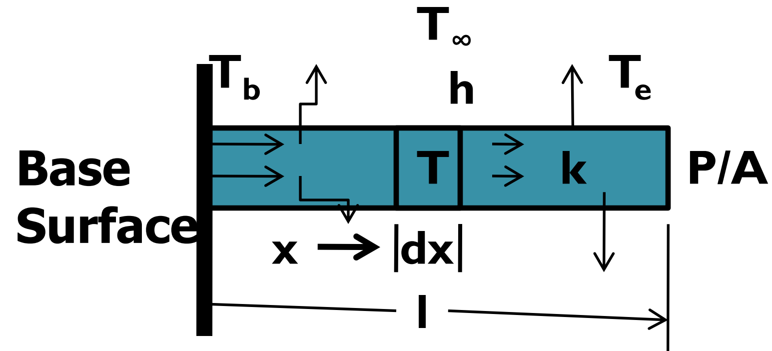


Also, θ_e approaches zero at ml ≈3.

…us, increasing ml (or l) beyond 3 will not give any …vantage. Hence Fin with ml≈3 is called adequately …ng fin or fin with insulated tip/end

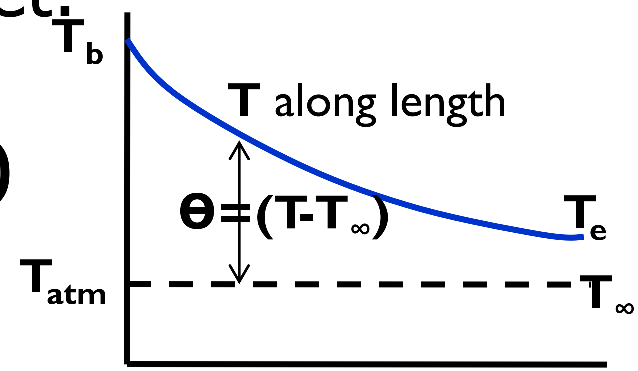
Case –II : Analysis of Very Long FIN

Boundary Conditions:



1. At x=0; $T=T_b$;$\theta=\theta_b$
2. At x=l; $\theta=\theta_e=0$

- Applying above BC 1), we get:



$$\theta_b = C_1 + C_2 \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Applying BC 2); We have;

$$\theta = C_1 e^{ml} + C_2 e^{-ml} = 0 \ldots\ldots(2)$$

$$= \frac{-\theta_b e^{-ml}}{e^{ml} - e^{-ml}} \quad \& \quad C_2 = \frac{\theta_b e^{ml}}{e^{ml} - e^{-ml}}$$

Case –II : Analysis of Very Long FIN

$$Substituting\ C_1\ and\ C_2\ in\ Eqn\ \theta = C_1 e^{mx} + C_2 e^{-mx};$$

$$\frac{\theta}{\theta_b} = \frac{-e^{-ml}.e^{mx} + e^{ml}.e^{-mx}}{e^{ml} - e^{-ml}} = \frac{e^{m(l-x)} - e^{-m(l-x)}}{e^{ml} - e^{-ml}}$$

$$\frac{\theta}{\theta_b} = \frac{Sinh\ m(l-x)}{Sinh\ ml}$$

Case –II : Analysis of Very Long FIN

$$Also,\ \theta = C_1 e^{ml} + C_2 e^{-ml} = 0......(2)$$

$$Substituting\ l = \infty : C_1 e^{m\infty} + C_2 e^{-m\infty} = 0$$

$$or\ C_1 e^{\infty} + 0 = 0$$

$$\Rightarrow C_1 = 0$$

$$Hence\ \ C_2 = \theta_b\ \ from\ eqn...(1)\ C_1 + C_2 = \theta_b$$

$$Substituting\ C_1\ and\ C_2\ in\ Eqn\ \theta = C_1 e^{mx} + C_2 e^{-mx}$$

herefore, we also have $\theta = \theta_b e^{-mx}$

Case –II : Analysis of Very Long FIN

Heat Flow Rate through Fin:

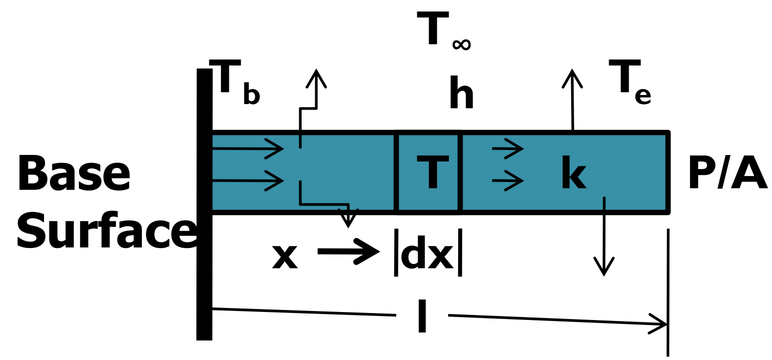
$$Q = -kA\left[\frac{d\theta}{dx}\right]_{x=0} = -kA(-m)\theta_b \frac{Cosh\ \ ml}{Sinh\ \ ml}$$

$$Q = kAm\theta_b Coth\, ml \approx kAm\theta_b$$

$$\text{Also, } Q = \theta_b \sqrt{hPkA}\, Coth\, ml$$

Case – III : Analysis of Short FIN

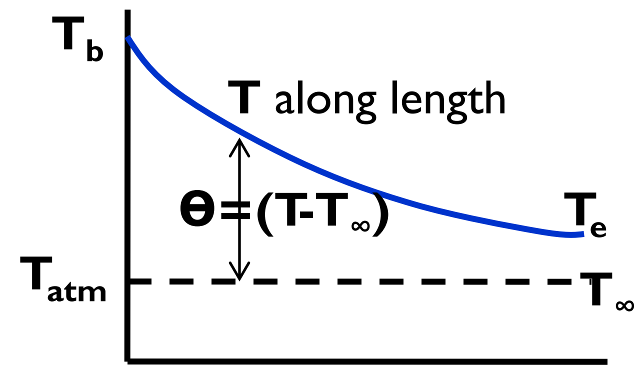
Boundary Conditions:



1. At x=0; $T=T_b$; $\theta=\theta_b$

2. At x=l;

Heat conducted to Fin Tip= Heat convected from Fin Tip



$$That\ \ is : -kA\left[\frac{d\theta}{dx}\right]_{x=l} = \left[hA\theta\right]_{x=l}$$

- Applying above BC 1), we get:

$$\theta_b = C_1 + C_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Case – III : Analysis of Short FIN

Applying BC2) in:

$$-kA\left[\frac{d\theta}{dx}\right]_{x=l} = [hA\theta]_{x=l}$$

$$-kA\left[mC_1e^{ml} - mC_2e^{-ml}\right] = hA\left[C_1e^{ml} + C_2e^{-ml}\right] ..(2)$$

From Eqn (1) & (2);

$$C_1 = -\left[\frac{\frac{h}{mk}-1}{\frac{h}{mk}+1}\right]\frac{\theta_b e^{-ml}}{e^{ml} - \left[\frac{\frac{h}{mk}-1}{\frac{h}{mk}+1}\right]e^{-ml}} \quad \& \quad C_2 = \frac{\theta_b e^{ml}}{e^{ml} - \left[\frac{\frac{h}{mk}-1}{\frac{h}{mk}+1}\right]e^{-ml}}$$

Case – III : Analysis of Short FIN

Putting C_1 & C_2 in eqn:

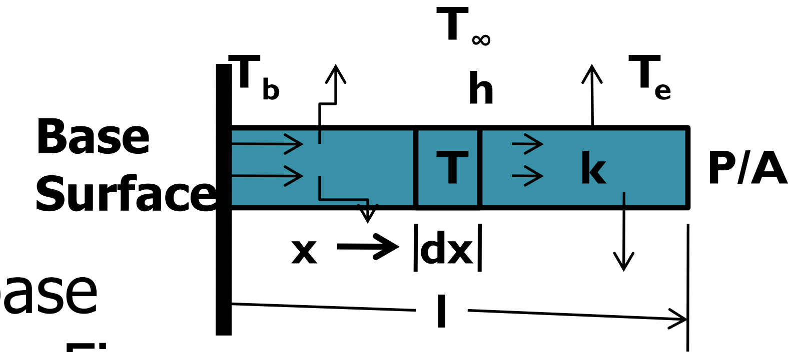
$$\theta = C_1 e^{mx} + C_2 e^{-mx}$$

$$We\ have \quad \frac{\theta}{\theta_b} = \frac{Cosh\, m(l-x) + \frac{h}{mk} Sinh\, m(l-x)}{Cosh\, ml + \frac{h}{mk} Sinh\, ml}$$

$$And \quad \frac{\theta_e}{\theta_b} = \frac{1}{Cosh\, ml + \frac{h}{mk} Sinh\, ml}$$

Case – III : Analysis of Short FIN

Heat Flow Rate from Fin:



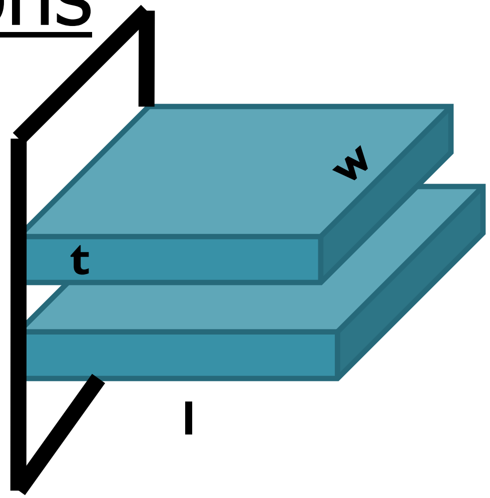
Heat conducted to Fin at the base shall be the heat Flow rate from Fin

$$Hence\ Q = -kA\frac{d\theta}{dx}\ at\ x = 0$$

$$Subsituting\ \frac{d\theta}{dx}\ at\ x = 0$$

$$= kAm\theta_b\left(\frac{\frac{h}{mk} + \tanh ml}{1 + \frac{h}{mk}\tanh ml}\right) = \theta_b\sqrt{hPkA}\left(\frac{\frac{h}{mk} + \tanh ml}{1 + \frac{h}{mk}\tanh ml}\right)$$

Fins of Different Cross Sections



Rectangular Fins

P=2t+2w;

For t<<w;

P=2w

And A=t x w

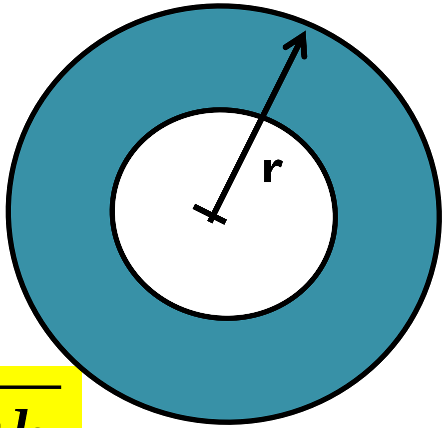
$$Hence\ m = \sqrt{\frac{Ph}{kA}} = \sqrt{\frac{2wh}{ktw}} = \sqrt{\frac{2h}{kt}}$$

Fins of Different Cross Sections

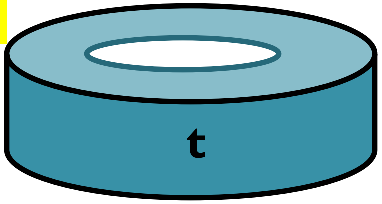
Circular/Disc Fins

P=2лr x 2;

And A=2лr x t



$$Hence\ m = \sqrt{\frac{Ph}{kA}} = \sqrt{\frac{4\pi rh}{k2\pi rt}} = \sqrt{\frac{2h}{kt}}$$



Fin Effectiveness

Fin Effectiveness (E) is defined as the ratio of Actual Heat Transfer Rate from finned surface to the Heat Transfer Rate from the area blocked by fin (or When fin was not there)

Naturally, use of fins is justified only when Effectiveness is greater than one
(and when h is very low)

Fin Effectiveness (E)

$$E = \frac{Q_{withFin}}{Q_{w/oFin}} = \frac{\theta_b \sqrt{PhkA}\tanh ml}{hA\theta_b} = \frac{\tanh ml}{\sqrt{\frac{hA}{Pk}}}$$

When ml is large (≥3), tanhml tends to become 1.

$$Hence \quad E = \sqrt{\frac{kP}{hA}};$$

It should be greater than 1 for fin to be effective

$$E = \frac{Q_{withfin}}{Q_{w/o\,Fin}} = \frac{\eta_{fin} Q_{max}}{Q_{w/o\,Fin}} = \frac{\eta_{fin} A_f h\theta_b}{hA\theta_b} = \frac{\eta_{fin} A_f}{A}$$

Fin Efficiency (η)

Fin Efficiency (η) is defined as the ratio of Actual Heat Transfer Rate to max possible heat transfer rate from the same fin.

Heat Transfer Rate Q from fin shall be max when fin material has infinite conductivity k so that temp of fin all along the length can be assumed to be same as that at the base of fin

So $Q_{max} = h.\ A_f\ .\theta_b = h.P.l.\ \theta_b$

$Q_{actual} = \theta_b \sqrt{hPkA}\ \tanh ml$ for sufficiently long fin

Fin Efficiency (η)

$$Hence \quad \eta = \frac{\theta_b \sqrt{hPkA}\tanh ml}{hPl\,\theta_b} = \frac{\tanh ml}{\sqrt{\frac{Ph}{kA}}.l} = \frac{\tanh ml}{ml}$$

for fin with insulated end

Fin Efficiency for Short Fin

$$\eta = \frac{\frac{h}{mk} + \tanh ml}{ml\left(1 + \frac{h}{mk}\tanh ml\right)}$$

Fin Efficiency for Long Fin $\eta = \dfrac{1}{ml}$

Overall Fin Effectiveness

Overall Fin Effectiveness for a finned surface is defined as:

$$E_{overall} = \frac{Total\ Heat\ Transfer\ from\ Finned\ Surface}{Heat\ Transfer\ from\ the\ base\ if\ there\ were\ no\ fins}$$

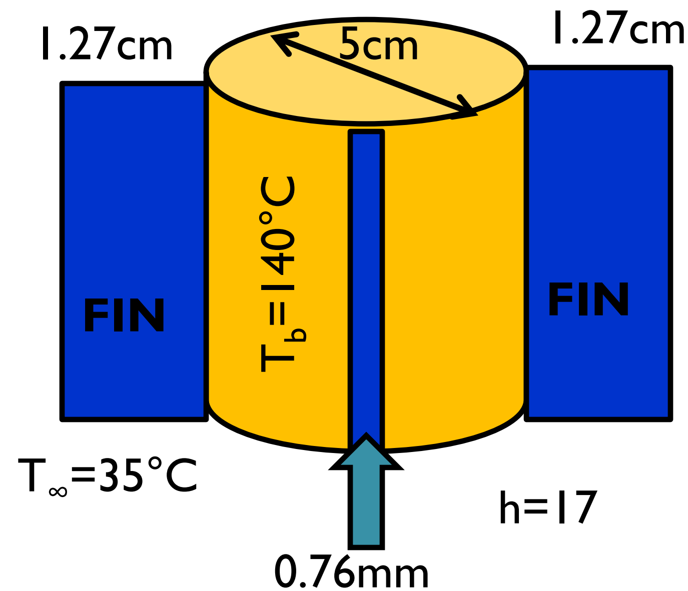
$$E_{overall} = \frac{h\left(A_{unfin} + \eta_{fin} A_{fin}\right)\left(T_b - T_\infty\right)}{h.A_{no\ fin}\left(T_b - T_\infty\right)}$$

Overall Fin Effectiveness thus depends on the fin density and individual fin effectiveness

…verall Fin Effectiveness is better measure of performance of …nned surface than effectiveness …individual fins

Q1. Thin fins of brass ($k=119.4\ W/mK$) are welded longitudinally on a 5cm brass cylinder, which stands vertically and is surrounded by air at 35°C with $h=17 W/m^2K$.

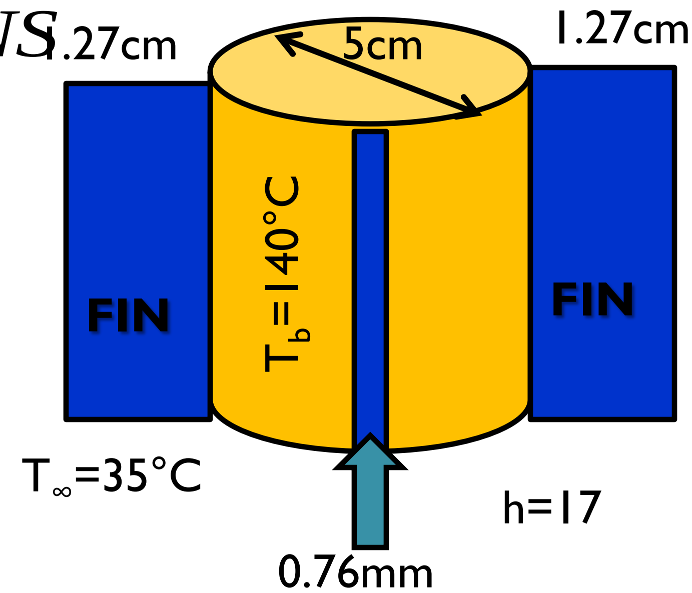
12 uniformly spaced fins are used, each 0.76mm thick, and extending 1.27cm radially from the cylinder.



Find heat transfer rate from the finned cylinder when its surface is maintained at 140°C.

Solution:

$Q = ?;$ $\quad$ *To apply Q formula, we should know whether FINS are Short or Long or Adequately Long.*



To do this, we should find $ml = ?$ *and compare with* $ml \approx 3$

$$m = \sqrt{\frac{hP}{kA}}$$

$$m = \sqrt{\frac{17x2}{119.4x0.00076}} = 19.36$$

Perimeter P=1+1=2m

A=1x0.00076m^2

ml=19.36x0.0127
=0.246

Hence case of Short Fin

$$Q = kAm\theta_b \frac{h/mk + \tanh ml}{1 + \frac{h}{mk}\tanh ml} \quad ; \frac{h}{mk} = \frac{17}{19.36x119.4} = 0.0074$$

$$Q = 119.4x0.00076x19.36(140-35)\frac{0.0074+\tanh(0.246)}{1+0.0074\tanh(0.246)}$$

$$= 45.72W$$

Hence Q for 12 fins=45.72x12=548.64 W

Cyl surface area=лDL=3.14x0.05x1=0.157m^2

Unfinned area=0.157-12x0.00076=0.148m^2

Heat Transfer from unfinned area
=hAθ_b=17x0.148x105=264W

nce total heat removal rate=548.64+264=812.64W

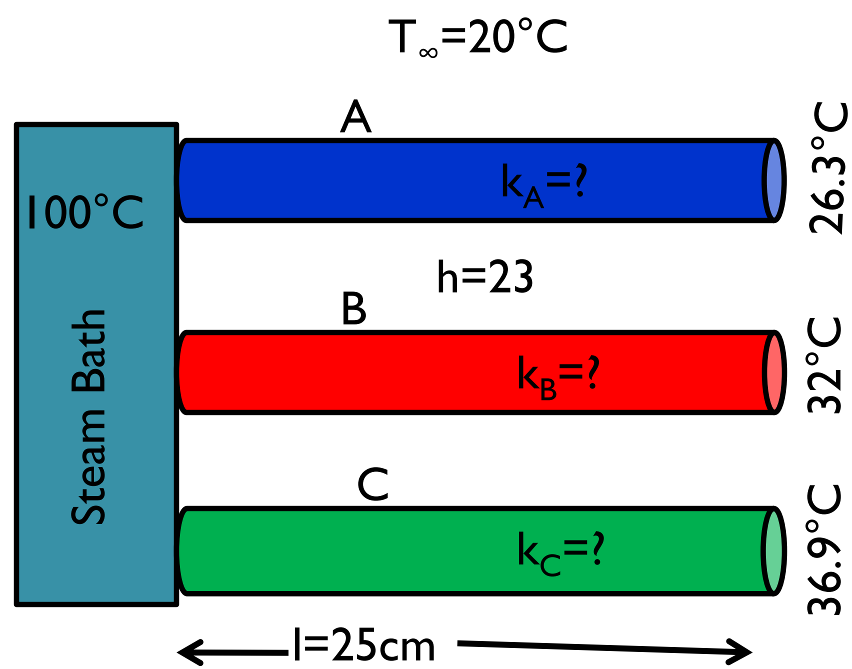
Answer

Q2 : Three 10mm dia rods A, B and C protrude from steam bath at 100°C up to a length of 25cm in to the atm at 20°C. The temp at other ends are found to be 26.7, 32 and 36.9°C respectively. Assuming surface heat transfer coefficient as 23 W/m²K, find their conductivities. Heat loss from the tips of the fins may be neglected.

<u>Solution:</u>

d=10mm

$T_A=26.7°C; T_B=32°C$
$T_C=36.9°C; T_\infty=20°C$



For Adequately Long Fin;

$$= \frac{Coshm(l-x)}{Coshml}$$

Solution:

d=10mm

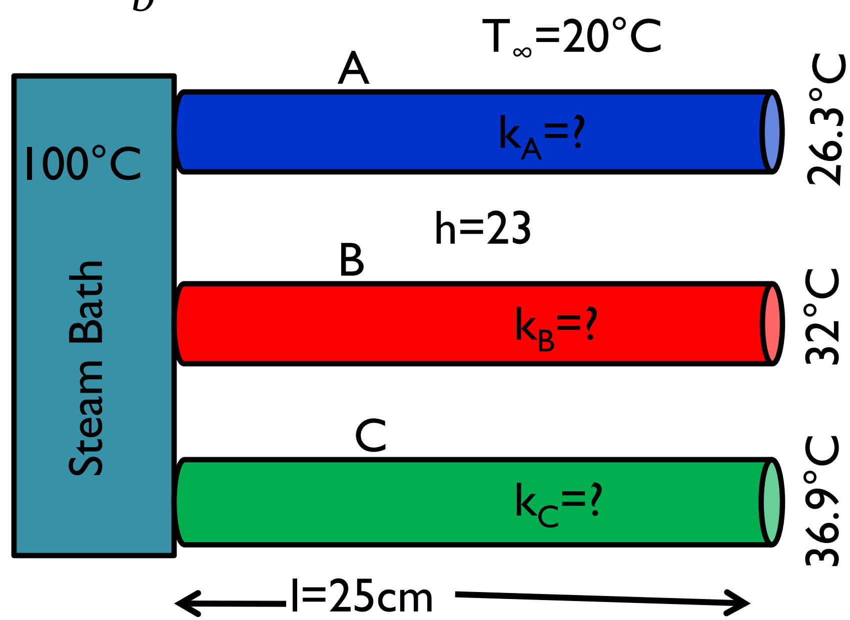
$T_A=26.7°C; T_B=32°C; T_C=36.9°C; T_\infty=20°C; T_b=100°C$

$$Conductivity\ k_A\ will\ appear\ in\ expression\ m=\sqrt{\frac{Ph}{k_A A}}$$

$$For\ Adequately\ Long\ Fin;\ \frac{\theta}{\theta_b}=\frac{Cosh\,m(l-x)}{Cosh\,ml}$$

For temp at tip of Fin, substituting x=l ;

$$\frac{\theta}{\theta_b}=\frac{1}{Cosh\,ml}$$



$$\frac{T_l-T_\infty}{T_b-T_\infty}=\frac{1}{Cosh\,ml};\ m=?$$

Solution: For Rod/ Fin A

$$\frac{T_A - T_\infty}{T_b - T_\infty} = \frac{1}{Coshml}$$

$$Substituting;\ \frac{26.7 - 20}{100 - 20} = \frac{1}{Cosh(m.0.25)}$$

$$Hence\ Cosh(m.0.25) = 11.94\ or\ 0.25m = 3.17$$

$$Therefore \quad m = 12.685 = \sqrt{\frac{Ph}{k_A.A}}$$

$$= \sqrt{\frac{\pi\,x\,0.01x23}{k_A.\frac{\pi}{4}0.01^2}} \Rightarrow k_A = 55.17\ W/mK\ Answer$$

Solution: For Rod/ Fin B

$$\frac{T_B - T_\infty}{T_b - T_\infty} = \frac{1}{Coshml}$$

$$Substituting; \frac{32-20}{100-20} = \frac{1}{Cosh(m \, x \, 0.25)}$$

$$Hence \; Cosh(m \, x \, 0.25) = 6.667 \; or \; 0.25m = 2.58$$

$$Therefore \quad m = 10.34 = \sqrt{\frac{Ph}{k_B.A}}$$

$$= \sqrt{\frac{\pi \, x \, 0.01 x 23}{k_B.\frac{\pi}{4}0.01^2}} \Rightarrow k_B = 86.07 \; W/mK \; Answer$$

Solution: For Rod/ Fin C

$$\frac{T_C - T_\infty}{T_b - T_\infty} = \frac{1}{Coshml}$$

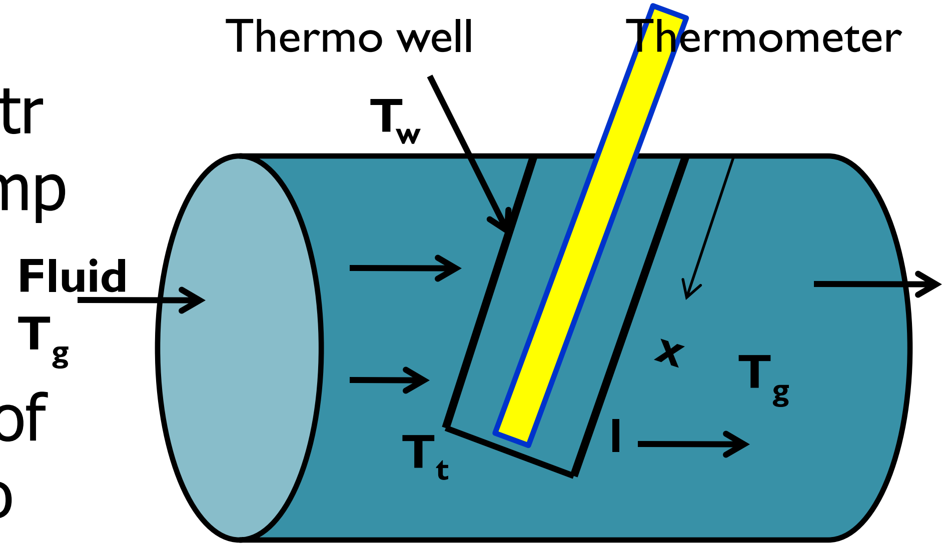
$$Substituting;\ \frac{36.9-20}{100-20} = \frac{1}{Cosh(m\,x\,0.25)}$$

$$Hence\ Cosh(m\,x\,0.25) = 4.734\ or\ 0.25m = 2.2365$$

$$Therefore\quad m = 8.95 = \sqrt{\frac{Ph}{k_C.A}}$$

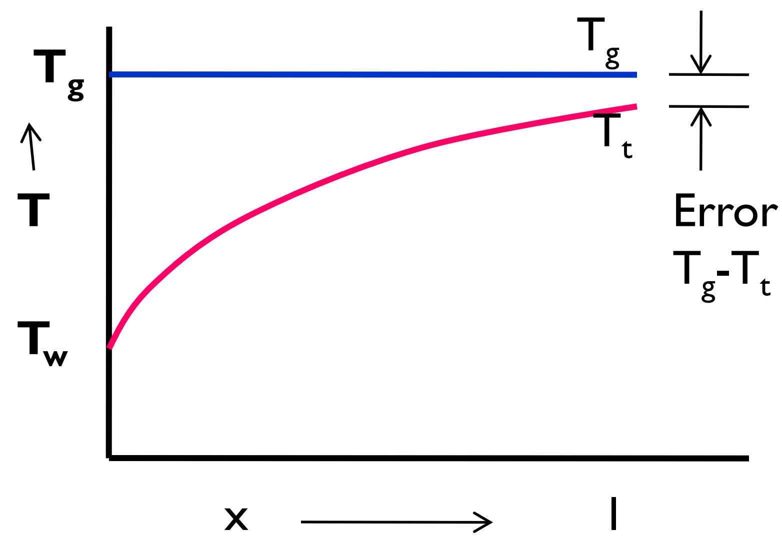
$$= \sqrt{\frac{\pi\,x\,0.01x23}{k_C.\frac{\pi}{4}0.01^2}} \Rightarrow k_C = 115\ W/mK\ \ Answer$$

Error in Temp Measurement by Thermometer



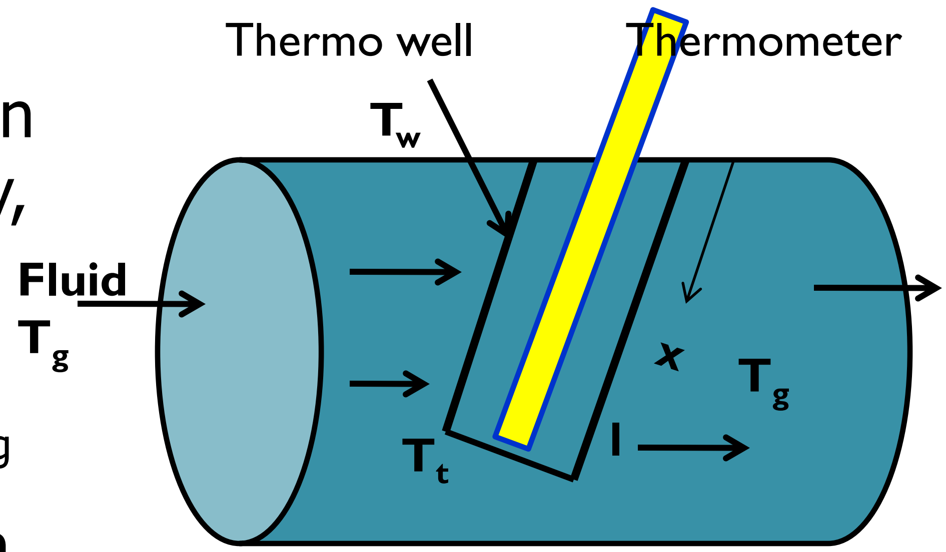
Principle: From temp distr for fin, we know that temp along the length of fin approaches surrounding fluid temp & at the end of Fin, it shall be nearest to fluid temp.

We also know that temp at the end of fin will become same as that of surrounding fluid, only
en length of fin will be
inite.



Error in Temp Measurement by Thermometer

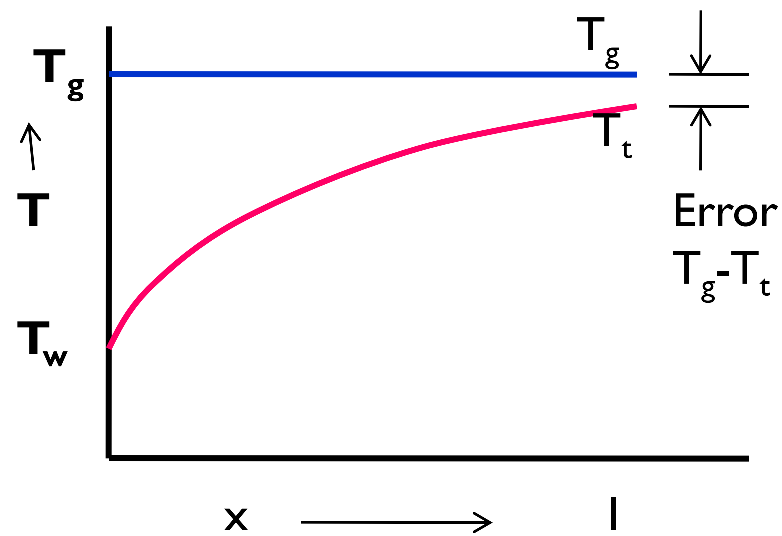
Since infinite length of fin is not possible practically, the recorded temp T_t is always different (lower) from actual fluid temp T_g



This is known as Error in Temp Measurement

As shown in Graph, Error in temp measurement shall be $(T_g - T_t)$.

s the wall Temp
he conduit



Error in Temp Measurement by Thermometer

Thermo well can be assumed as FIN and hence applying fin analysis, actual temp of fluid can be found by estimating the error.

Expression for temp distribution for adequately Long Fin

$$\frac{\theta}{\theta_b} = \frac{Cosh\,m(l-x)}{Cosh\,ml} \quad : where\ l\ is\ the\ Length\ of\ Thermowell$$

Temp at the End of Fin, that is Thermowell,

$$\theta_{end} = T_g - T_t \quad for\ x = l\ can\ be\ written\ as$$

$$\frac{T_g - T_t}{T_g - T_w} = \frac{1}{Cosh\,ml}$$

re $(T_g - T_t)$ *is the Error in Temp Measurement*

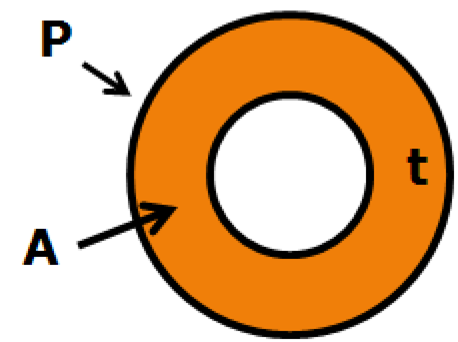
Error in Temp Measurement by Thermometer

$$\frac{T_g - T_t}{T_g - T_w} = \frac{1}{Coshml}$$

$$And \quad T_g - T_t \propto \frac{1}{Coshml} \propto \frac{1}{ml}$$

Because, with increase in ml, Coshml increases

If D is outer dia of Thermowell
and t the thickness of its wall
and assuming D>>t,

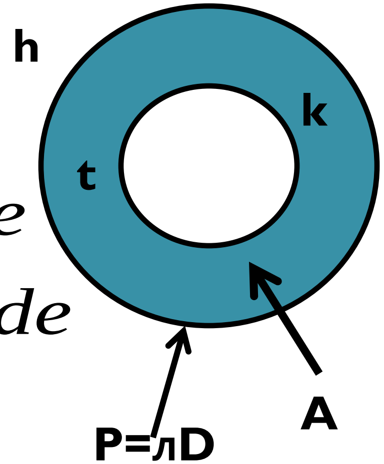


…en Perimeter P = лD
Area A = лDt

Error in Temp Measurement by Thermometer

If D is outer dia of Thermowell and t the thickness of its wall and assuming D>>t,

$$m = \sqrt{\frac{Ph}{kA}} = \sqrt{\frac{h.\pi D}{k.\pi Dt}} = \sqrt{\frac{h}{kt}}$$



$$Where\ P = \pi D\ as\ only\ outside\ surface\ of\ well\ is\ receiving\ heat\ and\ not\ inside$$

$$Then\ Error\ T_g - T_t \propto \frac{1}{ml} \propto \frac{1}{l\sqrt{\frac{h}{kt}}}$$

So, to minimize error in measurement (T_g-T_t);

1. Length of fin (l) should be as long as possible
2. Thickness of thermowell wall (t) should be as small as possible

Q3. Temp of air in reservoir is measured with the help of mercury-in-glass thermometer placed in a protective Well filled with oil. Thermometer shows a temp of 86°C at the end of the well. Find out error in measurement, if temp at the base of the well is 40°C. The well is 12cm long, 1.5mm thick having thermal conductivity of 56W/mK. h may be taken as 20W/m^{2}K.

Solution:

Error in temp measurement=T_g-T_t

$$\frac{T_g - T_t}{T_g - T_w} = \frac{1}{\cosh ml}$$

$$= \sqrt{\frac{h}{kt}} = \sqrt{\frac{20}{56 x 0.0015}} = 15.43$$

Solution (Contd):

$$\frac{T_g - T_t}{T_g - T_w} = \frac{1}{Coshml}$$

$$\Rightarrow \frac{T_g - 86}{T_g - 40} = \frac{1}{Cosh(15.43x0.12)} = 0.307$$

$$T_g - 86 = 0.307\left(T_g - 40\right)$$

$$= 0.307T_g - 12.28$$

$$or\ T_g = 106$$

$$Therefore,\ Error\ T_g - T_t = 106 - 86 = 20°C$$

$$rror = \frac{T_g - T_t}{T_g} x100 = \frac{106 - 86}{106} x100 = 18.8\%$$

End of Unit - II

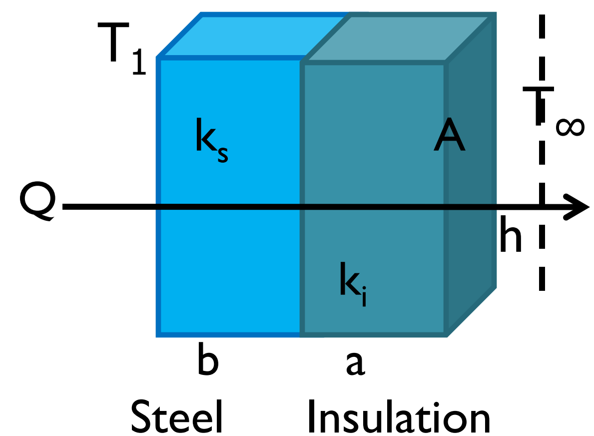
Critical Radius of Insulation

Insulation when applied to a surface, is expected to reduce heat transfer rate across it.

But, is it always true? Let us examine.

Take the example of heat flow across a steel plate, when a layer of insulation is applied to it as shown in Fig.

$$Without\ \ Insulation\quad Q=\frac{T_1-T_\infty}{\dfrac{b}{k_sA}+\dfrac{1}{hA}}$$



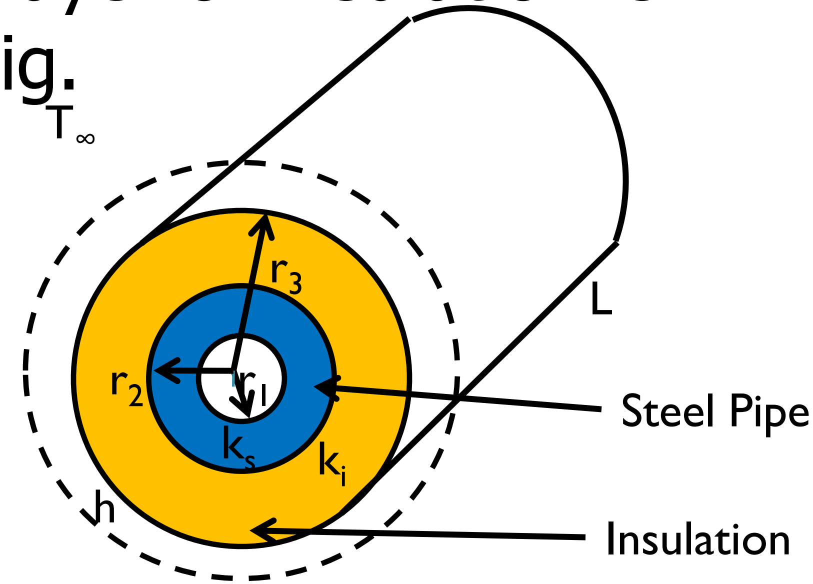
$T_l > T_\infty$

$$With\ insulation\quad Q=\frac{T_1-T_\infty}{\dfrac{b}{k_sA}+\dfrac{a}{k_iA}+\dfrac{1}{hA}}$$

nce Q will always reduce with insulation

Critical Radius of Insulation : Cylinder

Take the example of heat flow across a steel tube, carrying hot fluid, when a layer of insulation is applied to it as shown in Fig.



Hence;

$$Q = \frac{\Delta T}{\frac{\ln \frac{r_3}{r_2}}{2\pi k_i L} + \frac{1}{h 2\pi r_3 L}}$$

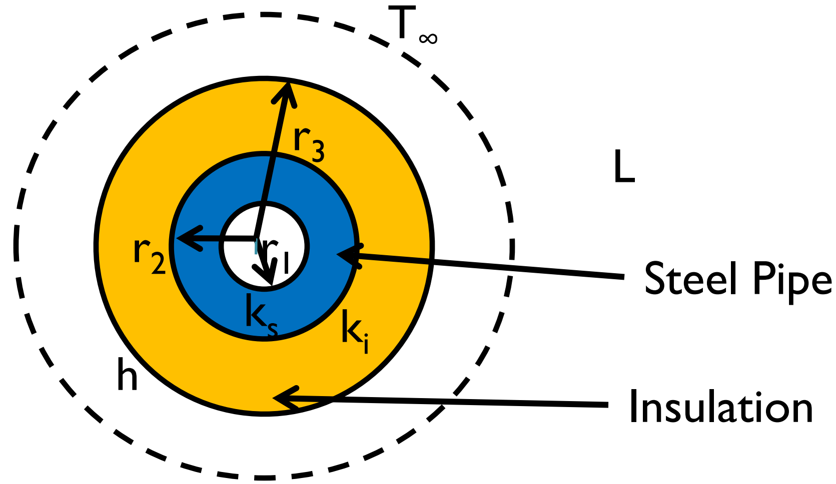
With increase in insulation radius r_3 , conductive istance increases but convective resistance creases, so we do not know whether Q will rease or decrease.

Critical Radius of Insulation: Sphere

Now, take another example of heat flow across a sphere, having hot fluid, when a layer of insulation is applied to it as shown in Fig.

Hence;

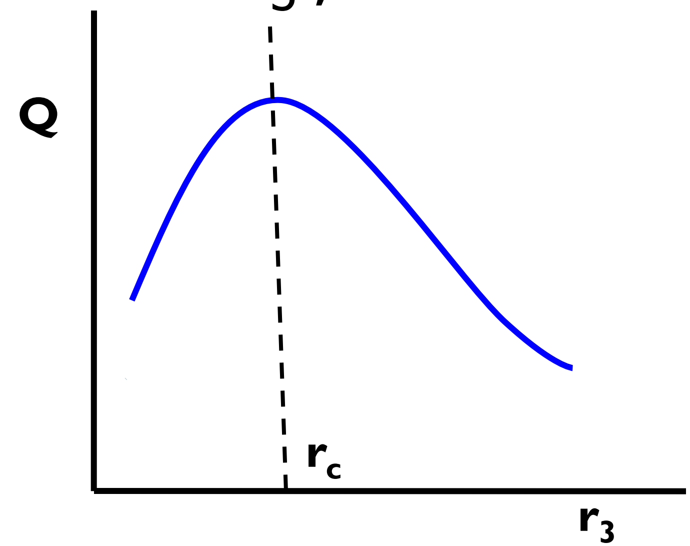
$$Q = \frac{\Delta T}{\dfrac{r_3 - r_2}{4\pi k_i r_3 r_2} + \dfrac{1}{h 4\pi r_3^{\,2}}}$$



Again, with increase in insulation thickness (r_3) , conductive resistance increases but convective istance decreases, so we do not know whether vill increase or decrease.

Critical Radius of Insulation: Sphere/Cylinder

From Q expression, it is found that while conductive resistance increases with r_3 ,convective resistance decreases.



It is seen from Q v/s r_3 plot that with increase in r_3 , Q first increases up to certain $r_3 = r_c$, and then starts decreasing.

Value of r_3 , for which Q is max or in other words, al resistance is minimum, is called critical radius insulation, denoted by r_c

Critical Radius of Insulation

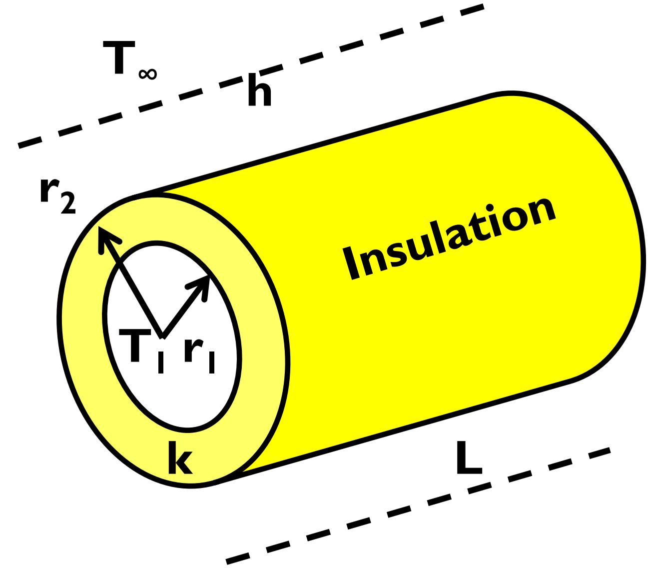
So, what is the conclusion?

- Whenever insulation is applied to wall, increase in its thickness will always reduce heat transfer
- Whenever insulation is applied to a cylinder or sphere, heat transfer may increase or decrease with application of insulation or with increase in its thickness.

Let us now derive the expressions for CRITICAL RADIUS of INSULATION

Critical Radius of Insulation for Cylinder

$$Q = \frac{T_1 - T_\infty}{\dfrac{\ln \dfrac{r_2}{r_1}}{2\pi k L} + \dfrac{1}{h.2\pi r_2 L}}$$



$$where\ \frac{\ln \dfrac{r_2}{r_1}}{2\pi k L}\ is\ conductive\ resistance$$

$$and\ \frac{1}{h 2\pi r_2 L}\ is\ convective\ resistance$$

Now it can be seen that with increase in thickness of insulation i.e, r_2 , conductive resistance increases logarithmically, while convective resistance decreases linearly; the net result is that l resistance first decreases up to certain r_2 , and then ts increasing. This shows that Q v/s r_2 plot first increases to a ima & then decreases

Critical Radius of Insulation for Cylinder

We have to find that value of r_2 , for which Q is maximum or total resistance is minimum.

To obtain maxima, we can either differentiate Q or resistance expression wrt r_2 and put it equal to zero.

$$Therefore we can write :$$

$$\frac{d}{dr_2}\left[\frac{\ln\frac{r_2}{r_1}}{2\pi kL}+\frac{1}{h2\pi r_2 L}\right]=0$$

Critical Radius of Insulation for Cylinder

$$Takingoutcommonwecanwrite:$$

$$\frac{d}{dr_2}\left[\frac{\ln\frac{r_2}{r_1}}{k}+\frac{1}{hr_2}\right]=0$$

Critical Radius of Insulation for Cylinder

$$On\ differentiation : \frac{1}{k}.\frac{1}{r_2} - \frac{1}{h}.\frac{1}{{r_2}^2} = 0$$

$$hence\ r_2 = \frac{k}{h} = r_c$$

This value $r_2 = r_c = \frac{k}{h}$ *is called*

CRITICAL RADIUS of INSULATION for CYLINDER

Critical Radius of Insulation

Similarly, it can be shown that critical radius of insulation for sphere is:

$$r_2 = r_c = 2k/h$$

Students will do this as Assignment

Critical Radius of Insulation

So, what are the deductions?

- Whenever insulation is applied to wall, increase in its thickness will always reduce heat transfer rate

- Whenever insulation is applied to a cylinder or sphere, heat transfer may increase or decrease with application of insulation.

- With increase in thickness of insulation, although conductive resistance always increases but convective resistance decreases as area exposed for heat convection increases; thereby reducing convective resistance ,1/hA

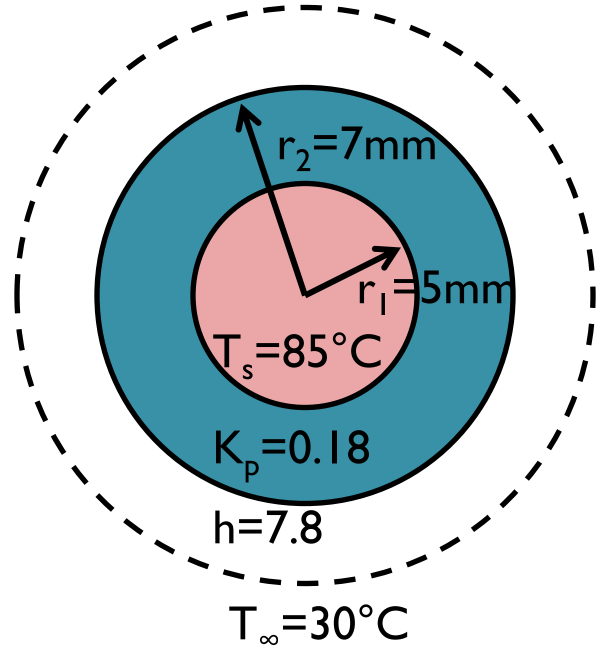
Critical Radius of Insulation

Deductions………?

- When outer radius r_1 , of cylinder/sphere < r_c , heat transfer will increase with increase in thickness of of insulation up to r_2=r_c , and then will decrease. So, if aim is to increase heat transfer rate, outer radius of cyl/sphere should be <r_c of insulation. Example: Current carrying conductor

- When outer radius r_1 , of cylinder/sphere > r_c , heat transfer rate will always decrease with increase in thickness of insulation. So, if aim is to reduce the heat transfer, outer radius of cyl/sphere should be >r_c of selected insulation. Example: Pipe carrying steam

Q1 :An electrical conductor of 10mm dia, insulated by PVC (k_p=0.18W/mK) is located in air at 30°C having h=7.8 W/m^{2}K. If the surface temp of the base conductor is 85°C, calculate:



a) Current carrying capacity of the conductor, when 2mm thick insulation is provided (Resistivity of the conductor is 70μΩcm)

$Q=I^2R; I=?$

b) Critical insulation thickness

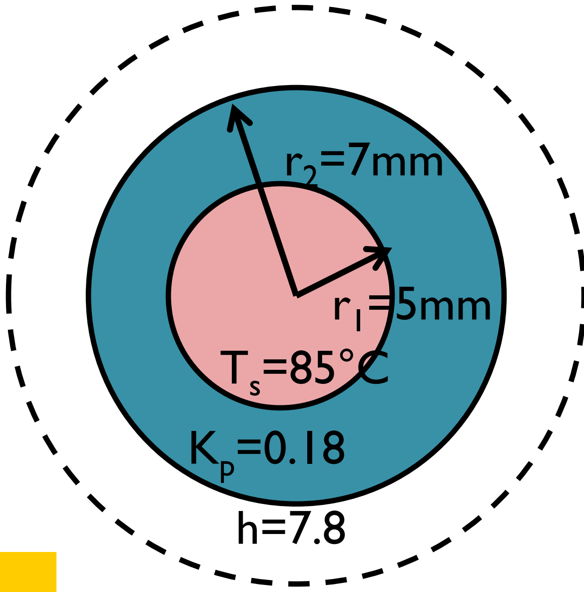
$r_c=k/h$

c) Max current carrying capacity

Percentage increase in current
rying capacity by providing
tical insulation

Solution: Q=I²R; I=? (43.8A)

$$Q = \frac{T_s - T_\infty}{\dfrac{\ln\frac{r_2}{r_1}}{2\pi k_p L} + \dfrac{1}{h 2\pi r_2 L}} \quad (17.12W)$$



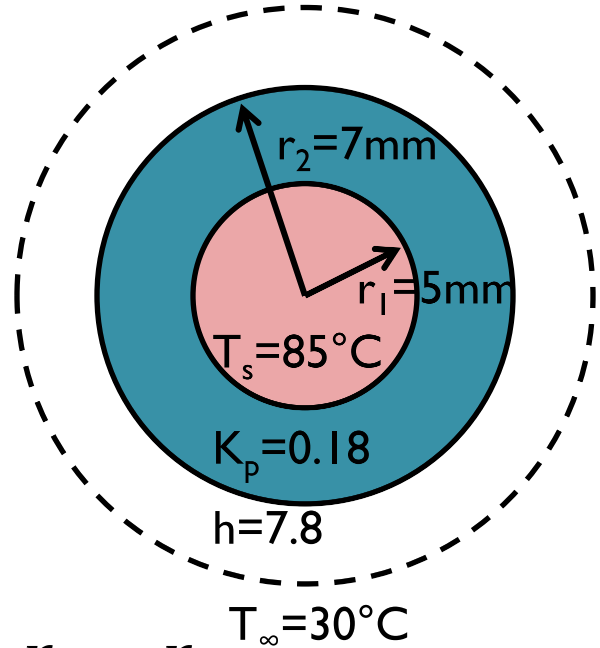
$$\Rightarrow Q = \frac{85-30}{\dfrac{\ln\frac{7}{5}}{2x3.14x0.18x1} + \dfrac{1}{7.8x2x3.14x0.007x1}}$$

$$= 17.12 W/m$$

$$\rho\frac{L}{A} = \frac{70x10^{-6}x10^{-2}}{\pi(0.005)^2} = 8.92x10^{-3}$$

$$I = \sqrt{\frac{Q}{R}} = \sqrt{\frac{17.12}{8.92x10^{-3}}} = 43.83\,Amp$$

Solution:



b) Critical Radius of Insulation

$$r_c = \frac{k}{h} = \frac{0.18}{7.8} = 0.023m = 23mm$$

$$hence\ thickness = 23 - 5 = 18mm$$

c) Max Current carrying capacity

$$for\ \max\ heat\ transfer\ will\ be\ with\ r_2 = r_c$$

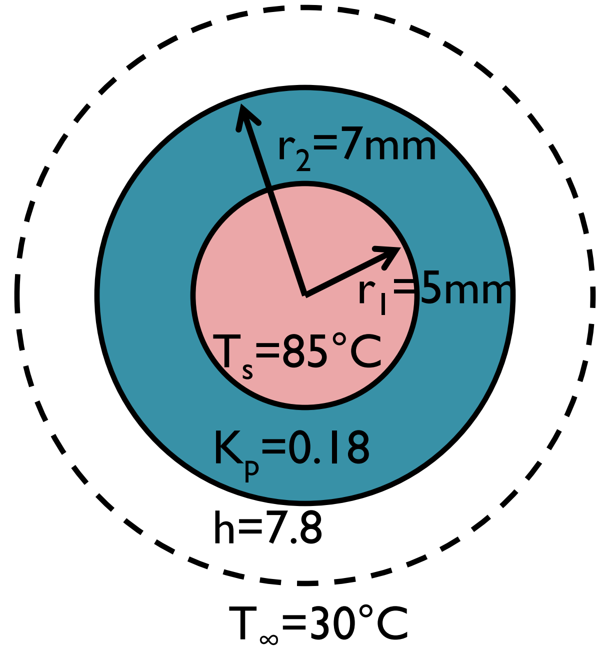
$$Q_{\max} = \frac{T_s - T_\infty}{\frac{\ln\frac{r_c}{r_1}}{2\pi k_p L} + \frac{1}{h 2\pi r_c L}} = \frac{85-30}{\frac{\ln\frac{0.023}{0.005}}{2x3.14x0.18x1} + \frac{1}{7.8x2x3.14x0.023x1}}$$

$$Q_{\max} = 24.56W$$

Solution:

$$Hence\ I_{max} = \sqrt{\frac{Q_{max}}{R}} = \sqrt{\frac{24.56}{8.92x10^{-3}}}$$

$$= 52.5 Amp$$

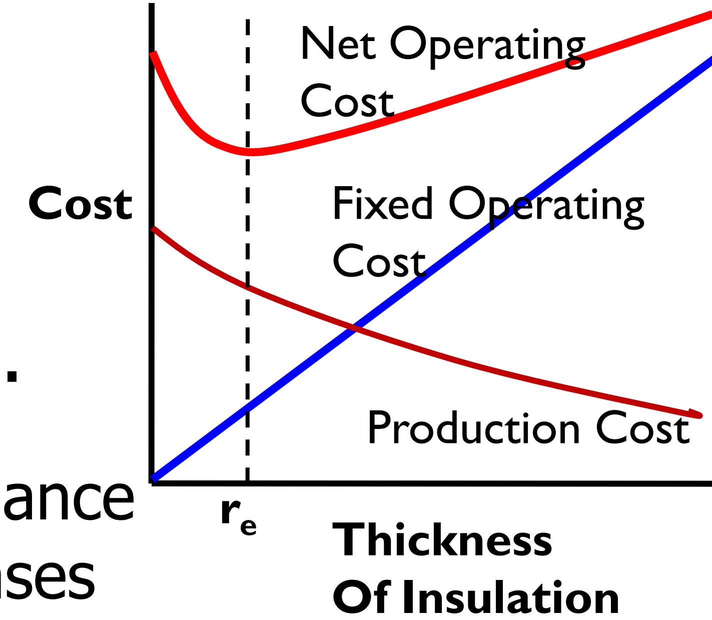


$$d)\ Percentage\ increase\ in$$
$$current\ carrying\ capacity$$

$$= \frac{I_{max} - I}{I} x100 = \frac{52.5 - 43.83}{43.83} x100 = 19.85\%$$

Economic Thickness of Insulation

- Concept based on economics
- As thickness of insulation increases, heat loss decreases, hence production cost decreases .
- However, depreciation & maintenance called fixed operating cost, increases



- Therefore, net operating cost, which is production cost plus fixed operating cost, initially decreases and then increases. The radius (thickness), at which net operating cost is minimum, is known as Economic Radius (Thickness) of Insulation (r_e).

Unsteady State Heat Conduction

- When heat transfer takes place from a body/ material, its temp changes. When temp of the body is function (fn) of location & time i,e. T(x,t), heat transfer process is called under Unsteady state conditions.

- When heat energy flows in or out of a body, its internal energy increases or decreases, which is indicated by increase or decrease in its temp. When temp of the body is a fn of time, heat nsfer process is known to be taking place under nsient conditions.

Unsteady State Heat Conduction

- Rate of heat transfer depends on temp gradient, and since temp changes with time, heat flow rate also changes with time continuously.

- Under transient conditions, characteristic equation for heat flow can be written as:

Rate of heat flow out Q=Rate of change of internal energy of the substance

$$= -\ mC_p\ dT/dt$$

- When heat flows out from a body, its surface temp changes. Thus a temp gradient is established from centre of the body to the surface.

Unsteady State Heat Conduction

- Generally, two types of problems are encountered:

1. When body has negligible internal temp gradient (ITG) <5%
2. When body has considerable ITG (>5%)

- To decide whether ITG is <5% (can be neglected)?
  Here, Biot No is defined. Bi No is measure of ITG

- If Biot No (Bi) is < 0.1; then ITG will be <5%

$$Bi = \frac{hL}{k} = \frac{hL.A}{k.A} = \frac{\frac{L}{kA}}{\frac{1}{hA}}$$

$$= \frac{Conductive\ Resistance\ of\ the\ object}{Convective\ Resistance\ at\ the\ surface\ of\ Object}$$

Unsteady State Heat Conduction

- Another Dimensionless Number utilized in transient heat transfer conditions is Fourier's No (Fo)

- Fo is dimensionless time, which is a measure of heat conduction compared to heat storage of a body

$$Fo = \frac{k}{\rho C_p} \frac{t}{L^2} = \frac{Heat\,Conduction}{Heat\,Storage\,with\,Time}$$

Where L is characteristic length of the object/body and given as:

V/A; where V is the volume of the body and A is the surface area of the body

Unsteady State Heat Conduction

Characteristic Lengths:

Sphere:

$$L = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3}; \quad R = Radius\ of\ Sphere$$

Cylinder

$$L = \frac{\pi R^2 L}{2\pi RL} = \frac{R}{2}; R = Radius\ of\ Cylinder$$

Cube

$$L = \frac{l^3}{6l^2} = \frac{l}{6}; \quad l = Length\ of\ Cube$$

ate

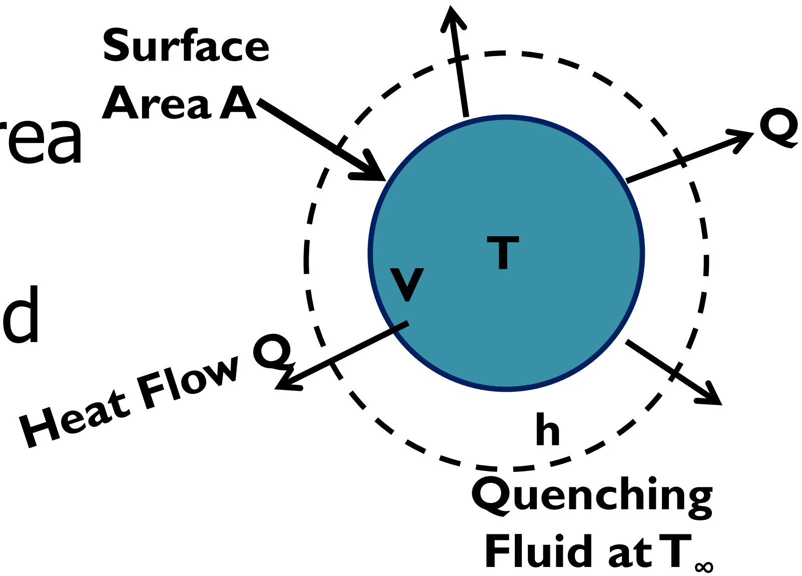
$$L = \frac{A\Delta x}{2A} = \frac{\Delta x}{2}; \quad \Delta x = Thickness\ of\ Plate$$

Practical Examples of Unsteady State Heat Transfer

1. Heat treatment of metals
2. Starting and shutting down of any Heat Transfer equipment like Lab Eqpt
3. Starting & shutting down of engines/motors
4. Starting & shutting down of Electric Furnace
5. Starting & shutting down of Electric heater

Quenching of Billet by Lumped Heat Capacity Method (For Heat Treatment)

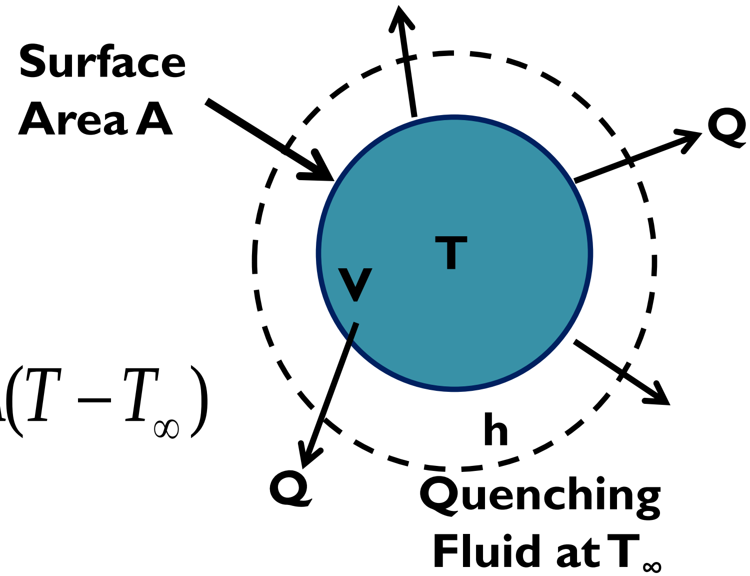
- Consider a solid of volume V and surface area A, initially at temp T_i , suddenly placed in a fluid at temp T_∞ ($T_i > T_\infty$)



- Lumped heat capacity of the solid will be $mC_p=\rho VC_p$.
 (Lump of heat energy is the heat required to raise/lower temp of mass m by 1°)

Quenching of Billet by Lumped Heat Capacity Method (For Heat Treatment)

- Heat flow from billet surface of area A at any time t can be given as:



$$Q = -mC_p \frac{dT}{dt} = -\rho V C_p \frac{dT}{dt} = hA(T - T_\infty)$$

- Putting $\theta = T - T_\infty$, the excess temp of solid above fluid, equation becomes:

$$\rho V C_p . \frac{d\theta}{dt} = hA\theta$$

Quenching of Billet

$$-\rho V C_p . \frac{d\theta}{dt} = hA\theta; \quad OR \quad \frac{d\theta}{\theta} = \frac{-hA}{\rho C_p V} . dt$$

$$Integrating, We\ have: \quad \ln\theta = -\frac{hA}{\rho C_p V} . t + C;$$

$$where\ \ C\ \ is\ \ Const\ \ of\ \ integration$$

$$Initial\ \ Conditions : At\ t = 0;\ \ T = T_i;$$

$$hence\ \theta = \theta_i = (T_i - T_\infty)$$

$$\therefore C = \ln\theta_i$$

Quenching of Billet by Lumped Heat Capacity Method (For Heat Treatment)

$$Hence\ \ln\theta = -\frac{hA}{\rho C_p V}.t + \ln\theta_i$$

$$\Rightarrow \ln\left(\frac{\theta}{\theta_i}\right) = -\frac{hA}{\rho C_p V}.t$$

$$\Rightarrow \frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = e^{-\frac{hA}{\rho C_p V}.t}$$

Quenching of Billet

$$Now\ \frac{hA}{\rho C_p V}.t = \frac{h}{\rho C_p L}.t \Rightarrow \left(\frac{hL}{k}\right)\left(\frac{k}{\rho C_p L^2}.t\right)$$

$$= \left(\frac{hL}{k}\right)\left(\frac{\alpha}{L^2}.t\right) = Bi.Fo;\ Hence \Rightarrow \frac{\theta}{\theta_i} = e^{-Bi.Fo}$$

$$For\ Plate\ of\ thickness\ \Delta x;\ L = \frac{\Delta x}{2}$$

$$\left(\frac{hL}{k}\right)\left(\frac{\alpha.t}{L^2}\right) = \left(\frac{h.\frac{\Delta x}{2}}{k}\right)\left(\frac{\alpha.t}{\left(\frac{\Delta x}{2}\right)^2}\right) = Bi.Fo$$

Instantaneous Rate of Heat Transfer

Instantaneous heat flow rate:

$$Q = h.A(T - T_\infty) \quad and \qquad \frac{T - T_\infty}{T_i - T_\infty} = e^{-\frac{hA}{\rho C_p V}.t}$$

$$Hence \quad Q = h.A\left[(T_i - T_\infty).e^{-\left(\frac{h.A}{\rho.C_p.V}\right).t}\right]$$

Quenching of Billet

1. For Plate $\Rightarrow$ $$\frac{\theta}{\theta_i} = e^{-Bi.Fo}$$

2. For Cylinder $\Rightarrow$ $$\frac{\theta}{\theta_i} = e^{-2BiFo}$$

3. For Sphere $\Rightarrow$ $$\frac{\theta}{\theta_i} = e^{-3BiFo}$$

For Cube of side $L \Rightarrow$ $$\frac{\theta}{\theta_i} = e^{-6BiFo}$$

Time Constant & Response of Thermocouple

- For measurement of temp of a fluid by thermocouple, thermocouple junction should attain fluid temp as quickly as possible.(Case of unsteady state heat transfer to thermocouple junction). If time taken to attain fluid temp is small, response of thermocouple is fast.

$$\frac{\theta}{\theta_i} = \frac{T_\infty - T}{T_\infty - T_i} = e^{-\frac{hA}{\rho C_p V}.t} \; : Here\ requirement\ is\ that$$

$$\theta = (T_\infty - T)\ should\ approach\ zero\ as\ quickly$$

$$as\ possible\ for\ fast\ response\ of\ Thermocouple.$$

Time Constant & Response of Thermocouple

$$If\ we\ define\ a\ term\ Time\ Constant\ as\ \tau = \frac{\rho C_p V}{hA};$$

$$Then \Rightarrow \frac{\theta}{\theta_i} = e^{-\frac{t}{\tau}} = \frac{1}{e^{\frac{t}{\tau}}}$$

$$For\ \theta \to 0; \quad \tau\ should\ be\ as\ small\ as\ possible$$

$$For\ convenience\ if\ we\ put\ \frac{t}{\tau} = \frac{hA.t}{\rho C_p V} = 1$$

$$Then \Rightarrow \frac{\theta}{\theta_i} = e^{-1} = 0.368;$$

ence $\theta = 0.368\theta_i$

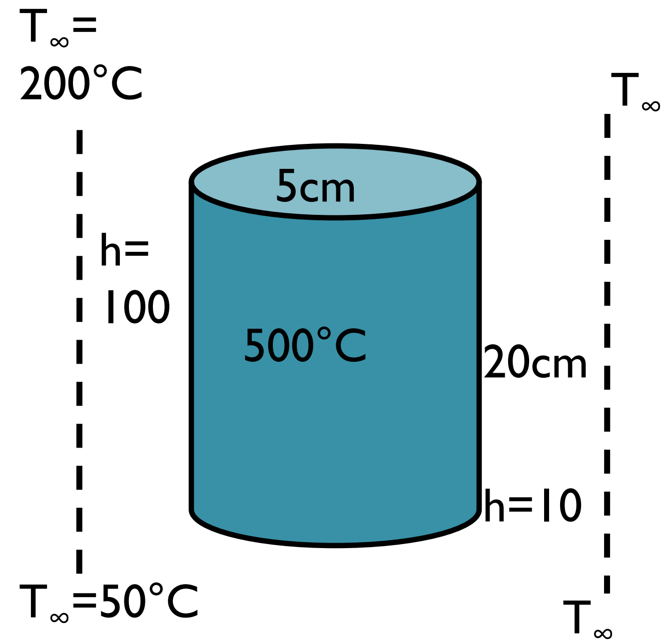
Time Constant of Thermocouple

$$Hence \quad \theta = 0.368\theta_i$$

- Therefore, time required by the thermocouple to achieve 63.2% of initial temp difference, is called Time Constant of Thermocouple

- Time Constant should be as small as possible for better response of thermocouple

Q4. A solid cylinder of steel of 5cm dia and 20cm length, initially at a uniform temp of 500°C is suddenly placed in a fluid at 200°C with h=100 W/m²K. After a period of 5 minutes, cylinder is taken out from this fluid and immediately immersed in another fluid at 50°C with h=10W/m²K.



Steel properties are: C_p=0.46kJ/kgK; ρ=7800kg/m³; K=35W/mK. Calculate the temp of cylinder when it was taken out from the first fluid and total time required for it to achieve the temp of 100°C.

er 5 min, temp of cylinder?

al time required for cylinder to achieve 100°C ?

ution: Check Biot No Bi=hL/k <0.1

Solution:

Stage I:

$$\frac{\theta}{\theta i} = e^{-\left(\frac{hA}{\rho C p V}\right)t} \qquad \& V = \pi R^2 L;\ A = 2\pi RL + 2\pi R^2$$

$$\theta = (500 - 200).e^{-\left(\frac{100x90}{7800x460}\right)300}$$

$$= 300.e^{-0.7525} = 141.35$$

$$\theta = T - 200 = 141.35 \Rightarrow T = 341.35^{o} C$$

Stage II: T_i=341.35°C;T=100°C; h=10

$$\frac{T - T_{\infty}}{T_i - T_{\infty}} = e^{-\left(\frac{h\,xA}{\rho C_p V}\right)t}$$

$$\frac{100 - 50}{1.35 - 50} = e^{-\left(\frac{10x90}{7800x460}\right)t} \Rightarrow t = 7028\sec$$

$$al\ Time = 7028 + 300 = 7328\sec = 2hr\ 2\min 8\sec$$

End of Unit - III

Forced Convection

Forced Convection

- Heat transfer between a fluid and solid surface is known as Convection and when the fluid is made to flow by external means like pump, fan, slope etc, the convection is called Forced Convection.

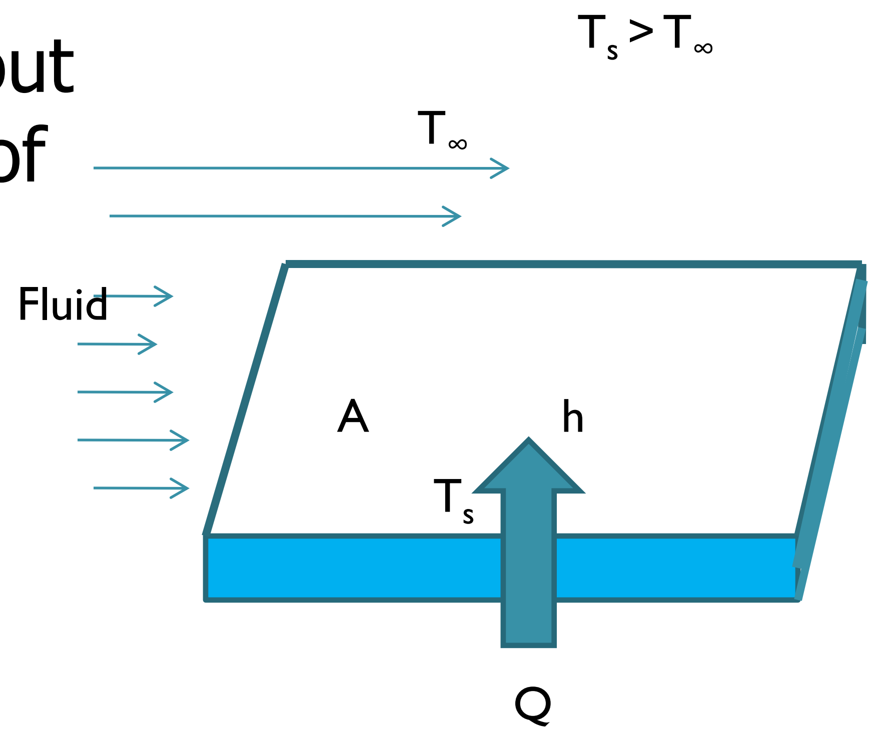
- Since energy transfer in convection takes place by movement of fluid molecules by picking up or giving out heat energy, parameters like nature of flow, fluid velocity, viscous forces, etc have significant effects on heat transfer process.

- Hence, knowledge of differential equations for fluid flow like Continuity Equation, Momentum Equation (Navier Stoke's Equations) is vital. Based on these equations, information on fluid flow through the pipe and over flat plate like velocity profile, pressure drop etc, have been worked out.

Convection

Heat Flow is found out from Newton's Law of Cooling as:

$Q = hA(T_s - T_\infty)$



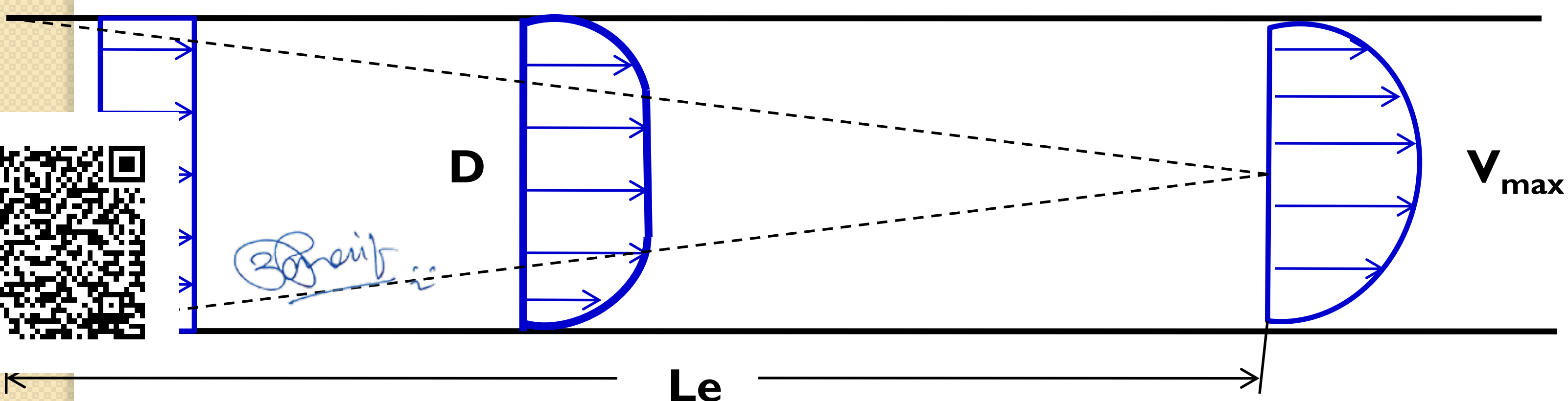
Here h is neither property of surface nor that of fluid but it is dependent on type of fluid flow, fluid properties, and vital dimension of surface pipe.

$$= f(\rho, V, D/L, \mu, C_p, k)$$

Fluid Flow Through Pipe

- At the entrance of the tube, all fluid layers will have same velocity. When flow progresses, fluid layer in contact with surface tends to become stationary due to friction between tube surface and this layer of fluid.

- Due to viscous forces, this stationary layer retards the velocity of second layer towards the centre. Second layer retards the velocity of third layer and so on.



Fluid Flow Through Pipe

- Velocity of layers is proportional to the distance from the tube surface.

- For Law of Conservation of Mass to hold good, since velo is almost zero at the surface, it has to increase towards the centre as mass flow rate remains same at all sections of pipe.

- After certain distance from entrance, velocity profile develops fully and becomes steady.

- Profile becomes parabolic and does not change there after till the time flow remains Laminar.

Fully Developed Laminar Flow Through Pipe (Re <2000)

Entrance Length:

$$\frac{Le}{D} = 0.0575\,\mathrm{Re}$$

Local Velocity:

$$\frac{V}{V_{\max}} = 1 - \left(\frac{r}{r_o}\right)^2;$$

$$where\ r_o\ is\ outer\ radius\ from\ centre$$

Average Velocity:

$$V_{av} = \frac{V_{\max}}{2}$$

iction Factor:

$$f = \frac{16}{\mathrm{Re}} = \frac{\Delta P}{4\left(\frac{L}{D}\right)\left(\frac{\rho V^2}{2}\right)}$$

Turbulent Flow Through Pipe (Re >4000)

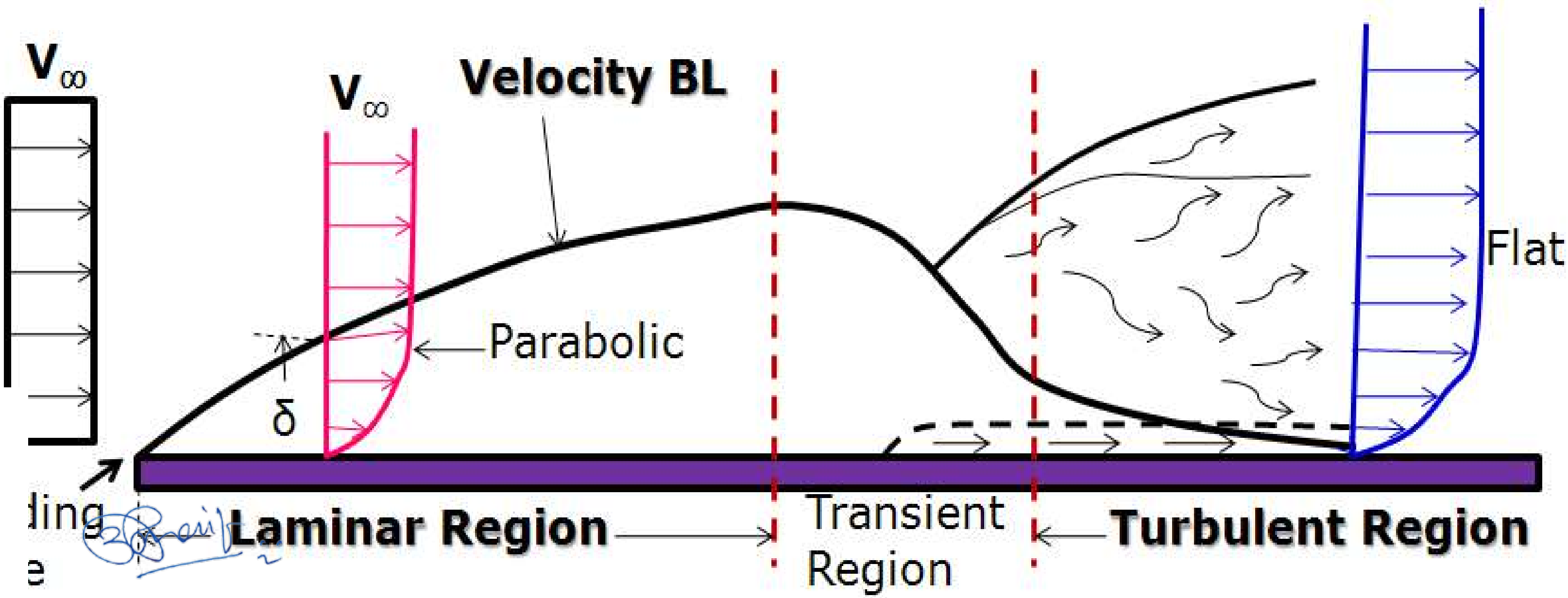
In turbulent flow, velocity profile quickly stabilizes due to large eddies formations. Hence entrance length is relatively smaller and velocity profile is flat in the core region of the pipe.

Friction Factor:

$$f = 0.046(\text{Re})^{-0.2} = \frac{\Delta P}{4\left(\frac{L}{D}\right)\left(\frac{\rho V^2}{2}\right)}$$

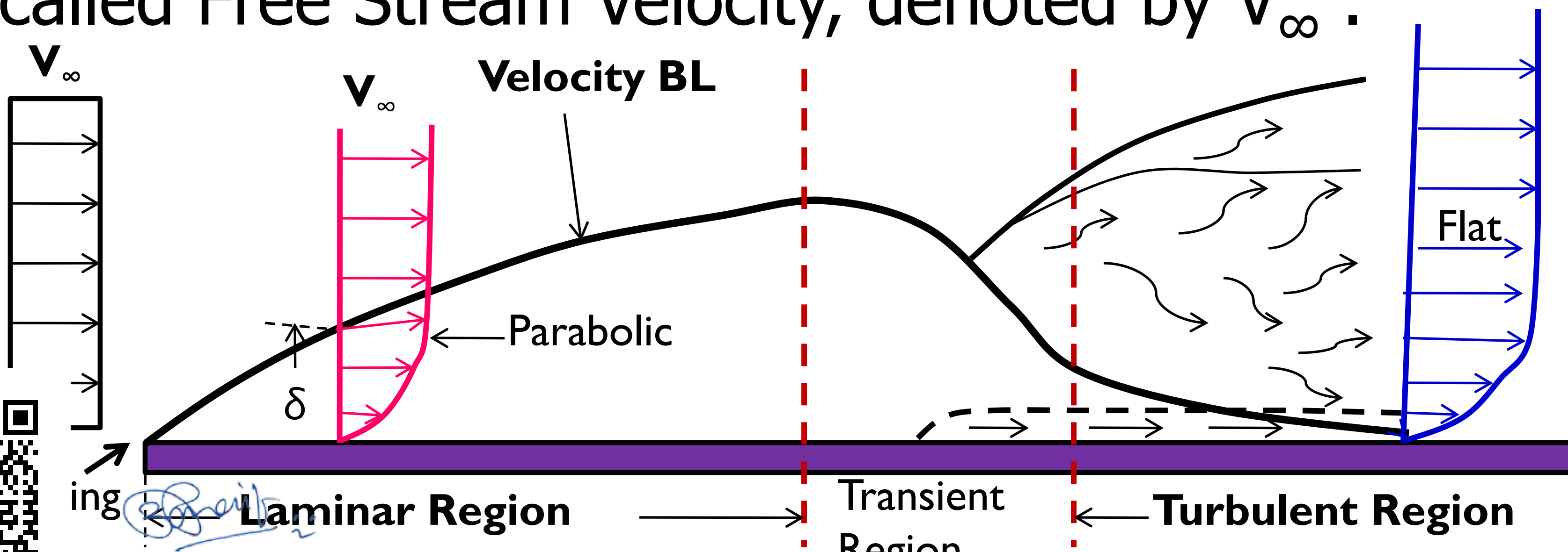
Flow Over Flat Plate : Velocity Boundary Layer

- When fluid flows over a flat plate, at the leading edge, all layers of fluid have same velocity.
- However, due to friction force, layer adjacent to plate comes to rest.



Flow Over Flat Plate : Velocity Boundary Layer

- Velocity of next layer is hence retarded by this stationary layer due to fluid viscous force.
- However, velocities of layers increase with distance from surface and beyond certain distance, it attains certain max steady value called Free Stream Velocity, denoted by V_∞ .



Flow Over Flat Plate : Velocity Boundary Layer

- The region normal to surface, in which velocity gradient exists, is known as Velocity BL / Hydrodynamic BL

- Thickness of Velo BL (δ) is defined as the distance normal to the surface, in which velo of layers varies from zero to 99% of the free stream velocity.

Fluid Flow in BL

Laminar BL
Transient BL
Turbulent BL

Laminar Flow Over Flat Plate ($Re<3x10^5$)

Drag Coeff (C_f):

$$C_{f\,av} = \frac{1.328}{\sqrt{R_{eL}}}$$

Drag Force (F_D):

$$F_D = C_f \cdot \frac{\rho A V^2}{2}$$

Thickness of BL (δ) at x from leading edge

$$\delta_x = \frac{4.64.x}{\sqrt{R_{ex}}}$$

Mass Flow Rate
ough BL at x

$$m_x = \frac{5}{8}\rho V \delta_x$$

Turbulent Flow Over Flat Plate ($Re > 5x10^5$)

Drag Coeff (C_f):

$$C_{f\,av} = \frac{0.455}{\ln\left(R_{eL}^{\;2.58}\right)} - \frac{C_1}{R_{eL}};$$

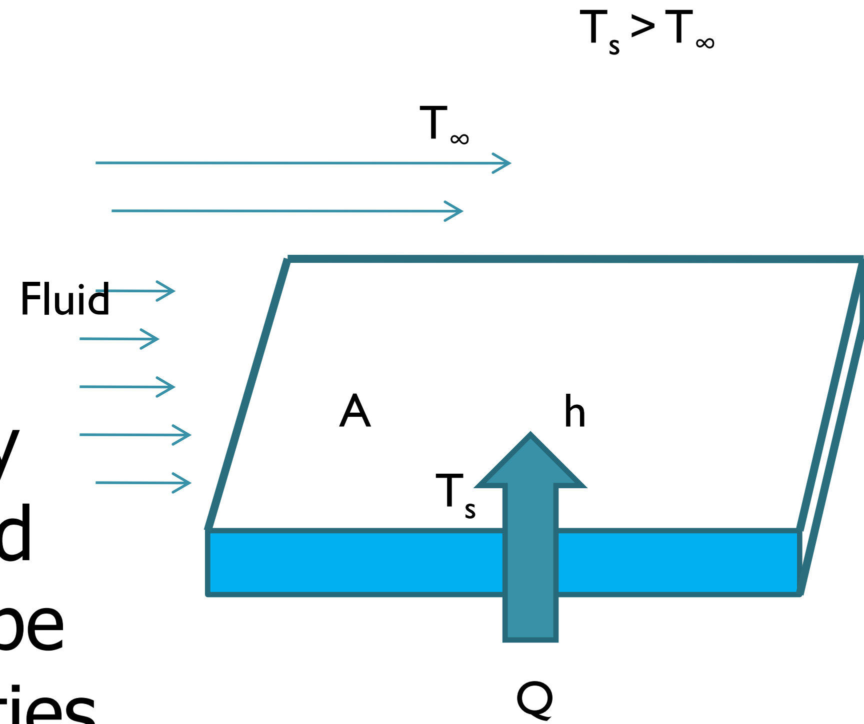
$$where\, C_1 = 1050$$

Thickness of BL (δ) at x from leading edge

$$\delta_x = \frac{0.39.x}{\mathrm{Re}_x^{\;0.2}}$$

Convection

Heat Flow is determined as: $Q = hA(T_s - T_\infty)$



Here h is neither property of surface nor that of fluid But it is dependent on type of fluid flow, fluid properties, and vital dimension of surface or pipe.

$$= f(\rho, V, D/L, \mu, C_p, k)$$

Convection

In practice, it is very difficult to estimate correct value of h and it becomes more complicated due to the fact that properties of all fluids vary with temp.

As the fluid flows over the surface, and if there is temp difference between fluid and surface, its temp in the BL changes. Accordingly, fluid properties change and hence h also changes. Thus on every loc on the surface along the fluid flow, we get different value of h. This value is known as Local Heat Transfer coefficient, denoted by h_x .

ence, average h is found out as: $h_{av} = \frac{1}{L}\int_0^L h_x .dx$

Values of h (W/m²K)

Free/Natural Convection with air:	5-15
Forced Convection with air :	10-500
Forced Convection with Water:	100-15000
Boiling of Water	1500-25000
Condensing Water Vapour	5000-100000

Forced Convection

- Since $h = f(\rho, V, D/L, \mu, C_p, k)$, it is very difficult to find relations of h because of large number of parameters involved.

- Such processes can be analyzed by Dimensional Analysis using Buckingham л Theorem.

- And we get the relations of the form :

$$\frac{hL}{k} = A\left(\frac{\rho VL}{\mu}\right)^a \left(\frac{\mu C_p}{k}\right)^b$$

- $Nu = A\ (Re)^a\ (Pr)^b$

Dimensional Analysis

- If large No of variables take part in a process, it is very difficult or almost impossible to study the effects of variation of one or more variables on others.

- By dimensional analysis, these variables can be grouped in to manageable No of groups, say four or three or less so that effect of variation of each on others can be studied.

Dimensional Analysis

Buckingham л Theorem

- This theorem is used as a thumb rule for determining number of independent dimensionless groups that can be obtained from a set of variables taking part in a process.

- This Theorem states that the number of independent dimensionless groups that can be formed from a set of n variables having basic/fundamental dimensions will be (n-r)

Dimensional Analysis For ‘h’

- From different experiments, it has been seen that h in forced convection depends on ρ, V, L, μ, C_p and k.

- Hence, we can write $h=f(\rho, V, L, \mu, C_p, k)$

Or $h=A(\rho^a, V^b, L^c, \mu^d, C_p{}^e, k^f)$

where A, a, b, c, d, e, f are constants

Dimensional Analysis For 'h'

Variables	Units	Dimensions
h	$W/m^2K=J/sm^2K=Nm/sm^2K$ $=kg.m.m/s^2.s.m^2K=kg/s^3K$	$M.T^{-1}.t^{-3}$
ρ	Kg/m^3	$M.L^{-3}$
V	m/s	Lt^{-1}
L	m	L
μ	Kg/m.s	$M.L^{-1}t^{-1}$
C_p	$J/kg.K=m^2/s^2K$	$L^2.T^{-1}.t^{-2}$
k	$W/mK=kg.m/s^3K$	$M.L.T^{-1}.t^{-3}$

Dimensional Analysis For 'h'

$\rightarrow MT^{-1}t^{-3}=A[(ML^{-3})^a(Lt^{-1})^b(L)^c(ML^{-1}t^{-1})^d(L^2T^{-1}t^{-2})^e(MLT^{-1}t^{-3})^f]$

Equating powers of:

M: $1=a+d+f$ ....(1)

L: $0=-3a+b+c-d+2e+f$ ....(2)

T: $-1=-e-f$ or $1=e+f$ ....(3)

$-3=-b-d-2e-3f$ ....(4)

Dimensional Analysis For 'h'

- Let us obtain values of all constants in terms of only 2 constants, say 'a' & 'e'

- Hence, we obtain the eqn as under:

  $h = A[\rho^a, V^a, L^{a-1}, \mu^{e-a}, C_p^{\,e}, k^{1-e}]$

$$\Rightarrow h = A\left[\left(\frac{\rho VL}{\mu}\right)^a \cdot \left(\frac{\mu Cp}{k}\right)^e \cdot \left(\frac{k}{L}\right)\right]$$

$$\frac{hL}{k} = A\left(\frac{\rho VL}{\mu}\right)^a \cdot \left(\frac{\mu Cp}{k}\right)^e \Rightarrow Nu = A.\mathrm{Re}^a . \mathrm{Pr}^e$$

Physical Significance of Dimensionless Parameters

Nusselt Number (Nu):

$$Nu = \frac{hL}{k} = \frac{hD}{k}$$

where L / D are characteristic length

$$= \frac{hL}{k} \cdot \frac{A\Delta T}{A\Delta T} = \frac{hA\Delta T}{\frac{kA\Delta T}{L}}$$

$$= \frac{Heat\ Transfer\ by\ Convection}{Heat\ Transfer\ by\ Conduction}$$

h can be found out from here

Prandtl Number:

$$\Pr = \frac{\mu C_p}{k} = \frac{\frac{\mu}{\rho}}{\frac{k}{\rho C_p}} = \frac{\nu}{\alpha} = \frac{Kinematic\ Viscosity}{Thermal\ Diffusivity}$$

$$= \frac{Diffusion\ of\ Momentum\ through\ Fluid}{Diffusion\ of\ Heat\ through\ Fluid}$$

High Pr No means higher Nu and hence higher h; higher heat transfer

No is the property of fluid as μ, C_p ,k are all properties of d and these are temp dependent

Prandtl Number:

For Liquid Metals: $Pr < 0.01$

For Air: $Pr \approx 1$

For Water: $Pr \approx 10$

r Heavy Oils: $Pr > 1$ lac

Reynold's No (Re):

$$Re = \frac{\rho VL}{\mu} = \frac{\rho VD}{\mu} = \frac{VL}{\nu} = \frac{VD}{\nu} \left(= \frac{4m}{\mu P} \right)$$

$$Re = \frac{\rho VL.V}{\mu.V} = \frac{\rho V^{2}}{\frac{\mu V}{L}}$$

$$= \frac{Inertia\ Force}{Viscous\ Force}$$

Péclet No (Pe):

$$Pe = \mathrm{Re}\,.\,\mathrm{Pr} = \frac{\rho V L}{\mu}\,.\,\frac{\mu C_p}{k} = \frac{\rho V C_p}{\dfrac{k}{L}}$$

$$= \frac{Mass\,Heat\,Flow\,Rate}{Heat\,Flow\,by\,Conduction\,per\,Unit\,Temp\,Diff}$$

When Pr is very small (of the order of 0.01), like for liquid metals, then as a practice, governing equation $Nu=A(Re)^a(Pr)^b$ is used as:

$$Nu=C(Pe)^n$$

s is only for convenience

Stanton No (St):

$$St = \frac{Nu}{\mathrm{Re}.\mathrm{Pr}} = \frac{hL}{\frac{k.\rho VL}{\mu}.\frac{\mu C_p}{k}} = \frac{h}{\rho V C_p}$$

$$= \frac{Heat\,Flux\,in\,Convection\,per\,Unit\,Temp\,Diff}{Mass\,Heat\,Flow\,Rate}$$

In such cases, governing equation is used as:

$$St^n = C \; or \left(\frac{Nu}{\mathrm{Re}.\mathrm{Pr}}\right)^n = C$$

Reynold's Numbers

Flow through conduit/pipe

Laminar Flow : $Re < 2000$

Turbulent Flow : $Re > 4000$

Flow over flat plate/surface

Laminar Flow : $Re < 3 \times 10^5$

rbulent Flow : $Re > 5 \times 10^5$

Correlations : Flow Through Pipe

For Laminar Flow (Re<2000)

Nu = 4.36 for const heat flux

Nu = 3.66 for const wall temp

Correlations : Flow Through Pipe

For Turbulent Flow (Re>4000)

$Nu = 0.023\, Re^{0.8}\, Pr^{0.4}$ for heating of fluid

$Nu = 0.023\, Re^{0.8}\, Pr^{0.3}$ for cooling of fluid

Above Equations are known as Dittus-Boelter Correlations

l properties of fluid are to be taken at
lk Mean Temp

Hydraulic Diameter:

Characteristic Length for flow through pipe or conduit of different cross sections is taken as its hydraulic diameter (D_h), which is defined as:

$$D_h = \frac{4\,x\,Cross\,Sectional\;\;Area\,of\;\;Flow}{Wetted\;\;Perimeter} = \frac{4A}{P}$$

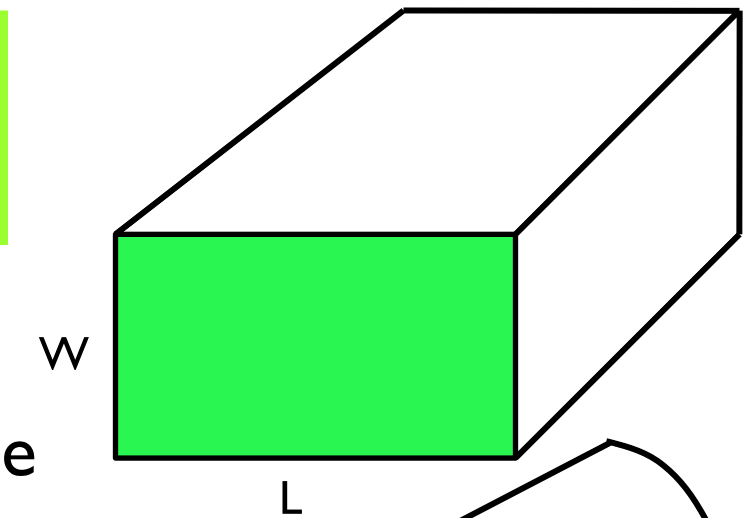
For circular tube of dia D:

$$D_h = \frac{4A}{P} = \frac{4.\dfrac{\pi}{4}D^2}{\pi D} = D$$

Hydraulic Diameter:

For rectangular cross section conduit

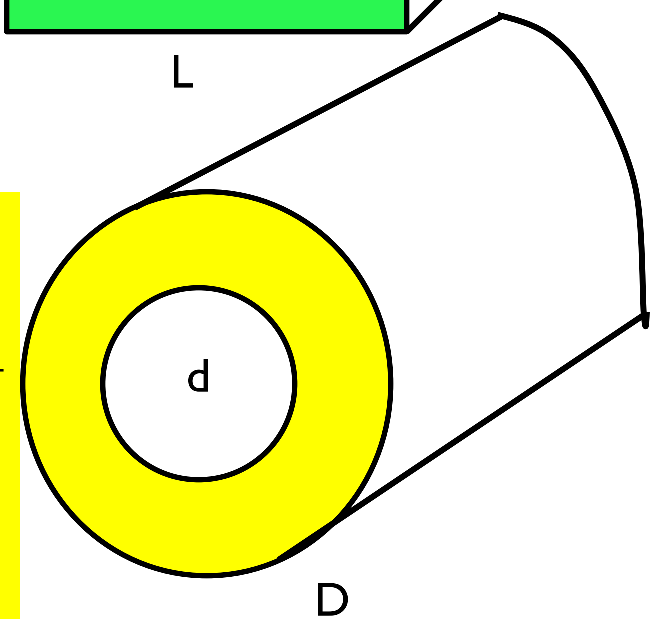
$$D_h = \frac{4A}{P} = \frac{4.LW}{2(L+W)}$$



For flow through annular space of outer dia D and inner dia d

$$D_h = \frac{4A}{P} = \frac{4.\left[\left(\frac{\pi}{4}D^2\right)-\left(\frac{\pi}{4}d^2\right)\right]}{\pi D + \pi d}$$

$$\frac{\pi(D^2-d^2)}{\pi(D+d)} = D-d$$



Flow of Liquid Metals Through Pipe (Low Pr)

$$Nu = 5+0.025(Re.Pr)^{0.8}$$ for const wall temp

$$Nu = 4.82+0.0185(Pe)^{0.827}$$ for const heat flux

Flow of Heavy Oil Through Pipe(High Pr)

$$\ldots u=0.027Re^{0.8}Pr^{0.33}(\mu/\mu_w)^{0.14}$$

…eder & Tate Relation)

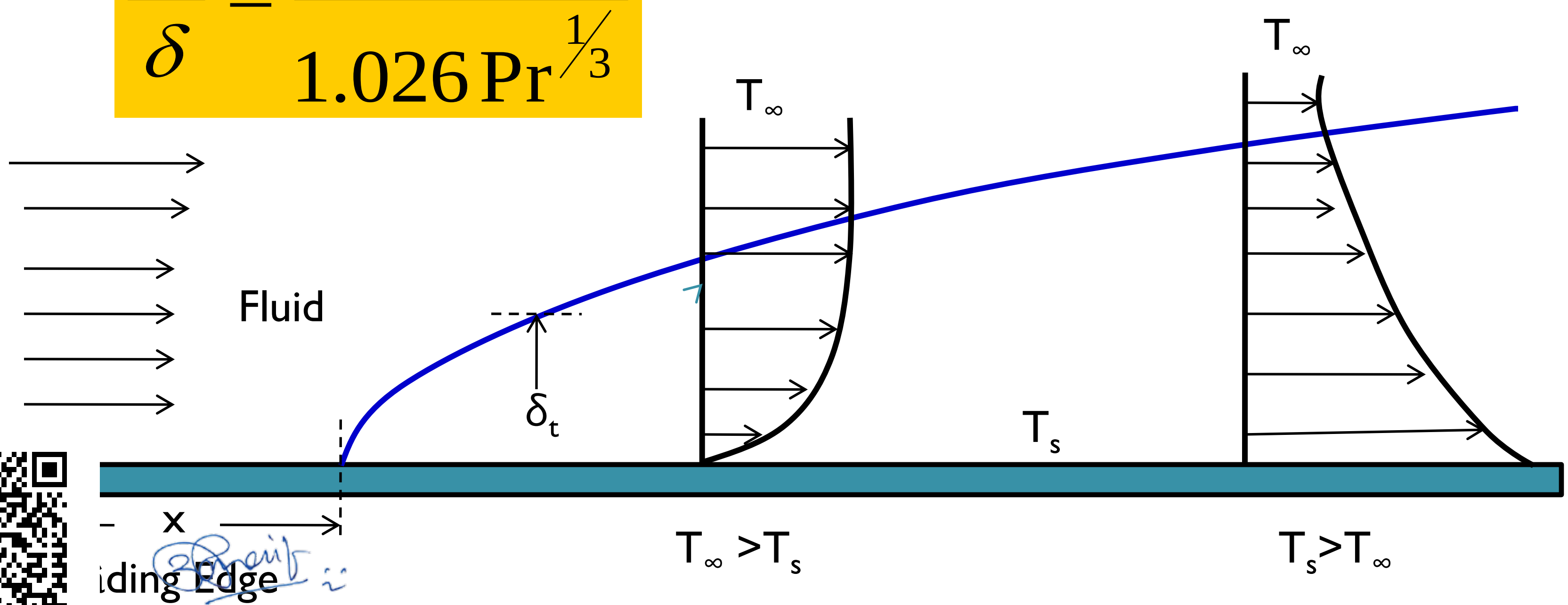
Flow Over Flat Plate

Thermal Boundary Layer

Thermal Boundary layer is the thin region over the surface, in which temp gradient exist.

Thickness of Thermal BL is found out as:

$$\frac{\delta_t}{\delta} = \frac{1}{1.026\,Pr^{1/3}}$$



Laminar Flow Over Flat Plate

Local Nusselt No (at distance x from leading edge)

$$Nu_x = 0.332\,\mathrm{Re}_x^{1/2}.\mathrm{Pr}^{1/3} \text{ from dimensional analysis}$$

To find Nu_{av} *: We have*

$$Nu_x = \frac{h_x.x}{K} = 0.332\,\mathrm{Re}_x^{1/2}.\mathrm{Pr}^{1/3}$$

$$\text{or } h_x = 0.332\frac{K}{x}\left(\frac{Vx}{\nu}\right)^{1/2}\mathrm{Pr}^{1/3}$$

$$h_x = 0.332.K.\left(\frac{V}{\nu}\right)^{1/2}\mathrm{Pr}^{1/3}.x^{-1/2}$$

Laminar Flow Over Flat Plate

$$h_{av} = \frac{1}{L}\int_0^L h_x\,dx = \frac{1}{L}\int_0^L \left[0.332K\left(\frac{V}{\nu}\right)^{1/2} \Pr^{1/3} .x^{-1/2} \right] dx$$

$$= \frac{1}{L}\left[0.332.K\left(\frac{V}{\nu}\right)^{1/2} \Pr^{1/3} \frac{x^{1/2}}{1/2} \right]_0^L$$

$$= 0.332.\frac{K}{L}\left(\frac{VL}{\nu}\right)^{1/2} \Pr^{1/3} .2 = 2h_L$$

$$\frac{h_{av}.L}{K} = Nu_{av} = 0.664\,\mathrm{Re}^{\frac{1}{2}}.\Pr^{\frac{1}{3}}$$

Turbulent Flow Over Flat Plate

$Nu_x = 0.029\ Re_x^{0.8} . Pr^{0.334}$

$Nu = 0.0366\ Re^{0.8} . Pr^{0.334}$

Characteristic Length is the plate length (L) in the direction of fluid flow

All the fluid properties to be taken at mean film temp $T_{mean} = (T_s + T_\infty)/2$

Flow Across Horizontal Cylinder

$Nu_D = C\,(Re_D)^n$ for const heat flux

Hilpert's Relations

Re_D	C	n
40-4000	0.615	0.466
4000-40000	0.174	0.618

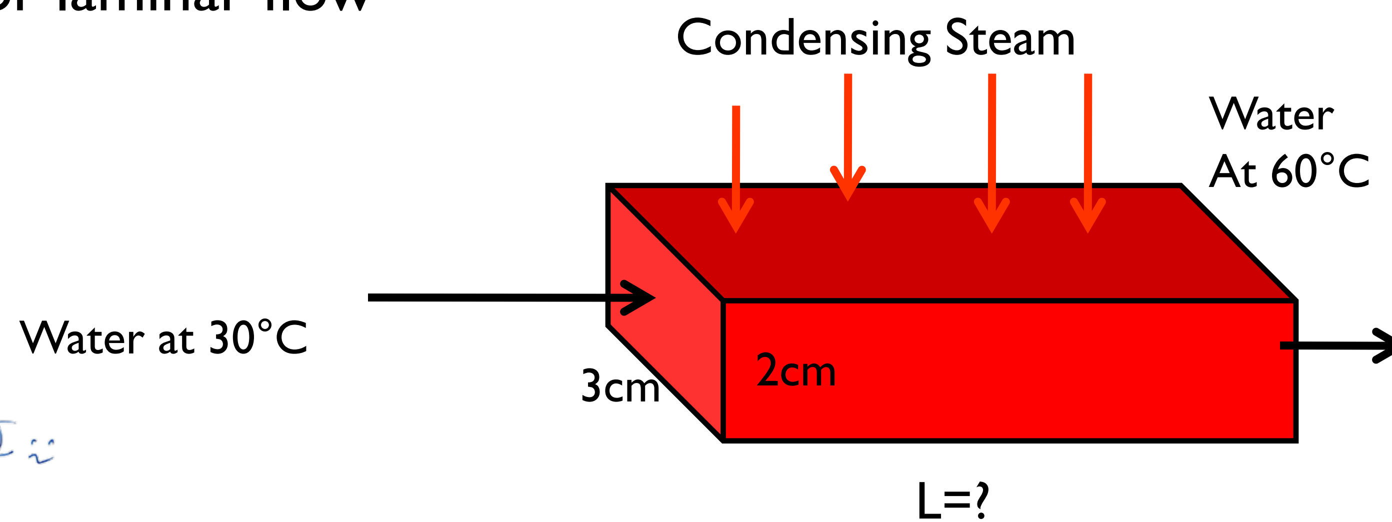
Q1: 65 kg/min of water is heated from 30°C to 60°C by passing it through a rectangular duct of 3cm x 2cm. The duct is heated by condensing the steam on its outer surface. Find the length of the duct required.

Properties of Water: ρ=995kg/m^3; μ=7.65x10^{-4}kg/ms; C_p=4.174kJ/kgK; k=0.623W/mK; Conductivity of the Duct material=35W/mK

Use the following correlations:

$Nu=0.023Re^{0.8}Pr^{0.4}$ for turbulent flow

$Nu=4.36$ for laminar flow



Solution:

We know that $Q = h\,A\,\Delta T = m\,C_p(T_e - T_i)$

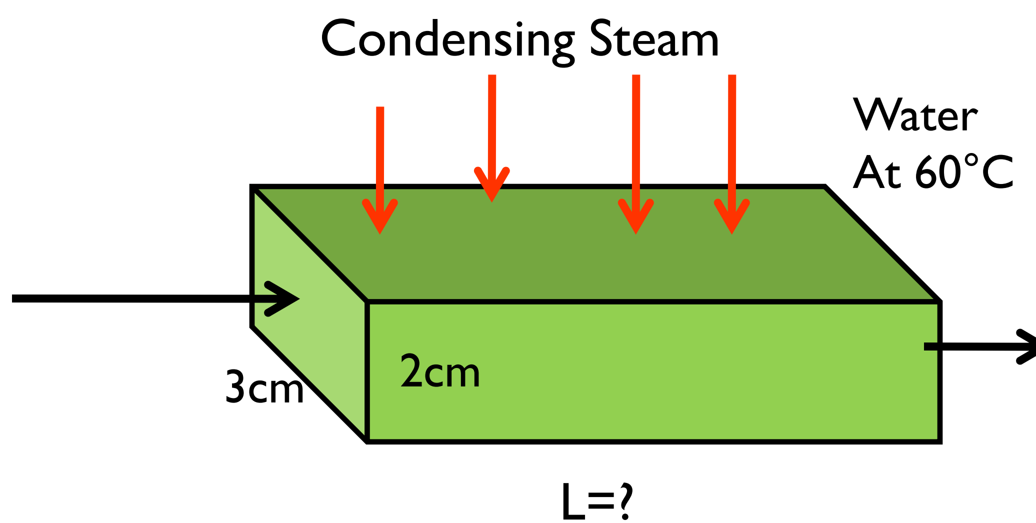
So Q=h (0.03+0.02)*2*L*[100-(30+60)/2]
=65/60[4174*(60-30)]

Hence L can be determined, provided h is known.

To determine h, we can use Nu relation, if we can Know which one to be used.

To find that, we should know whether flow is Laminar Or Turbulent

For that,
Re to be found out.



Solution (Contd):

$$Re = \frac{\rho VD}{\mu}; \text{ and we have to find } V \text{ from } m = \rho AV$$

$$\text{and } D \text{ from } D_h = \frac{4A}{P} \text{ as conduit is NOT circular}$$

$$D_h = \frac{4A}{P} = \frac{4*0.03*0.02}{(0.03+0.02)*2} = 0.024$$

$$m = \rho AV \Rightarrow V = \frac{65}{60*995*0.03*0.02} = 1.81 m/s$$

$$Re = \frac{\rho VD_h}{\mu} = \frac{995*1.81*0.024}{7.65x10^{-4}} = 5.65x10^4$$

ıce $Re = 5.65x10^4 \succ 4000$ *Flow is Turbulent*

Solution (Contd):

$$Hence\ \ we\,have\,to\,use\ Nu = 0.023\,\mathrm{Re}^{0.8}\ \mathrm{Pr}^{0.4}$$

$$\mathrm{Pr} = \frac{\mu C_p}{k} = \frac{7.65x10^{-4}\,x4174}{0.623} = 5.125$$

$$Nu = \frac{hD_h}{k} = 0.023(5.65x10^{4})^{0.8}(5.125)^{0.4}$$

$$\therefore h = \frac{0.623}{0.024}x0.023x6333.43x1.923 = 7271.48W/m^2K$$

$$7271.48(0.03+0.02)x2xL(100-45) = \frac{65}{60}x4174x(60-30)$$

$$L = 3.38m\ Answer$$

Q2. Air at 20°C is flowing along a heated plate at 134°C with a velocity of 3m/s. The plate is 2m long. Heat transferred from first 40cm from the leading edge is 1.45kW. Determine the width of the plate.

Properties of air at 77°C: ρ=0.998kg/m^3; ν=20.76x10^{-6} m^2/s; C_p=1.009kJ/kgK; k=0.03W/mK.

Use the following correlation:

$Nu_x=0.332\ Re^{0.5}\ Pr^{0.33}$

Solution:

(LINE OF APPROACH)

To determine width of the plate, we should find out area A transferring heat, since A=Width x Length (Length is given as 0.4m)

Area can be found out from Q=h A ΔT

Since Q & ΔT are known, we should find out h, which can be found out from given Nu_x relation.

Solution (Contd):

$$Re_{0.4} = \frac{VL}{\nu} = \frac{3x0.4}{20.76x10^{-6}} = 0.57803x10^{5}$$

$$\Pr = \frac{\mu C_p}{k}; \; Since \; \frac{\mu}{\rho} = \nu \Rightarrow \mu = \rho \nu$$

$$Hence \; \Pr = \frac{\rho \nu C_p}{k}$$

$$\frac{0.998x20.76x10^{-6}x1009}{0.03} = 0.697$$

Solution (Contd):

$$N_{uL} = \frac{h_L . L}{k} = 0.332(57803)^{0.5}(0.697)^{0.33}$$

$$h_L = \frac{0.03}{0.4} x 0.332 x 240.4 x 0.887 = 5.313 W/m^2 K$$

We know that $h_{av}=2h_L$ =2 x 5.313 =10.626

Hence Q=h A ΔT

=10.626x0.4xWx(134-20) =1450 (given)

erefore, width W=2.99m **Answer**

Free / Natural Convection

Natural Convection

- When a fluid comes in contact with a hot surface, its molecules in the immediate vicinity receive heat from hot surface.
- Due to this, temp of molecules rise and their volume increases.
- Therefore fluid molecules become lighter and start rising.
- Their places are taken by heavier molecules, which also rise in similar way on taking energy from hot surface.

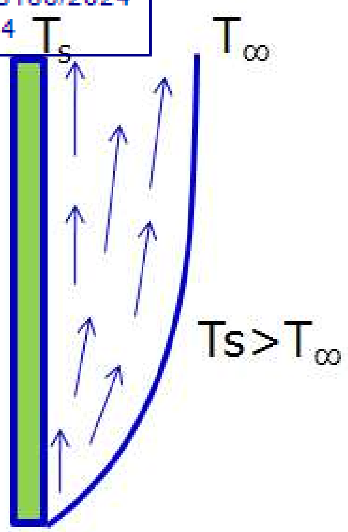
Natural Convection

- This way, natural motion in fluid molecules is set-in.

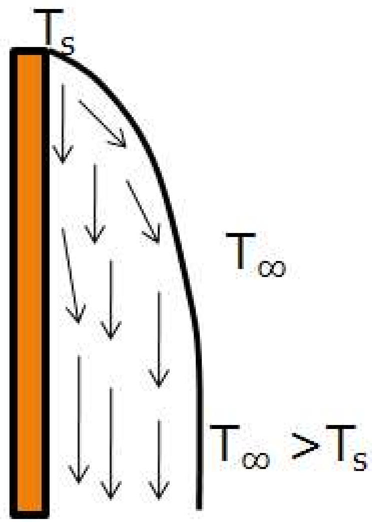
- Transfer of heat from solid surface to fluid in this manner is called Free/Natural Convection.

- When surrounding fluid is hotter than surface, heat transfer will be from fluid to surface.

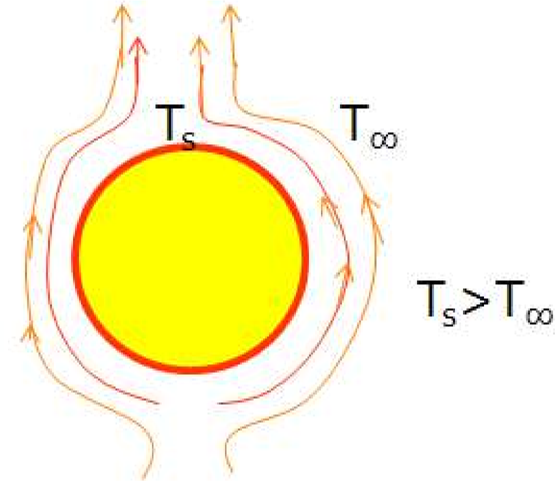
Natural Fluid Motion from Standard Surfaces



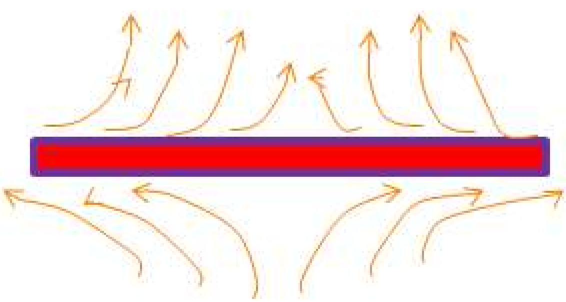
Vertical Hotter Plate



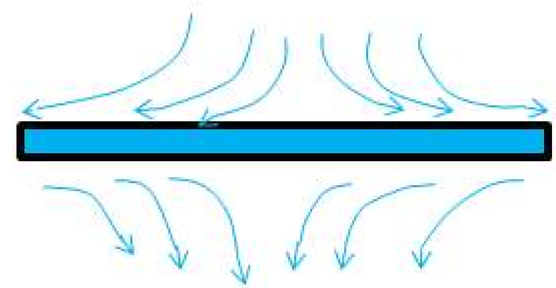
Vertical Colder Plate



Horizontal Hotter Cylinder



izontal Hotter Plate



Horizontal Colder Plate

Governing Equation In Natural Convection

In Natural Convection, $h=f(\rho, g, \beta, \Delta T, L, \mu, C_p, k)$

From dimensional analysis, we get the relation of following form:

$$\frac{hL}{k} = C\left[\left(\frac{\rho^2 . g . \beta . \Delta T . L^3}{\mu^2}\right)^a \left(\frac{\mu C_p}{k}\right)^b\right]$$

$$Nu = C(Gr)^a (\Pr)^b \quad or$$

$$Nu = C(Gr.\Pr)^n$$

This is the Governing Equation for Natural Convection

Physical Significance of Grashof No (Gr)

$$Gr = \frac{g.\beta.\Delta T.L^3.\rho^2}{\mu^2} = \frac{g.\beta.\Delta T.L^3}{\nu^2}$$

$$Rearranging\ terms, we\ get;$$

$$Gr = \frac{\left(\rho g \beta \Delta T L^3\right)\left(\rho V^2\right)}{\left(\mu V\right)^2}$$

$$Gr = \frac{Buoyancy\ Force\ x\ Inertia\ Force}{\left(Viscous\ Force\right)^2}$$

Grashof No is the ratio of product of Buoyancy
rce and Inertia Force to square of Viscous
rce acting on fluid.

Correlations:Natural Convection

Vertical Plate & Cylinder

$Nu=0.56(Gr_L.Pr)^{1/4}$ for $10^4 < Gr.Pr < 10^8$

$=0.13(Gr_L .Pr)^{1/3}$ for $10^8 < Gr.Pr < 10^{12}$

Horizontal Cylinder

$Nu=0.53(Gr_D.Pr)^{1/4}$ for $10^4 < Gr.Pr < 10^8$

$=0.13(Gr_D.Pr)^{1/3}$ for $10^8 < Gr.Pr < 10^{12}$

Correlations:Natural Convection

From Upper Surface of Square/Circular Plates

$Nu = 0.54(Gr.Pr)^{1/4}$ for $10^5 < Gr.Pr < 2 \times 10^7$

$= 0.14(Gr.Pr)^{1/3}$ for $2 \times 10^7 < Gr.Pr < 2 \times 10^{10}$

From Lower Surface of Square/Circular Plates

$Nu = 0.27(Gr.Pr)^{1/4}$ for $3 \times 10^5 < Gr.Pr < 3 \times 10^{16}$

Notes:-

1. Characteristic Length L=A/P
2. $\beta = 1/T_{mean}$ in Kelvin
3. All properties of fluid to be taken at $T_{mean} = (T_{surface} + T_{fluid})/2$

Summary : Dimensionless Numbers

Conduction :

$$1.\quad B_i = \frac{hL}{k} \qquad 2.\quad F_o = \frac{\alpha . t}{L^2}$$

Forced Convection:

$$3.\ Nu = \frac{hL}{k} \qquad 4.\ \mathrm{Re} = \frac{\rho VL}{\mu} \qquad 5.\ \mathrm{Pr} = \frac{\mu C_p}{k} \qquad 6.\ Pe = \mathrm{Re}.\mathrm{Pr} \qquad 7.\ St = \frac{Nu}{\mathrm{Re}.\mathrm{Pr}}$$

Natural Convection:

$$8.\ Gr = \frac{g\beta\Delta TL^3}{\nu^2} \qquad 9.\ Ra = Gr.\mathrm{Pr} \qquad Nu = \frac{hL}{k};\ \mathrm{Pr} = \frac{\mu C_p}{k}$$

Mixed Convection: (0.3m/s≤ V ≤30m/s)

$$Graetz\ No \quad Gz = (Gr.\mathrm{Pr})\frac{d}{L}$$

Q3. A circular disc insulated from other side of dia of 25cm is exposed to air at 20°C. If the disc (Open Surface) is maintained at 120°C, estimate heat transfer rate from it, when;

a) Disc is kept horizontal with (open) hot surface facing upwards

b) Disc is kept horizontal with (open) hot surface facing downwards

c) Disc is kept vertical

For air at 70°C, $k=0.03$; $Pr=0.697$; $\nu=2.076\text{x}10^{-6}$

Use the following correlations:

$Nu=0.14(Gr.Pr)^{0.334}$ for upward/top surface

$=0.27(Gr.Pr)^{0.25}$ for downward/bottom surface

$=0.59(Gr.Pr)^{0.25}$ for vertical surface

Solution: Horizontal Plate-Convection from Top Surface

Heat Flow Rate Q=h.A.ΔT; h=? Nu=hL/k

Nu=0.14(Gr.Pr)$^{0.334}$

$$Gr = \frac{g\beta \Delta T L^3}{\nu^2}$$

$$\beta = \frac{1}{T_{mean}(K)} \qquad \Rightarrow \beta = \frac{1}{273+70} = 0.0029$$

$$= \frac{A}{P} \Rightarrow L = \frac{\pi/4\, D^2}{\pi D} = \frac{D}{4} = \frac{0.25}{4} = 0.0625$$

Solution: Horizontal Plate-Convection from Top Surface

$$Gr = \frac{9.81x1x(120-20)(0.0625)^3}{(273+70).(2.076x10^{-6})^2} = 1.62x10^8$$

$$Nu = 0.14(1.62x10^8\,x0.697)^{1/3} = 68.51$$

$$= \frac{hL}{k} = \frac{h.x0.0625}{0.03}$$

$$\Rightarrow h = 32.88W/m^2K$$

$$= hA\Delta T = 32.88x\frac{\pi}{4}(0.25)^2(120-20) = 161W$$

Solution: Horizontal Plate Convection from Lower Surface

Heat Flow Rate Q=h.A.ΔT; h=? Nu=hL/k

Nu=0.27(Gr.Pr)$^{0.25}$

$$Gr = \frac{g\beta \Delta T L^3}{\nu^2}$$

$$\beta = \frac{1}{T_{mean}(K)} \qquad \Rightarrow \beta = \frac{1}{273+70} = 0.0029$$

$$L = \frac{A}{P} \Rightarrow L = \frac{\pi/4 \, D^2}{\pi D} = \frac{D}{4} = \frac{0.25}{4} = 0.0625$$

Solution: Horizontal Plate-Convection from Lower Face

$$Gr = \frac{9.81x1x(120-20)(0.0625)^3}{(273+70).(2.076x10^{-6})^2} = 1.62x10^8$$

$$Nu = 0.27(1.62x10^8\,x0.697)^{0.25} = 27.83$$

$$\Rightarrow \frac{hL}{k} = \frac{h.0.0625}{0.03} = 27.83$$

$$\Rightarrow h = 13.36W/m^2K$$

$$= hA\Delta T = 13.36\frac{\pi}{4}(0.25)^2(120-20) = 65.6W$$

Solution:Vertical Plate

Heat Flow Rate Q=h.A.ΔT; h=? Nu=hL/k

$Nu=0.59(Gr.Pr)^{0.25}$

$$Gr = \frac{g\beta \Delta T L^3}{\nu^2}$$

$$\beta = \frac{1}{T_{mean}} \Rightarrow \beta = \frac{1}{273+70} = 0.0029$$

$$L = D = 0.25$$

Solution: Vertical Plate

$$Gr = \frac{9.81 x 1 x (120 - 20)(0.25)^3}{(273 + 70).(2.076 x 10^{-6})^2} = 103.6 x 10^8$$

$$Nu = 0.59(103.6 x 10^8 \, x 0.697)^{0.25} = 172$$

$$\Rightarrow \frac{hL}{k} = \frac{h.0.25}{0.03} = 172$$

$$\Rightarrow h = 20.64 W / m^2 K$$

$$= hA\Delta T = 20.64 \frac{\pi}{4}(0.25)^2 (120 - 20) = 101.3 W$$

Q4. A hot rectangular plate 5cm X 3cm maintained at 200°C is exposed to still air at 30°C. Calculate percentage increase in convective heat transfer rate if smaller side of the plate is held vertical than the bigger side. Neglect ITG of the thickness.

Use Correlation $Nu=0.59(Gr.Pr)^{0.25}$

Air properties at 115°C: density=0.91kg/m^3; C_p=1.009kJ/kgK; $\mu=22.65\times10^{-6}$; k=0.0331

Solution: Bigger Side (L=5cm) Vertical

$$Gr = \frac{\rho^2 g\beta\Delta T L^3}{\mu^2} = \frac{0.91^2 x9.81x1x(200-30)(0.05)^3}{(115+273)(22.65x10^{-6})^2}$$

$$= 8.67x10^5$$

$$\Pr = \frac{\mu C_p}{k} = \frac{22.65x10^{-6}x1009}{0.0331} = 0.69$$

$$Nu = 0.59(Gr.\Pr)^{0.25}$$

$$0.59(8.67x10^5 x0.69)^{0.25} = 16.41$$

Solution: Bigger Side (L=5cm) Vertical

$$Nu = \frac{h_L . L}{k} = 16.41$$

$$\Rightarrow h_L = 16.41x\frac{0.0331}{0.05} = 10.86W/m^2K$$

$$Q = hA\Delta T$$

$$= 10.86x0.05x0.03x2(200-30) = 5.54W$$

Solution: Smaller Side (L=3cm) Vertical

Since Characteristic length has changed, Grashof No will change, hence

$$Gr = \frac{\rho^2 g\beta\Delta TL^3}{\mu^2} = \frac{0.91^2 x9.81x1x(200-30)(0.03)^3}{(115+273)(22.65x10^{-6})^2} = 1.87x10^5$$

$$Nu = 0.59(Gr.\Pr)^{0.25} = 0.59(1.87x10^5 x0.69)^{0.25} = 11.18$$

$$Nu = \frac{h_s.L}{k} = 11.18$$

$$h_s = 11.18x\frac{0.0331}{0.03} = 12.33W/m^2K$$

Solution: Smaller Side (L=3cm) Vertical

$$Q = h_s A \Delta T$$

$$= 12.33 x 0.05 x 0.03 x 2 (200 - 30)$$

$$= 6.288 W$$

Increase in Heat Transfer Rate

$$Q = \frac{6.288 - 5.54}{5.54} x 100 = 13.5\%$$

Q5: A solid cylinder of steel (density=8000 Kg/m^3, C_p=0.42kJ/kgK) of 12cm dia and 30cm length at 380°C is suspended vertically in a large room at temp 20°C. If the emissivity of cylinder surface is 0.8, find total heat loss rate by the cylinder and initial rate of cooling.

Take properties of air at 200°C as follows:
C_p=1026J/kgK; ρ=0.746kg/m^3; k=0.0393W/mK
ν=34.85x10^{-6} m^2/s

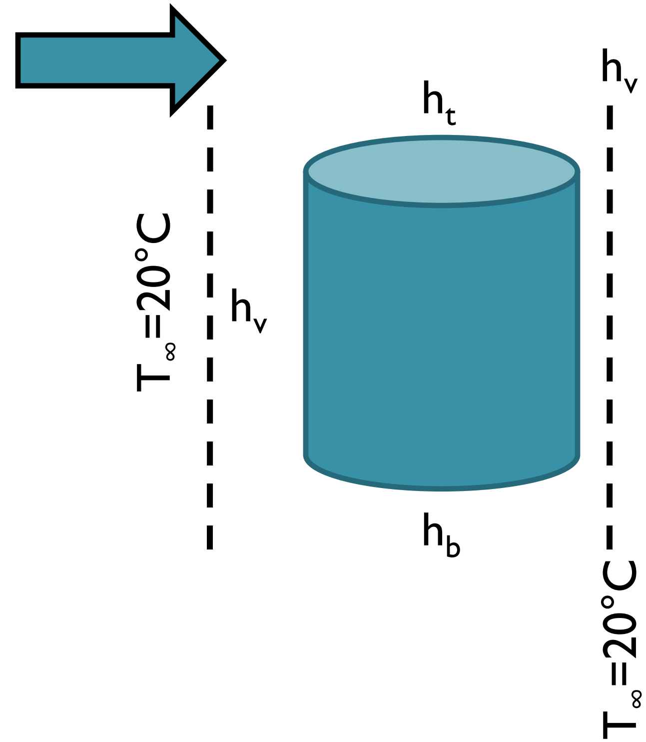
Use the following correlations:

$Nu=0.56(Gr.Pr)^{0.25}$ for vertical surface

$Nu=0.27(Ra)^{0.25}$ for lower horizontal surface

$Nu=0.54(Ra)^{0.25}$ for upper horizontal surface

Solution: Line of Approach



We have to find out
heat flow rate Q=?

Heat flow will take place by convection and radiation.

Radiant heat flow $Q_r = \varepsilon_1 \sigma A_1 (T_s^4 - T_\infty^4)$

Heat flow by convection $Q_c = h\,A\,\Delta T$

Since h will be different for different surfaces i,e. h_t for top, h_b for bottom and h_v for vertical surfaces, we should first find out h_t, h_b and h_v by given Nu co-relations.

e can now find out Q_c for different surfaces. Add up Q_c and Q_r to get total heat flow rate Q

Solution: For Vertical Surface

Mean Film Temp=(380+20)/2=200°C =473K

$$Gr = \frac{g\beta \Delta T L^3}{\nu^2}$$

$$= \frac{9.81x(380-20)x0.3^3}{473x(34.85x10^{-6})^2} = 1.66x10^8$$

$$\Pr = \frac{\mu C_p}{k} = \frac{\rho \nu C_p}{k}$$

$$\frac{0.746x34.85x10^{-6}x1026}{0.0393} = 0.679$$

Solution: For Vertical Surface

$$Nu = 0.56(Gr\,\Pr)^{0.25}$$

$$Nu = \frac{h_V L}{k}$$

$$= 0.56(1.66x10^8\,x0.679)^{0.25} = 57.69$$

$$h_V = \frac{0.0393x57.69}{0.3} = 7.56W/m^2K$$

Solution: For Top Horizontal Surface

$$Charac\,Length\ L = \frac{A}{P} = \frac{\pi D^2}{4\pi D} = \frac{D}{4} = \frac{12}{4} = 3cm = 0.03m$$

$$Gr = \frac{g\beta\Delta TL^3}{\nu^2}$$

$$= \frac{9.81x(380-20)x0.03^3}{473x(34.85x10^{-6})^2} = 1.66x10^5$$

$$\Pr = \frac{\mu C_p}{k} = \frac{\rho\nu C_p}{k}$$

$$\frac{0.746x34.85x10^{-6}x1026}{0.0393} = 0.679$$

Solution: For Top Horizontal Surface

$$Nu = 0.54(Gr\,\Pr)^{0.25}$$

$$Nu = \frac{h_t L}{k} = 0.54(1.66x10^5\,x0.679)^{0.25} = 9.89$$

$$\Rightarrow h_t = \frac{0.0393x9.89}{0.03} = 12.96\,W/m^2K$$

Solution: For Bottom Horizontal Surface

$$Charac\,Length\ L = \frac{A}{P} = \frac{\pi D^2}{4\pi D} = \frac{D}{4} = \frac{12}{4} = 3cm = 0.03m$$

$$Gr = \frac{g\beta\Delta TL^3}{\nu^2}$$

$$= \frac{9.81x(380-20)x0.03^3}{473x(34.85x10^{-6})^2} = 1.66x10^5$$

$$\Pr = \frac{\mu C_p}{k} = \frac{\rho\nu C_p}{k}$$

$$\frac{0.746x34.85x10^{-6}x1026}{0.0393} = 0.679$$

Solution: For Bottom Horizontal Surface

$$Nu = 0.27(Gr\,\mathrm{Pr})^{0.25}$$

$$Nu = \frac{h_b L}{k} = 0.27(1.66x10^5\,x0.679)^{0.25}$$

$$h_b = 6.48W / m^2 K$$

Hence total heat flow by convection

$$Qc = h_v.\pi DL(T_s - T_\infty) + h_t.\frac{\pi}{4}D^2.(T_s - T_\infty) + h_b\frac{\pi}{4}D^2.(T_s - T_\infty)$$

$$= 7.56x\pi x0.12x0.3(380-20) + 12.96x\frac{\pi}{4}.0.12^2(380-20)$$

$$+ 6.48x\frac{\pi}{4}.0.12^2(380-20) = 386.76W$$

$$Heat\ loss\ by\ Radiation \quad Q_r = \varepsilon_1\sigma A_1(T_s^4 - T_\infty^4)$$

$$Q_r = 0.8x5.67x10^{-8}x(\pi DL + 2x\frac{\pi}{4}D^2)(653^4 - 293^4)$$

$$= 1073.35W$$

Hence total heat flow by convection and Radiation ⇒

$$Q = Qc + Qr = 386.76 + 1073.35 = 1460W$$

$$To\,obtain\ Initial\ Rate\ of\ Cooling\quad Q = -mC_p\frac{dT}{dt}$$

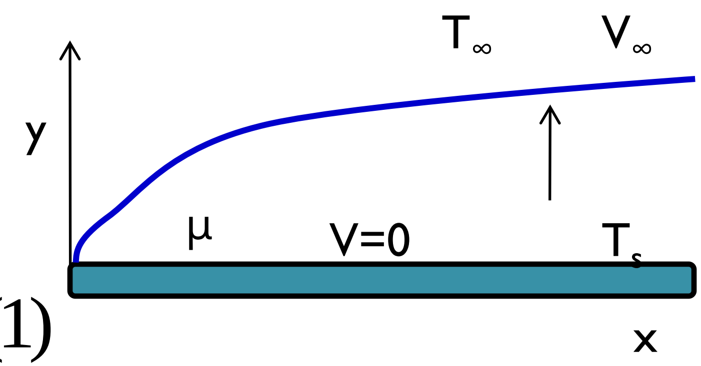
$$m = \rho V = \frac{8000 x \pi (0.12)^2 x 0.3}{4} = 27.13kg$$

$$\therefore \frac{dT}{dt} = \frac{1460}{420x27.13} = 0.128\,^{\circ}C/\sec = 7.69\,^{\circ}C/\min$$

Reynold's Analogy

Reynold's Analogy is the relationship between C_f & h (heat transfer by convection) between plate surface and fluid for Laminar Flow over Flat Plate

As per Newton's Law of Viscosity, Shear Stress in Laminar Flow in the normal direction to the Plate is given as:



$$\tau_s = \mu \frac{dV}{dy} \Rightarrow dy = \frac{\mu dV}{\tau_s} \ldots\ldots\ldots(1)$$

Heat Flow along y direction is given by Fourier's Law

$$= -KA\frac{dT}{dy} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Reynold's Analogy

$$We\ know\ that\ \Pr = \frac{\mu C_p}{K}$$

$$Assuming\ \Pr \approx 1;\ we\ have\ K = \mu C_p \ ..(3)$$

$$On\ substitution\ in\ Eqn(2)\ from\ (1)\ \&\ (3)$$

$$Q = -\mu C_p A \frac{dT}{dy}$$

$$= -\mu C_p A \frac{dT}{\mu dV} \tau_s = -C_p A \frac{dT}{dV} \tau_s \ ......(4)$$

Reynold's Analogy

BC 1) For V=0 at plate surface, $T=T_s$

BC 2): For $V=V_\infty$ on outer edge of BL; $T=T_\infty$

Separating Variables and Integrating,
we have:

$$\frac{Q}{C_p .A\tau_s}\int_{0}^{V\infty} dV = -\int_{Ts}^{T\infty} dT \Rightarrow \frac{Q}{C_p .A\tau_s}.V_\infty = (T_s - T_\infty)$$

Reynold's Analogy

$$\Rightarrow \frac{Q}{A(T_s - T_\infty)} = \tau_s \frac{C_p}{V_\infty} \Rightarrow h = \tau_s \frac{C_p}{V_\infty}$$

Skin Friction is defined in Drag Force as :

$$F_D = C_f . \frac{1}{2} \rho A V^2$$

$$Hence \quad \tau_s = \frac{F_D}{A} = C_f . \frac{1}{2} \rho V^2$$

Reynold's Analogy (Pr=1)

$$\text{Substituting in equation} \Rightarrow h = \tau_s \cdot \frac{C_p}{V_\infty}$$

$$h = C_f \cdot \frac{1}{2} \rho V_\infty{}^2 \cdot \frac{C_p}{V_\infty}$$

$$\frac{C_f}{2} = \frac{h}{\rho C_p V_\infty} = St \quad REYNOLD's\ ANALOGY$$

Chilton & Colburn Analogy

Reynold's Analogy assumes Pr=1; hence when Pr≠1, poor results are obtained. This analogy was modified Chilton & Colburn

We know that : $$Nu = 0.664\,\mathrm{Re}^{1/2}\,.\,\mathrm{Pr}^{1/3}$$

$$\textit{Dividing both sides by } \mathrm{Re}\,\mathrm{Pr}^{1/3}\,;\textit{We have}$$

$$\frac{Nu}{\mathrm{Re}\,.\,\mathrm{Pr}^{1/3}} = \frac{0.664}{\mathrm{Re}^{1/2}} = \frac{1}{2}\cdot\frac{1.328}{\sqrt{\mathrm{Re}}} = \frac{C_f}{2}$$

Chilton & Colburn Analogy

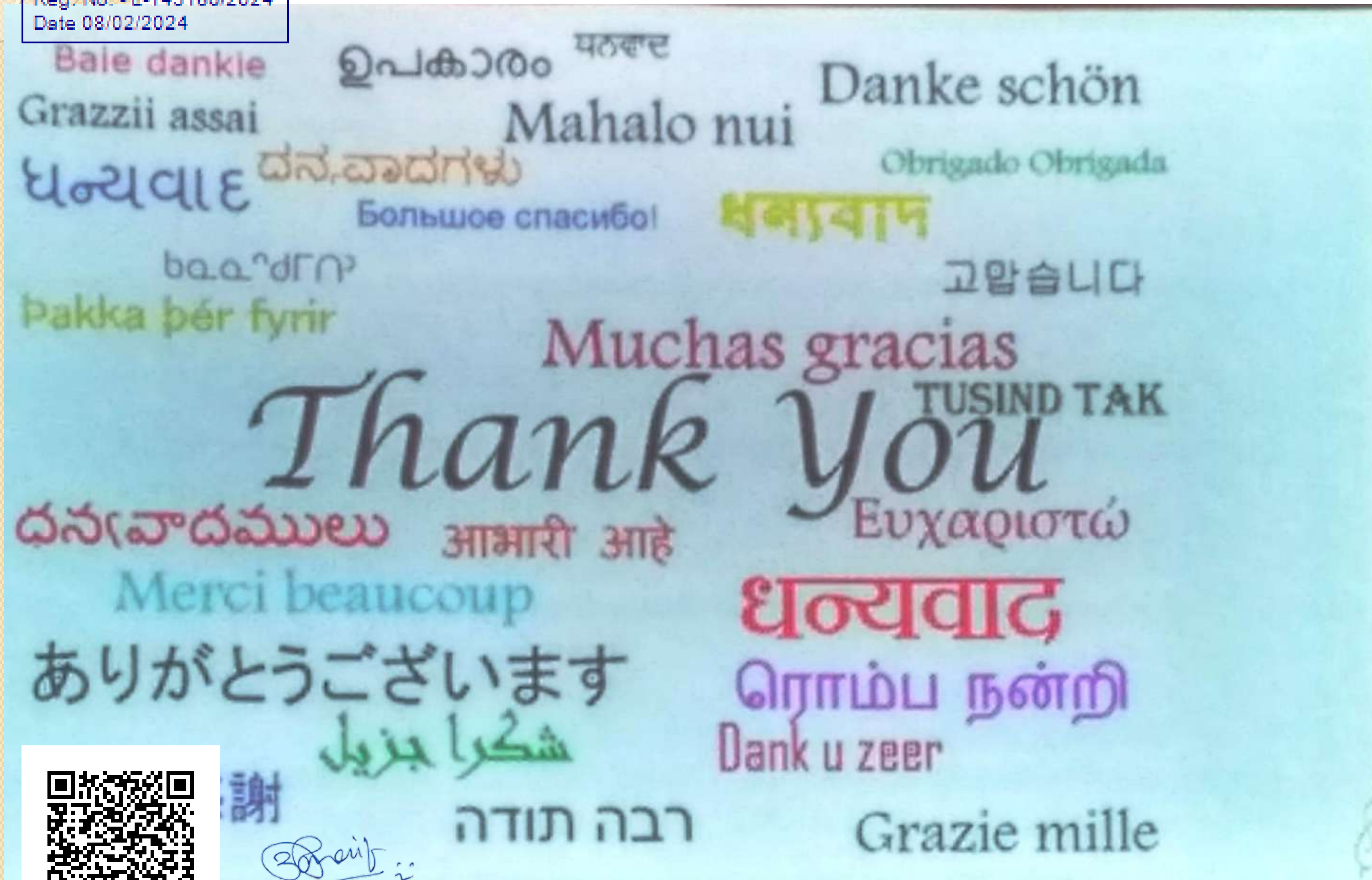
$$\frac{C_f}{2} = \frac{Nu}{Re.Pr^{1/3}} = St.Pr^{2/3}$$

CHILTON & COLBURN ANALOGY

$$(Holds\ good\ for\ Pr\ from\ 0.5\ to\ 50)$$

$$(Put\ Pr = 1 \Rightarrow Reynold's\ Analogy)$$

End of Unit - IV


Baie dankie
ഉപകാരം
Danke schön
Grazzii assai
Mahalo nui
ಧನ್ಯವಾದಗಳು
Obrigado Obrigada
ધન્યવાદ
Большое спасибо!
고맙습니다
Þakka þér fyrir
Muchas gracias
TUSIND TAK
Thank You
ధన్యవాదములు
आभारी आहे
Ευχαριστώ
Merci beaucoup
धन्यवाद
ありがとうございます
ரொம்ப நன்றி
شكرا جزيلا
Dank u zeer
תודה רבה
Grazie mille

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"Heat Transfer"

Nutan Maharashtra Institute of Engineering & Technology Talegaon Dabhade

UNIT NO: 05

CREATED BY

Prof. Rohit R. Jadhao

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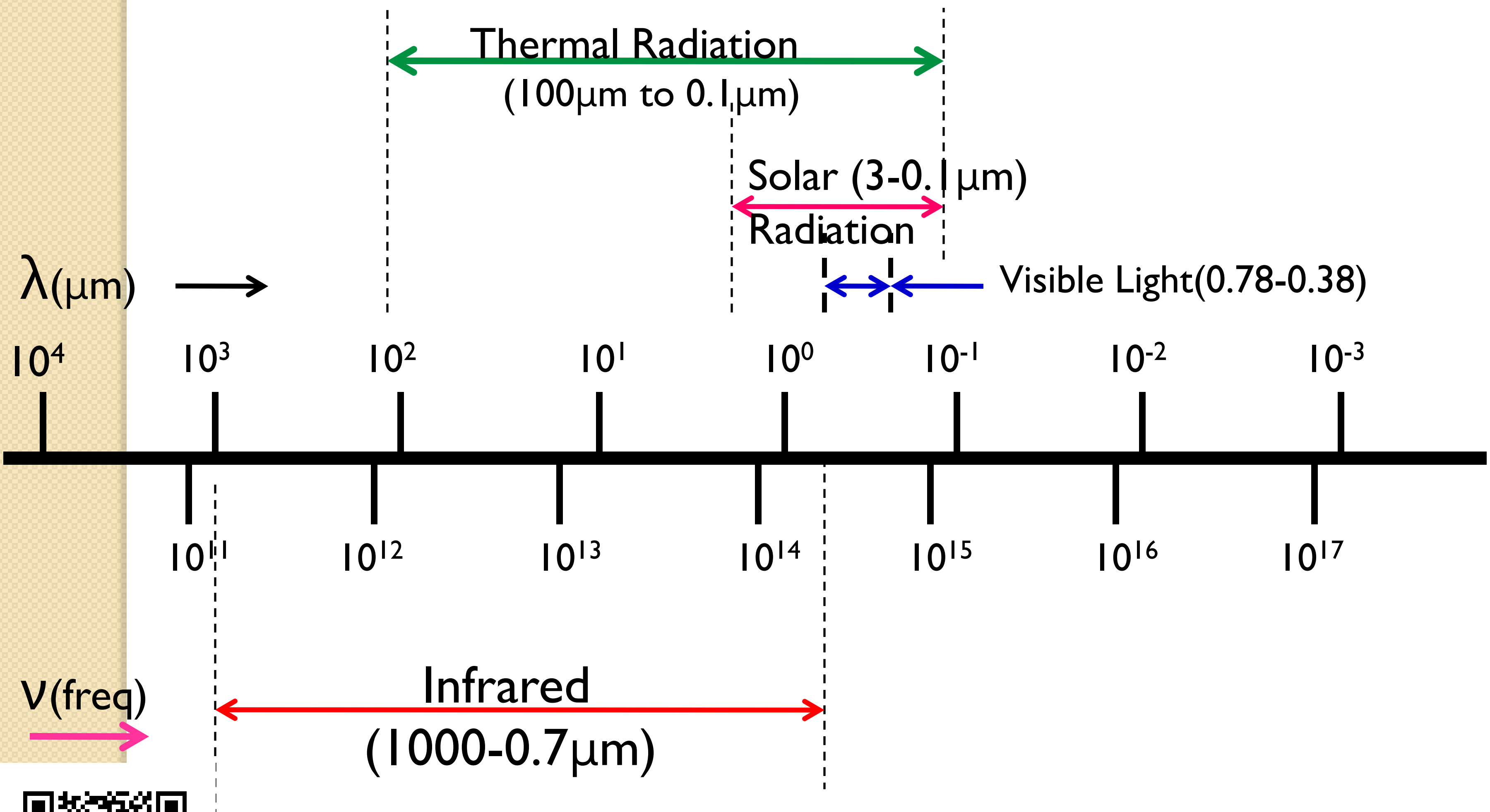
Radiation

- Every surface emits electromagnetic waves continuously in all possible directions due to its temp.

- These electromagnetic waves carry energy when they propagate and transfer thermal energy when they impinge on a substance/ body. This kind of energy transfer is called Heat Transfer by RADIATION.

- Heat transfer by emissions of radiation is explained by two theories:

Spectrum of Electromagnetic Radiation



Radiation

1. Wave Theory or Maxwell's Classical Theory

- Radiation emissions propagate in the form of waves. Since waves propagate through some medium, this theory assumes that Universe is filled with a hypothetical medium ETHER.

- Waves travel with the speed of light

- Every wave possesses certain amount of energy, a part of which is transferred on eing impinged by some object in its route f travel

Radiation

2. Quantum Theory or Planck's Theory

- Radiation emissions are in the form of series of entities known as quanta.

- Each quanta possesses certain amount of energy, which is proportional to its frequency of emission.

- Quanta moves with the speed of light and releases its energy on being impinged by some object in its route of travel

Properties of Surface

Reflectivity (ρ):

Fraction of total energy falling on the surface, which is reflected

Absoptivity (α):

Fraction of total energy falling on the surface, which is absorbed

Transmissivity (τ):

Fraction of total energy falling on the surface, which is transmitted (through)

Hence, $\rho + \alpha + \tau = 1$

Some Definitions

Black Body:

A body which absorbs all incident energy and does not transmit and reflects at all, is called Black Body. It is also the highest emitter of radiation

$$\tau = 0; \ \ \rho = 0; \ \alpha = 1; \ \varepsilon = 1$$

Examples: Surface coated with lamp black,
lk, ice, water, white paper etc

Some Definitions

White Body:

A body which reflects the entire radiation falling on it, is called White Body

$$\tau = 0; \quad \alpha = 0; \; \varepsilon = 0; \; \rho = 1$$

Gray Body:

The body having same value of emissivity at all wavelengths , which is equal to average emissivity, is known as Grey body.

enerally, all engg metals are grey bodies, which ε=α, when in thermal equilibrium

Some Definitions

Emissive Power (q):

It is the rate, at which the radiant flux is emitted from the surface at certain temp

Monochromatic Emissive Power (q_λ):

It is the rate, at which radiant flux is emitted with a specific wave length at certain temp; it is λ dependent emissive power

Some Definitions

Emissivity (ε):

It is the ratio of emissive power of a surface to that of black body when both at same temp

$$\varepsilon = \frac{q}{q_b}$$

Monochromatic Emissivity (ε_λ):

It is the ratio of monochromatic emissive power of a surface to that of black body when both are at same temp for same given wavelength

$$\varepsilon_\lambda = \frac{q_\lambda}{q_{b\lambda}}$$

Some Definitions

Radiosity (J):

It is the net energy leaving the surface.
It consists of the radiant energy emitted and energy reflected out of the incident radiation from the surface.

$$J = \varepsilon_1 q_b + (1-\varepsilon_1)G$$

Irradiation (G):

It is the net energy incident/falling on the rface (need not necessarily be absorbed)

Planck's Law

Planck's law is based on Quantum theory and it gives the relationship among monochromatic Emissive power of black body, the absolute Temp of the surface and corresponding Wavelength and is given as:

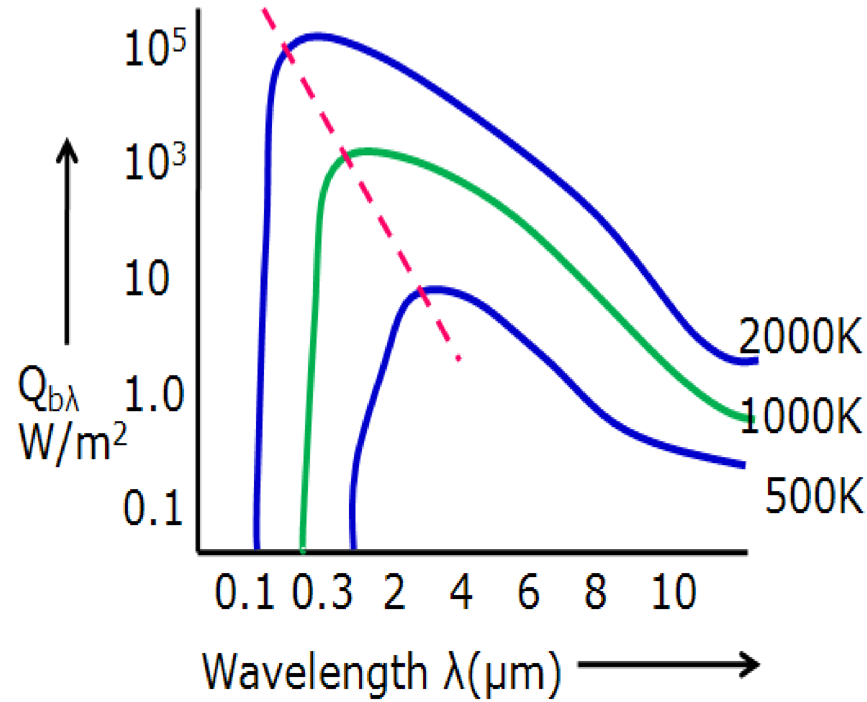
$$q_{b\lambda} = \frac{2\pi C_1}{\lambda^5 . \left(e^{C_2 / \lambda T} - 1 \right)} \, W/m^2;$$

$$where \, C_1 = 0.596 x 10^{-16} \; \&$$

$$C_2 = 0.014387$$

Planck's Law

Plot shows the following:



- $q_{b\lambda}$ at certain temp first increases with λ, attains some max value and then decreases

- For specific wavelength, $q_{b\lambda}$ of black surface increases with temp

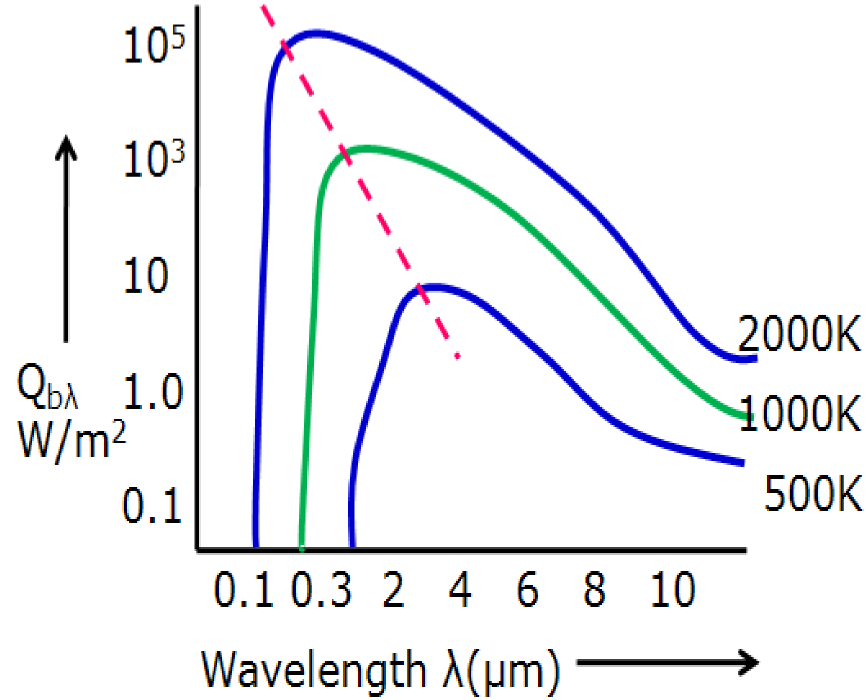
- Most of the thermal radiations lie in wavelength region from 0.3 to 10 μm

- Wavelength (λ_m) , at which peak $q_{b\lambda}$ obtained, decreases with increase in temp

Wien's Displacement Law

Wien's Law gives the relationship between the wavelengths (λ_m), at which peak ($q_{b\lambda}$) monochromatic emissive power is obtained and the absolute temp and given as:

$$\lambda_m . T = 0.0029 \text{ mK}$$



Plot and above relation show that the value of wavelength, at which peak/max monochromatic issive power is obtained, decreases (displaces/shifts) :h increase in surface temperature of the black body.

Derivation of Wien's Law

As per Planck's law,

$$q_{b\lambda} = \frac{2\pi C_1}{\lambda^5 \left(e^{C_2 / \lambda T} - 1 \right)}$$

$$Putting \;\; \frac{C_2}{\lambda T} = x \;\; \Rightarrow \lambda = \frac{C_2}{xT}$$

$$Substituting \quad q_{b\lambda} = \frac{2\pi C_1}{\frac{C_2{}^5}{x^5 T^5} \left(e^x - 1 \right)}$$

$$Or \;\; q_{b\lambda} = \frac{2\pi C_1 . x^5 . T^5 . \left(e^x - 1 \right)^{-1}}{C_2{}^5}$$

This eqn expresses $q_{b\lambda}$ of black body as a function of x

Derivation of Wien's Law

For obtaining the wavelength (λ_m) for specified temp, at which max $q_{b\lambda}$ occurs, we have to differentiate this equation wrt x and equate it to zero.

$$\therefore \quad \frac{d}{dx}\left[\frac{2\pi C_1 . x^5 . T^5 .\left(e^x - 1\right)^{-1}}{C_2{}^5}\right] = 0$$

$$Or \ \frac{2\pi C_1 T^5}{C_2{}^5} \cdot \frac{d}{dx}\left[x^5 .\left(e^x - 1\right)^{-1}\right] = 0$$

$$r \quad \frac{d}{dx}\left[x^5\left(e^x - 1\right)^{-1}\right] = 0$$

Derivation of Wien's Law

$$\frac{d}{dx}\left[x^{5}\left(e^{x}-1\right)^{-1}\right]=0$$

$$\Rightarrow \left(e^{x}-1\right)^{-1}.\left(5x^{4}\right)+\left(x^{5}\right).\left(-1\right).\left(e^{x}-1\right)^{-2}.e^{x}=0$$

$$\Rightarrow \frac{5x^{4}}{\left(e^{x}-1\right)}-\frac{x^{5}.e^{x}}{\left(e^{x}-1\right)^{2}}=0$$

$$\Rightarrow \frac{x^{4}}{\left(e^{x}-1\right)}\left[5-\frac{x.e^{x}}{\left(e^{x}-1\right)}\right]=0$$

$$\frac{5e^{x}-5-x.e^{x}}{e^{x}-1}=0 \Rightarrow e^{x}\left(5-x\right)-5=0$$

Derivation of Wien's Law

$$We\ now\ have \quad e^{x}(5-x)-5=0$$

This eqn is satisfied by putting x=4.96

$$Hence, \quad x = 4.96 = \frac{C_2}{\lambda T}$$

$$\therefore \; \lambda_m T = \frac{0.014387}{4.96} = 0.0029$$

$$Therefore, \quad \lambda_m T = 0.0029\, mK$$

Stefan Boltzmann's Law

Emissive power of a black body is directly proportional to fourth power of its absolute temperature:

$$q_b \propto T^4 \; or \;\; q_b = \sigma T^4;$$

$$where \;\; \sigma = 5.67x10^{-8} W/m^2K^4$$

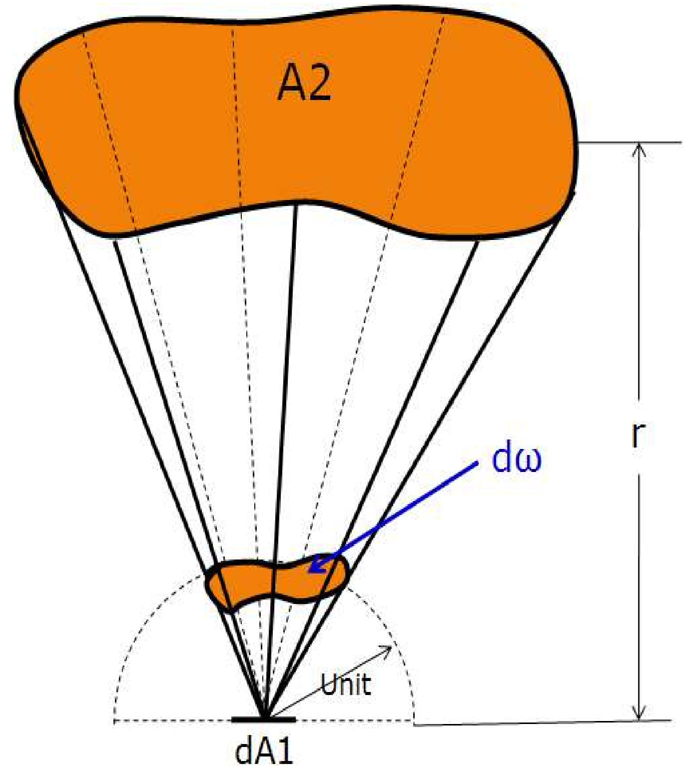
Kirchhof's Law

When a surface is in thermal equilibrium with its surroundings, the emissivity of the surface equal to its absoptivity

$$That \; is \; \alpha = \varepsilon$$

Solid Angle

Solid angle subtended by surface A_2 at surface dA_1 (elementary surface) is numerically equal to the area on a surface of sphere with unit radius and centre at elementary area, which is cut by conical surface having its base as perimeter of A_2 and vertex at dA_1



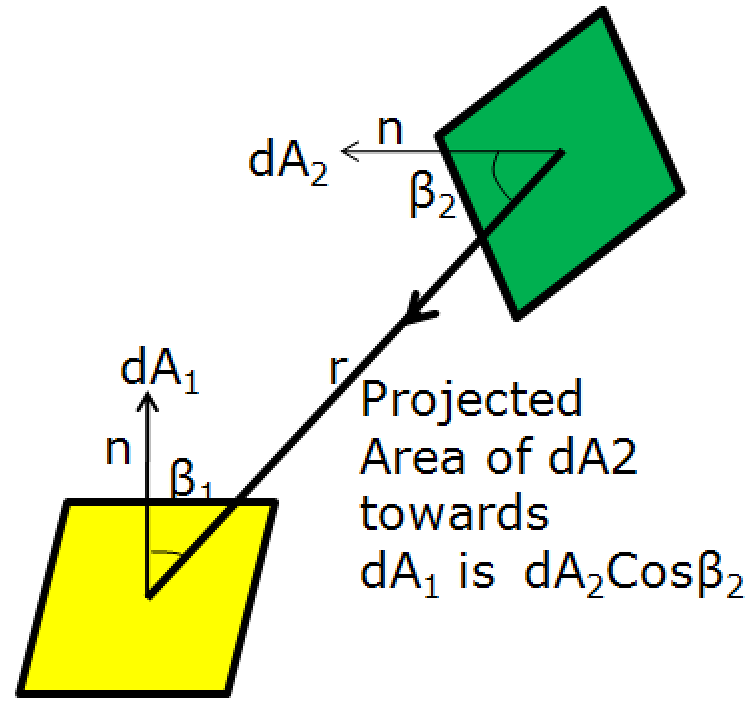
Solid angle is measured in Steradians (Sr) and noted by symbol ω

$$\therefore d\omega = \frac{A_2}{r^2}$$

Solid Angle Between Two Elementary Areas

Solid Angle subtended by elementary area dA_2 at dA_1 can be given as:

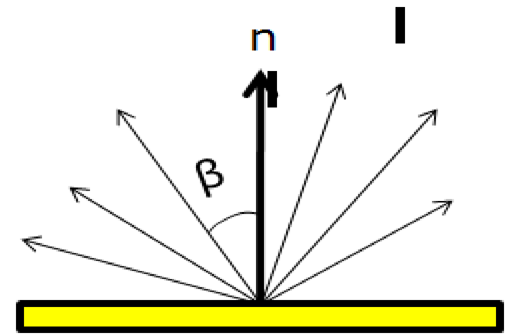
$$d\omega = \frac{dA_2 Cos\beta_2}{r^2}$$



Similarly, solid angle ... tended by area dA_1 ... A_2 can be given as:

$$d\omega = \frac{dA_1 Cos\beta_1}{r^2}$$

Intensity of Radiation



- Intensity of radiation emitted by a surface is equal to the radiant energy passing in a specified direction per unit solid angle
- Intensity of radiation varies in different directions and is max in the direction normal to the surface

Lambert Cosine Law:

Intensity of radiation in any direction is proportional to the Cosine of the angle made by that direction with the normal.

That is, $I = I_n \cos\beta$; where I_n is the intensity (max) in the normal direction and β is the angle made by that ...ection with the normal

$$\text{...tal emissive power } q = \pi I_n \Rightarrow I_n = \frac{\sigma T^4}{\pi}$$

Shape Factor/Geometric Factor

Shape factor is defined as the fraction of energy emitted by one surface and directly intercepted by the other.

$$Shape\ Factor : F_{12} = \frac{1}{A_1}\left[\int_{A_1}\int_{A_2}\frac{Cos\beta_1 Cos\beta_2 dA_1 dA_2}{\pi r^2}\right]$$

Shape Factor depends upon:

- Shape and size of surfaces
- Orientation of surfaces w.r.t each other
- Distance between the surfaces

Relations/ Theorems of Shape Factors

1. Reciprocal Relation:

$F_{12}.A_1 = F_{21}.A_2$

2. Enclosure Relation: If n no of surfaces form an enclosure, then:

$F_{11}+F_{12}+F_{13}+\ldots\ldots\ldots+F_{1n}=1$

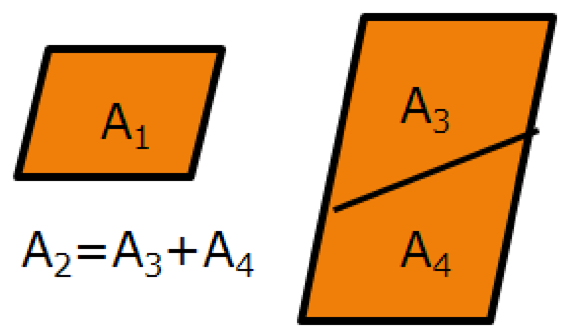
$F_{21}+F_{22}+F_{23}+\ldots\ldots\ldots+F_{2n}=1$

…………………………………………………

$F_{n1}+F_{n2}+F_{n3}+\ldots\ldots\ldots+F_{nn}=1$

3. Summation Relation:

Shape Factor F_{12} between two surfaces A_1 and A_2 is equal to the sum of shape factors F_{13} & F_{14}, if the two areas A_3 … $_4$ together make up area



$$F_{12} = F_{13} + F_{14}; \quad However,\ F_{21} \neq F_{31} + F_{41}$$

Relations/ Theorems of Shape Factors

4. Shape factor depends on geometry and orientation of surfaces and it does not change with temp.

5. Shape Factor wrt itself (F_{11}, F_{22}, F_{33}…) means radiation emitted by a portion of a surface falling on the other portion of itself directly

Example : Shape Factor for concave surface

Shape factor for convex or Flat surface wrt itself is zero.

Radiation Heat Exchange Between Two Parallel Plates

Consider two grey opaque parallel plates maintained at temperatures T_1 & T_2 having emissivities ε_1 & ε_2 respectively

For grey bodies, absoptivity α = emissivity ε

T_1, ε_1

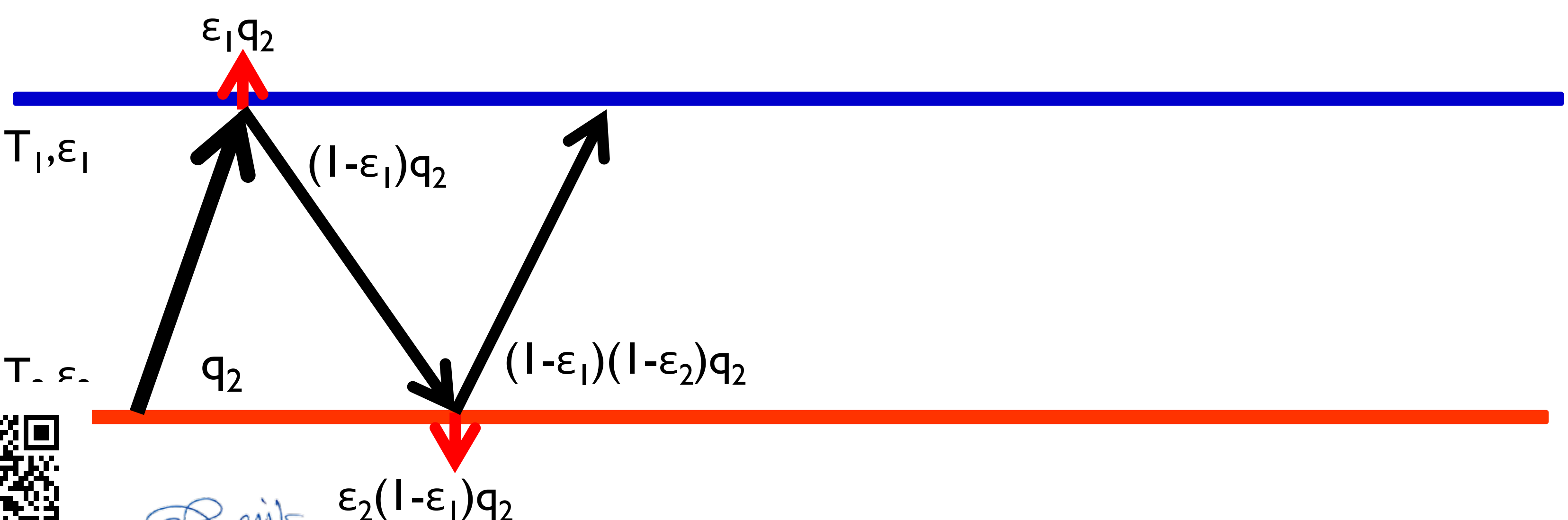
T_2

Radiation Heat Exchange Between Two Parallel Plates

Consider radiant flux q_2 emitted by surface 2.

Out of q_2, a fraction $\varepsilon_1 q_2$ will be absorbed by surface 1 and rest $(q_2-\varepsilon_1 q_2)$ will be reflected towards surface 2

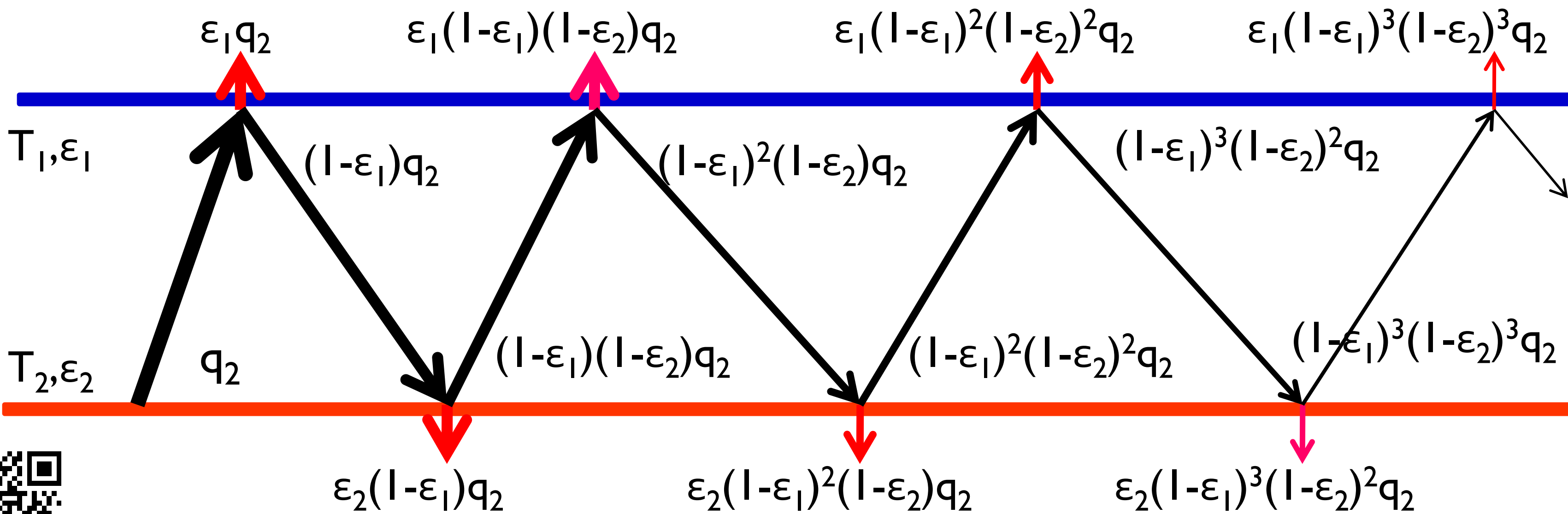
Out of this, $\varepsilon_2(1-\varepsilon_1)q_2$ will be absorbed by surface 2 and balance $(1-\varepsilon_1)(1-\varepsilon_2)q_2$ will be reflected to 1



Radiation Heat Exchange Between Two Parallel Plates

Out of this, $\varepsilon_1(1-\varepsilon_1)(1-\varepsilon_2)q_2$ will be absorbed by surface 1 and balance $(1-\varepsilon_1)^2(1-\varepsilon_2)q_2$ will be reflected back to 2

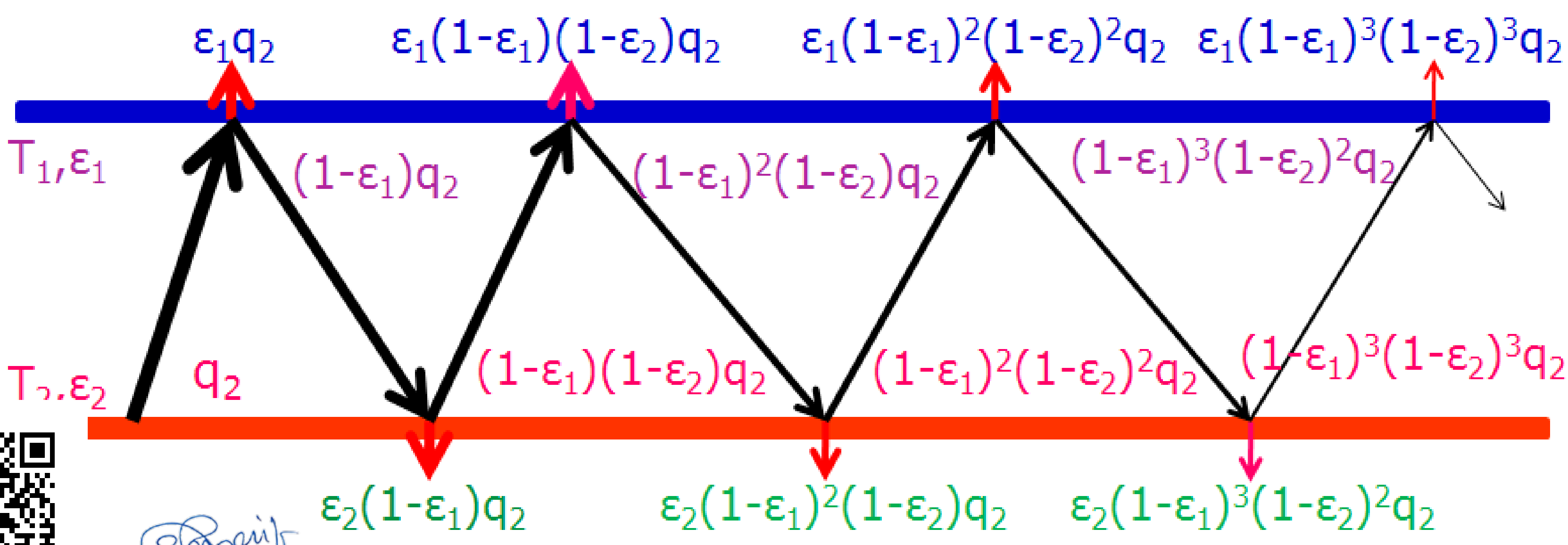
This process of absorption and reflection goes on indefinitely, the quantities involved being successively smaller.



Radiation Heat Exchange Between Two Parallel Plates

Thus, total radiant flux absorbed by surface 1 out of q_2 emitted by surface 2 will be:

$$= \varepsilon_1 q_2 + \varepsilon_1(1-\varepsilon_1)(1-\varepsilon_2)q_2 + \varepsilon_1(1-\varepsilon_1)^2(1-\varepsilon_2)^2 q_2 + \varepsilon_1(1-\varepsilon_1)^3(1-\varepsilon_2)^3 q_2 + \ldots\ldots\ldots\ldots\infty$$



Radiation Heat Exchange Between Two Parallel Plates

$$= q_2\varepsilon_1\left[1+(1-\varepsilon_1)(1-\varepsilon_2)+(1-\varepsilon_1)^2(1-\varepsilon_2)^2+(1-\varepsilon_1)^3(1-\varepsilon_2)^3+\ldots\ldots\ \infty\right]$$

$$= \frac{q_2\varepsilon_1}{1-(1-\varepsilon_1)(1-\varepsilon_2)} \qquad = \frac{q_2\varepsilon_1}{1-(1-\varepsilon_1-\varepsilon_2+\varepsilon_1\varepsilon_2)}$$

$$= \frac{q_2\varepsilon_1}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2}$$

Similarly, considering radiation flux q_1 emitted by surface 1 and fraction out of which absorbed by surface 2 can be given as:

$$: \frac{q_1\varepsilon_2}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2}$$

Radiation Heat Exchange Between Two Parallel Plates

Assuming $T_1 > T_2$, net radiant flux absorbed by 2:

$$q_{12} = \frac{q_1\varepsilon_2}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2} - \frac{q_2\varepsilon_1}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2}$$

$$Since\ \ q_1 = \varepsilon_1\sigma T_1{}^4 \ \ \&\ q_2 = \varepsilon_2\sigma T_2{}^4$$

$$q_{12} = \frac{\varepsilon_2\varepsilon_1\sigma T_1{}^4 - \varepsilon_1\varepsilon_2\sigma T_2{}^4}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2}$$

$$q_{12} = \frac{\sigma\left(T_1{}^4 - T_2{}^4\right)}{\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1\right)}$$

$$Or\ \ Q_{12} = \frac{\sigma A\left(T_1{}^4 - T_2{}^4\right)}{\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1\right)}$$

Electrical Analogy of Radiation

Shape/Space Resistance:

Heat flow between two black surfaces at temps T_1 & T_2 can be written as:

$$Q_{12} = F_{12} A_1 \sigma \left(T_1^{\,4} - T_2^{\,4}\right) = \frac{\sigma\left(T_1^{\,4} - T_2^{\,4}\right)}{\dfrac{1}{A_1 F_{12}}}$$

$$Here,\ \ Equivalent\ \ Potential\ \ Diff\ = \sigma\left(T_1^{\,4} - T_2^{\,4}\right)$$

$$And\ \ Equivalent\ \ \mathrm{Resistance} = \frac{1}{A_1 F_{12}} = \frac{1}{A_2 F_{21}}$$

Due to finite dimensions of the surfaces, 100% of emitted radiation from surface 1 does not fall on surface 2, hence some part of emitted energy go to surroundings, thus lost. This loss is conceptually ...plained to be caused due to resistance offered by ...teness of dimensions of surfaces and their ...entation. Hence, it is called Shape/Space Resistance

Electrical Analogy of Radiation

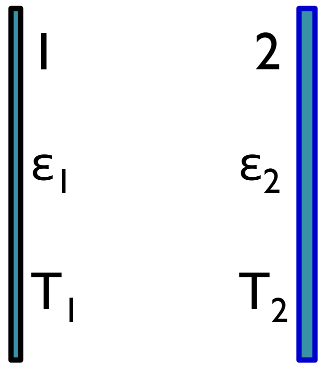
Surface Resistance:

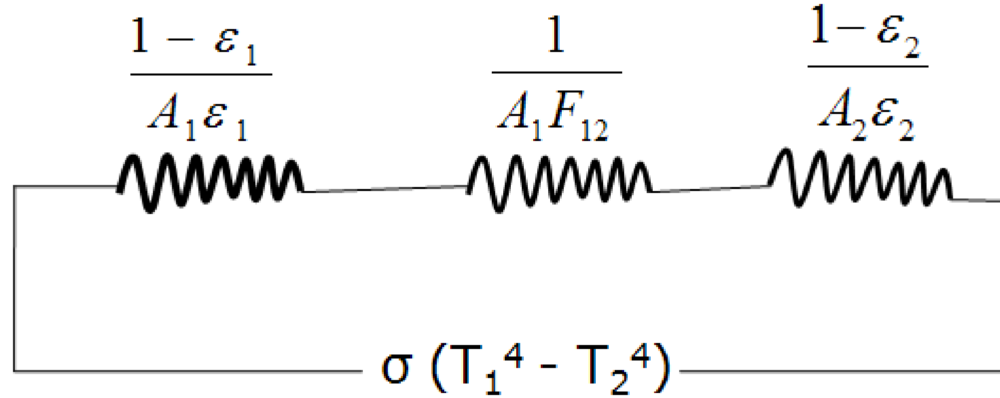
Black body emits max possible radiation and its emissivity is taken as 1(the datum). However, Grey bodies emit less due to surface properties; and hence their emissivities are taken as less than 1 (in comparison).

Therefore, emission of radiation from grey bodies is always less than that of black body. This lesser emission is conceptually assumed to be caused due to a resistance offered by surface of the body as it depends on surface property; the emissivity. This resistance is called Surface Resistance and given as:

$$_{surface} = \frac{1-\varepsilon_1}{A_1\varepsilon_1}\ (of\ the\ surface\ 1)\ \&\ \frac{1-\varepsilon_2}{A_2\varepsilon_2}\ (of\ surface\ 2)$$

Radiation Heat Exchange Between Two Parallel Plates (By Other Method)





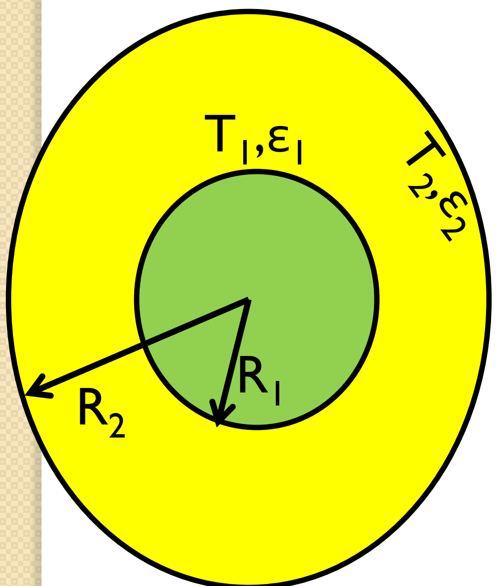
$$Q_{12} = \frac{\sigma\left(T_1^{\,4} - T_2^{\,4}\right)}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1F_{12}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

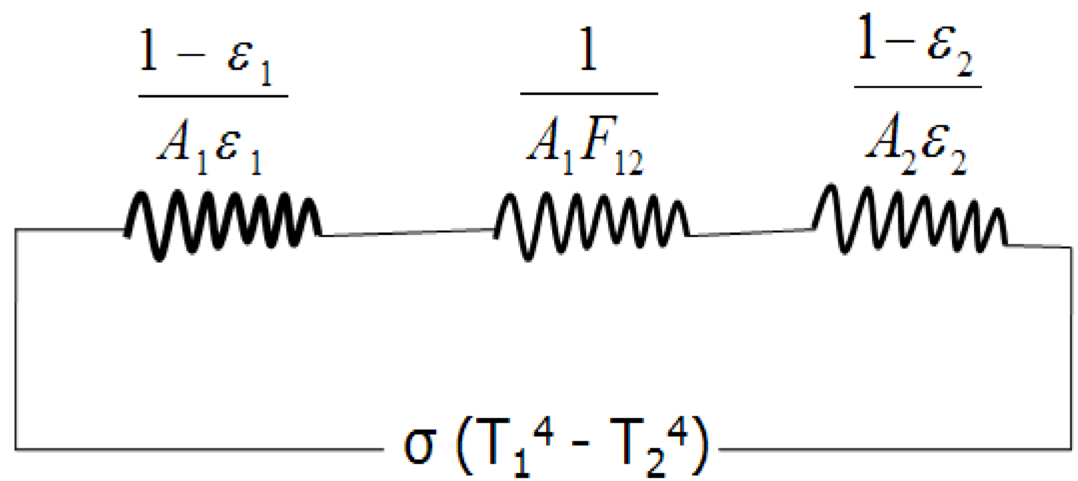
Since $F_{12}=1$ & $A_1=A_2=A$;

$$Q_{12} = \frac{\sigma\left(T_1^{\,4} - T_2^{\,4}\right)}{\frac{1}{\varepsilon_1} - 1 + 1 + \frac{1}{\varepsilon_2} - 1}$$

$$\Rightarrow q_{12} = \frac{\sigma\left(T_1^{\,4} - T_2^{\,4}\right)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right)}$$

Radiation Heat Exchange Between Two Concentric Infinitely Long Grey Cylinders





$$Q_{12} = \frac{\sigma\left(T_1^{\ 4} - T_2^{\ 4}\right)}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1F_{12}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

=1 as inner cylinder is completely enclosed by 2

Radiation Heat Exchange Between Two Concentric Infinitely Long Grey Cylinders

$$Q_{12} = \frac{\sigma\left(T_1^{\,4} - T_2^{\,4}\right)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1 F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

Putting $F_{12}=1$,
We have:

$$Q_{12} = \frac{\sigma\left(T_1^{\,4} - T_2^{\,4}\right)}{\dfrac{1}{A_1}\left[\dfrac{1}{\varepsilon_1} - 1 + 1 + \dfrac{A_1}{A_2}\left(\dfrac{1}{\varepsilon_2} - 1\right)\right]}$$

$$\Rightarrow Q_{12} = \frac{\sigma A_1\left(T_1^{\,4} - T_2^{\,4}\right)}{\dfrac{1}{\varepsilon_1} + \dfrac{A_1}{A_2}\left(\dfrac{1}{\varepsilon_2} - 1\right)}$$

$A_1=2\pi R_1 L$

$A_2=2\pi R_2 L$

Radiation Heat Exchange Between Two Surfaces

$$\Rightarrow Q_{12} = \frac{\sigma A_1\left(T_1^{\,4} - T_2^{\,4}\right)}{\dfrac{1}{\varepsilon_1} + \dfrac{A_1}{A_2}\left(\dfrac{1}{\varepsilon_2} - 1\right)}$$

This expression is very useful as it can be applied to so many situations:

1. For heat exchange between two concentric spheres;
   Only diff will be : $A_1 = 4\pi R_1^2$ & $A_2 = 4\pi R_2^2$

2. For eccentric cylinders and spheres

3. For heat exchange between two parallel plates
   as $A_1 = A_2 = A$

4. For convex/Flat surface completely enclosed by other body as $F_{12} = 1$ and $F_{21} = A_1/A_2$

   If enclosure (A_2) is very large, $A_1/A_2 \approx 0$;
   Hence, $Q = \sigma\, \varepsilon_1 A_1 (T_1^4 - T_2^4)$

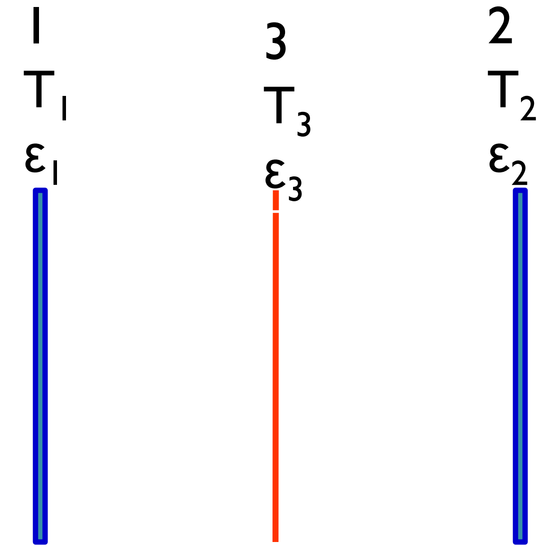
Radiation Shield

- In order to reduce the radiation heat transfer rate between two surfaces, a third surface is inserted between them. This surface is known as Radiation Shield.

- Requirements of Shield (Surface):
  - Highly reflecting
  - Lowest emissivity (also absorptivity)
  - Lowest thickness (thinnest)

- Applications in more effective thermos flasks, for reducing error in temp measurement by thermocouples etc

Radiation Shield



Heat Flow Rate
assuming $T_1 > T_2$:

$$q_{12} = \frac{\sigma\left(T_1^{\,4} - T_2^{\,4}\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

Now, a shield having both side emissivity ε_3 is placed between the surfaces 1 & 2.

On achieving steady state, the shield will attain steady temp T_3 between T_1 & T_2.

ce T_3 remains steady, that means whatever liation, the shield is receiving from surface 1, same it is giving out to surface 2.

Radiation Shield

$$Hence,\ q_{13} = q_{32} \Rightarrow \frac{\sigma\left(T_1^{\ 4} - T_3^{\ 4}\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} = \frac{\sigma\left(T_3^{\ 4} - T_2^{\ 4}\right)}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1}$$

$$Substituting\ \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1 = x\ and\ \frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1 = y$$

$$We\ have, \frac{T_1^{\ 4} - T_3^{\ 4}}{x} = \frac{T_3^{\ 4} - T_2^{\ 4}}{y} \Rightarrow \frac{T_3^{\ 4}}{y} + \frac{T_3^{\ 4}}{x} = \frac{T_1^{\ 4}}{x} + \frac{T_2^{\ 4}}{y}$$

$$xT_3^{\ 4} + yT_3^{\ 4} = yT_1^{\ 4} + xT_2^{\ 4} \Rightarrow T_3^{\ 4} = \frac{yT_1^{\ 4} + xT_2^{\ 4}}{x + y}$$

Radiation Shield

Substituting $T_3^{\,4}$ in q_{13} expression;

$$q_{13} = \frac{\sigma\left[T_1^{\,4} - \dfrac{yT_1^{\,4} + xT_2^{\,4}}{x+y}\right]}{x}$$

$$= \frac{\sigma\left(xT_1^{\,4} + yT_1^{\,4} - yT_1^{\,4} - xT_2^{\,4}\right)}{x\left(x+y\right)}$$

$$q_{13} = \frac{\sigma.x.\left(T_1^{\,4} - T_2^{\,4}\right)}{x\left(x+y\right)} = \frac{\sigma\left(T_1^{\,4} - T_2^{\,4}\right)}{x+y}$$

Radiation Shield

$$Substituting\ x\ \&\ y;\ q_{13} = \frac{\sigma\left(T_1^{\,4} - T_2^{\,4}\right)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1\right) + \left(\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1\right)}$$

On simplification:

$$q_{13} = \frac{\sigma\left(T_1^{\,4} - T_2^{\,4}\right)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{2}{\varepsilon_3} - 1\right)}$$

Since ε_3 will be very small, hence denominator of q_{13} be very large, therefore, there shall be large uction of q_{12} to q_{13}.

Radiation Shield

If $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon$;

$$q_{13} = \frac{\sigma\left(T_1^{\,4} - T_2^{\,4}\right)}{\left(\frac{1}{\varepsilon} + \frac{1}{\varepsilon} - 1\right) + \left(\frac{2}{\varepsilon} - 1\right)} = \frac{\sigma\left(T_1^{\,4} - T_2^{\,4}\right)}{\left(\frac{2}{\varepsilon} - 1\right) + \left(\frac{2}{\varepsilon} - 1\right)}$$

This means, $\left(\frac{2}{\varepsilon} - 1\right)$ *is used twice with one shield*

Hence, $q_{13} = \frac{\sigma\left(T_1^{\,4} - T_2^{\,4}\right)}{(n+1)\left(\frac{2}{\varepsilon} - 1\right)}$ *with n shields*

Radiation Shield

$$Hence, \quad q_{13} = \frac{\sigma\left(T_1^{\,4} - T_2^{\,4}\right)}{(n+1)\left(\frac{2}{\varepsilon} - 1\right)} \quad with \;\; n \;\; shields$$

$$With \;\; ONE \;\; shield\,;$$

$$q_{13} = \frac{\sigma\left(T_1^{\,4} - T_2^{\,4}\right)}{2\left(\frac{1}{\varepsilon} + \frac{1}{\varepsilon} - 1\right)} = \frac{q_{12}}{2}$$

ence, q_{13} now becomes half of q_{12}

Radiation Shield

Home Assignment:

$$\text{Prove that } \quad Q_{13} = \frac{\sigma A_1\left(T_1^{\,4} - T_2^{\,4}\right)}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right) + \frac{A_1}{A_3}\left(\frac{2}{\varepsilon_3} - 1\right)},$$

when a shield having ε_3 *emissivity is placed between TWO Cylinders / Sphheres 1& 2 having emissivities* ε_1 & ε_2 *maintained at temps* T_1 & T_2 *having areas* A_1 & A_2.

Q1: Effective temp of a body having an area of $0.12 m^2$ is 527°C. Calculate the following:

a) Rate of radiation energy emission

b) Intensity of normal radiation

c) Wavelength of max monochromatic emissive power

Solution:

a) Total emission of radiation $Q = \sigma A T^4$

$$Q = 5.67x10^{-8} x0.12x(527+273)^4 = 2786.9W$$

b) Intensity of Normal Radiation

$$= \frac{q_b}{\pi} = \frac{\sigma T^4}{\pi} = 5.67x10^{-8} x(527+273)^4 = 7392.5W/m^2.sr$$

c) Wavelength of max monochromatic emissive power:

From Wien's Displacement Law;

$$\lambda_m T = 0.0029\, mK$$

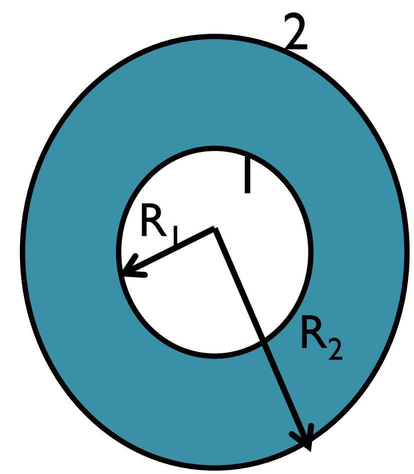
$$\Rightarrow \lambda_m = \frac{0.0029}{T} = \frac{0.0029}{527 + 273}$$

$$\therefore \ \lambda_m = 3.625x10^{-6}\, m = 3.625\, \mu m$$

Answer

Q2. A sphere of radius 5cm is concentric with another sphere. Find the radius of the outer sphere so that shape factor of outer sphere wrt inner sphere is 0.6.

<u>Solution:</u>



$A_1=4\pi r_1^2$ $A_2=4\pi r_2^2$ $F_{21}=0.6$

Since sphere 1 is completely enclosed by sphere 2, hence $F_{12}=1$

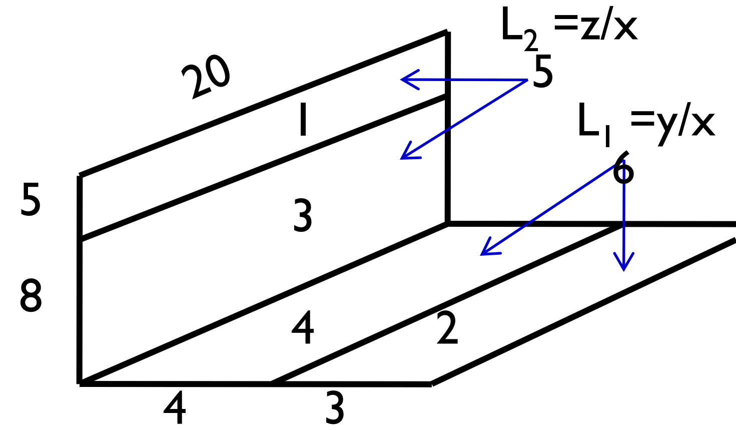
We know that $F_{12}.A_1=F_{21}.A_2$

Substituting values, we have;

$$1.\ 4\pi(0.05)^2 = 0.6x4\pi R_2^{\ 2}$$

$$=6.45cm \ \ Answer$$

Q3: Find F_{12}.



$$F_{12} = F_{16} - F_{14}$$

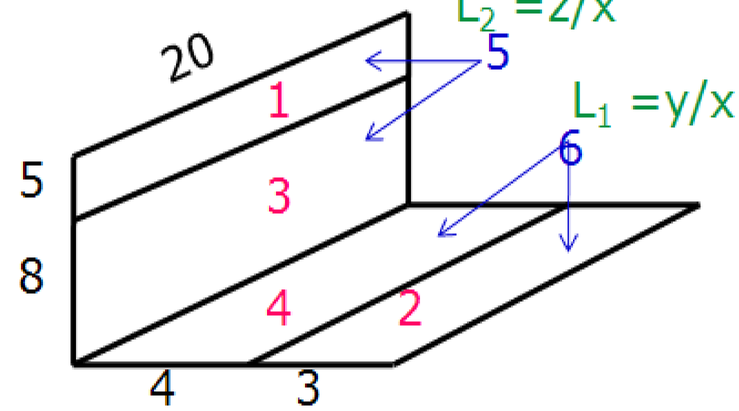
$$= \frac{A_6}{A_1} F_{61} - \frac{A_4}{A_1} F_{41}$$

$$= \frac{A_6}{A_1}\left(F_{65} - F_{63}\right) - \frac{A_4}{A_1}\left(F_{45} - F_{43}\right)$$

$$F_{65} : \frac{L_1}{W} = \frac{7}{20} = 0.35 \;\&\; \frac{L_2}{W} = \frac{13}{20} = 0.65$$

om graph: $F_{65}=0.32$

Solution (Contd):



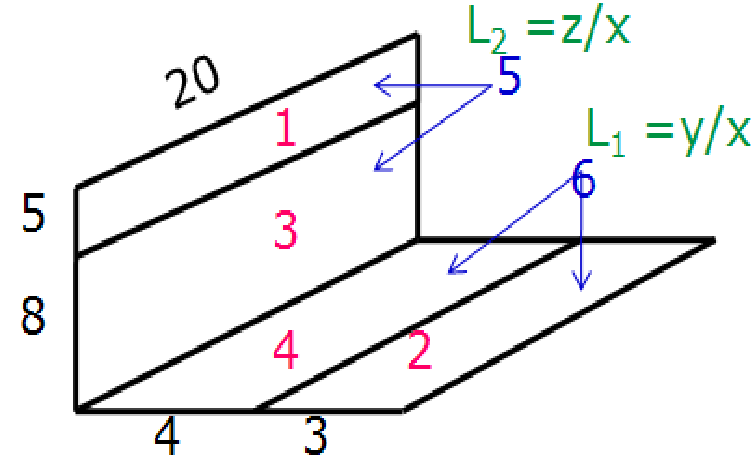
$$F_{63}\ :\ \frac{L_1}{W}=\frac{7}{20}=0.35\ \&\ \frac{L_2}{W}=\frac{8}{20}=0.4$$

From graph: F_{63}=0.26

$$F_{45}\ :\ \frac{L_1}{W}=\frac{4}{20}=0.2\quad \&\quad \frac{L_2}{W}=\frac{13}{20}=0.65$$

om graph: F_{45}=0.36

Solution (Contd):



$$F_{43} : \frac{L_1}{W} = \frac{4}{20} = 0.2 \quad \& \quad \frac{L_2}{W} = \frac{8}{20} = 0.4$$

From graph: $F_{43}=0.33$

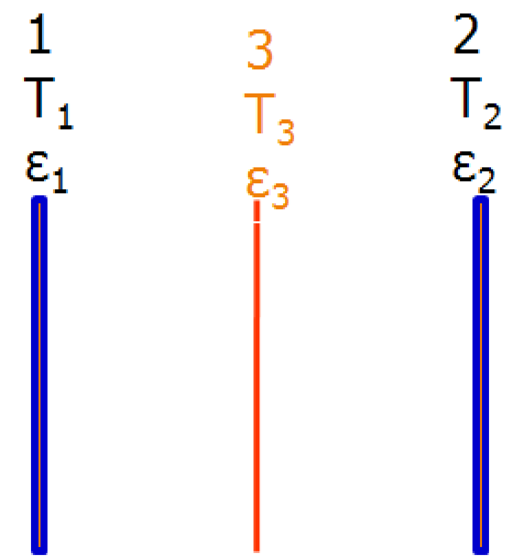
$$F_{12} = \frac{A_6}{A_1}\left(F_{65} - F_{63}\right) - \frac{A_4}{A_1}\left(F_{45} - F_{43}\right)$$

$$= \frac{7x20}{5x20}\left(0.32 - 0.26\right) - \frac{4x20}{5x20}\left(0.36 - 0.33\right) = 0.06$$

Q3: Find out heat transfer rate due to radiation between two infinitely long parallel planes. One plane has emissivity of 0.4 and is maintained at 200°C. Other plane has emissivity of 0.2 and is maintained at 30°C. If a radiation shield (ε=0.5) is introduced between the two planes, find percentage reduction in heat transfer rate and steady state temp of the shield.

Solution:

$$q_{12} = \frac{\sigma\left(T_1{}^4 - T_2{}^4\right)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}$$



$$q_{12} = \frac{5.67x10^{-8}\left[(200+273)^4 - (30+273)^4\right]}{\dfrac{1}{0.4} + \dfrac{1}{0.2} - 1}$$

$= 363\ W/m^2$

Solution (Contd):

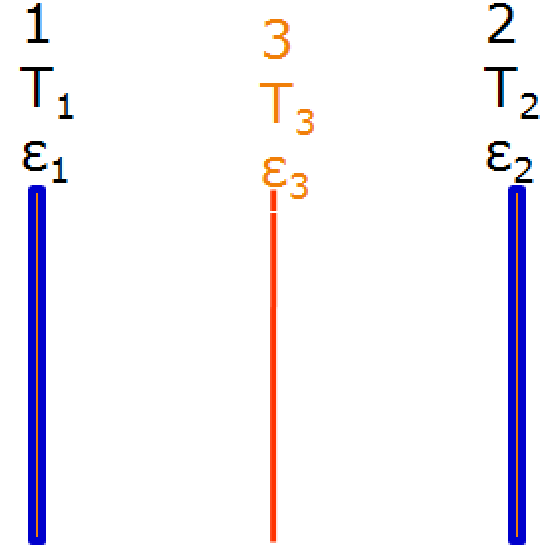
$$When\ shield\ is\ inserted;\ q_{13} = \frac{\sigma\left(T_1^{\,4} - T_2^{\,4}\right)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{2}{\varepsilon_3} - 1\right)}$$

$$q_{13} = \frac{5.67x10^{-8}\left[(200+273)^4 - (30+273)^4\right]}{\left(\frac{1}{0.4} + \frac{1}{0.2} - 1\right) + \left(\frac{2}{0.5} - 1\right)} = 248.4\ W/m^2$$

$$Percentage\ reduction = \frac{q_{12} - q_{13}}{q_{12}} x100$$

$$= \frac{363 - 248.4}{363} x100 = 31.57\%$$

Solution (Contd):



Under Steady State Conditions, we have:

$$q_{13} = q_{32} \Rightarrow \frac{\sigma\left(T_1^{\,4} - T_3^{\,4}\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} = \frac{\sigma\left(T_3^{\,4} - T_2^{\,4}\right)}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1}$$

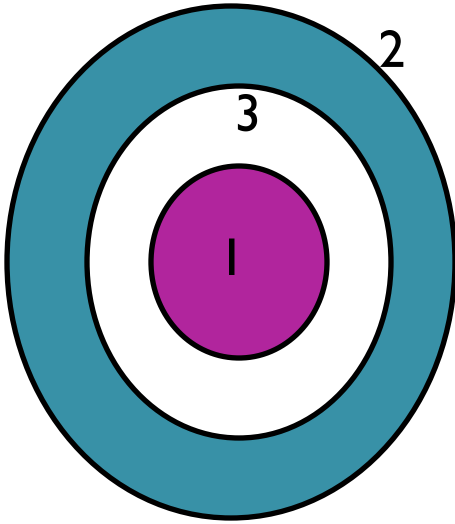
$$or \quad \frac{\sigma\left[(200+273)^4 - T_3^{\,4}\right]}{\left(\frac{1}{0.4} + \frac{1}{0.5} - 1\right)} = \frac{\sigma\left[T_3^{\,4} - (30+273)^4\right]}{\left(\frac{1}{0.5} + \frac{1}{0.2} - 1\right)}$$

$$T_3 = 431.67\,K$$

Q1. Cryogenic fluid flows through annular space of inner tube dia of 30mm and outer tube of 90mm dia. Surface emissivities of inner and outer tubes are 0.2 and 0.5, while respective temps are 100 and 300K. Find Heat gain rate by the fluid per meter length. Also, percentage reduction in heat gain, if a radiation shield of tubular shape having 45mm diameter and emissivities of 0.1 on inner surface and 0.05 on outer surface is introduced between the two tubes.

<u>Solution:</u>

$$Q_{12} = \frac{\sigma A_1 \left(T_1^{\,4} - T_2^{\,4}\right)}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$$



$$= \frac{5.67x10^{-8}\, x \pi x 0.03 x 1 \left(100^4 - 300^4\right)}{\frac{1}{0.2} + \frac{\pi x 0.03 x 1}{\pi x 0.09 x 1}\left(\frac{1}{0.5} - 1\right)} = -8\ W$$

Solution (Contd):

With shield:

$$Q_{13} = \frac{\sigma A_1 \left(T_1^{\ 4} - T_2^{\ 4}\right)}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right) + \frac{A_1}{A_3}\left(\frac{1}{\varepsilon_{31}} + \frac{1}{\varepsilon_{32}} - 1\right)}$$

$$= \frac{5.67 x 10^{-8} x \pi x 0.03 x 1 \left(100^4 - 300^4\right)}{\frac{1}{0.2} + \frac{\pi x 0.03 x 1}{\pi x 0.09 x 1}\left(\frac{1}{0.5} - 1\right) + \frac{\pi x 0.03 x 1}{\pi x 0.045 x 1}\left(\frac{1}{0.1} + \frac{1}{0.05} - 1\right)}$$

$$= -1.732\ W$$

$$Percentage\ \ reduction = \frac{Q_{12} - Q_{13}}{Q_{12}} x 100$$

$$\frac{8 - 1.732}{8} x 100 = 78.38\% \ \ ANSWER$$

Q5. A pipe carrying steam having an outside dia of 20cm passes through a large room and is exposed to air at temp of 30°C. Pipe surface temp is 200°C. Find the heat loss per meter length of pipe both by convection and radiation taking emissivity of the pipe surface as 0.8.

Use following relations:

$Nu = 0.53(Gr.Pr)^{0.25}$ for horizontal pipe

Air properties:

Temp °C	K (W/mK)	$\nu \times 10^6$ (m^2/s)	Pr
30	0.0267	18.60	0.701
115	0.0330	24.93	0.687
200	0.0393	26.00	0.680

Solution:

For Radiation:

$$Q_r = \frac{\sigma A_1 \left(T_1^{\,4} - T_2^{\,4}\right)}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$$

$$Since\ \ A_2\ \ (Room) \succ\succ\succ A_1 (Pipe)\ \Rightarrow \frac{A_1}{A_2} \rightarrow 0$$

$$\therefore Qr = \varepsilon_1\, \sigma\, A_1 \left(T_1^{\,4} - T_2^{\,4}\right)$$

$$= 0.8x5.67x10^{-8}\, x\pi\, x0.2x1\left[(200+273)^4 - (30+273)^4\right]$$

$$1185.6\ W/m$$

Solution (Contd):

For Convection:

$$T_{mean} = \frac{200+30}{2} = 115°C$$

$$G_r = \frac{g\beta \Delta T L^3}{\nu^2} = \frac{9.81x1x(200-30)x0.2^3}{(115+273)(24.93x10^{-6})^2} = 5.53x10^7$$

$$Gr.\Pr = 5.53x10^7\, x0.687 = 38x10^6$$

$$Hence, selecting \quad Nu = 0.53(Gr.\Pr)^{0.25} = \frac{hD}{k}$$

$$\therefore h = \frac{0.033x0.53(38x10^6)^{0.25}}{0.2} = 6.87\, W/m^2K$$

$$Q = hA(T_w - T_\infty) = 6.87x\pi x0.2x1(200-30) = 733.8\, W/m$$

al $Q = Q_c + Q_r = 733.8 + 1185.6 = 1919.4\, W/m$

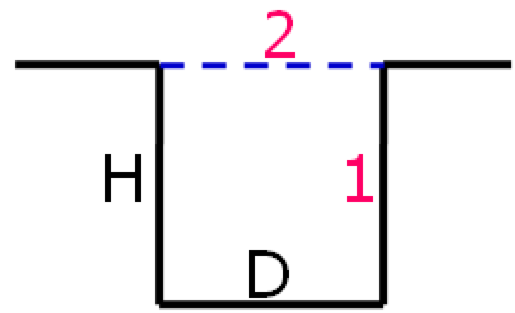
Q7: Find shape factor for the following wrt itself:

a) Cylindrical cavity of dia D and depth H

b) Conical hole of dia D and depth H

c) Hemispherical hole of dia D

<u>Solution</u>: a) We know $F_{21} = 1$



As $F_{12}A_1 = F_{21}A_2 \quad » F_{12} = A_2/A_1$

We know that $F_{11} + F_{12} = 1$

as surfaces 1 and 2 form enclosure

$$Hence \; F_{11} = 1 - F_{12} = 1 - \frac{A_2}{A_1}$$

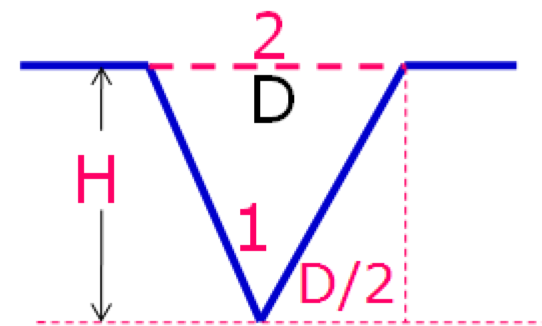
$$Now \; A_1 = \pi DH + \frac{\pi}{4} D^2$$

d $A_2 = \frac{\pi}{4} D^2$

$$\Rightarrow F_{11} = 1 - \frac{\pi/_4 D^2}{\pi DH + \pi/_4 D^2} = \frac{4H}{4H + D}$$

Solution:

b) We know that $F_{12}A_1 = F_{21}A_2$



$$\therefore F_{12} = \frac{A_2}{A_1} F_{21} = \frac{A_2}{A_1} \; as \; F_{21} = 1$$

Also, we know that : $F_{11} + F_{12} = 1$

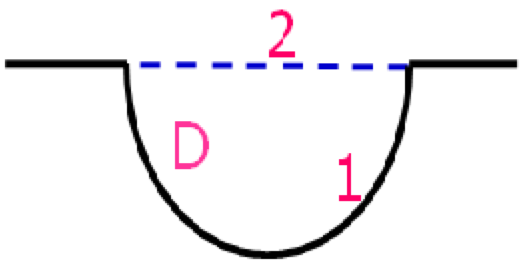
as surfaces 1 & 2 form enclosure

$$\therefore F_{11} = 1 - \frac{A_2}{A_1} \qquad Now \; A_2 = \frac{\pi}{4} D^2; \; A_1 = \frac{\pi D L}{2}$$

$$Now \; L^2 = H^2 + \left(\frac{D}{2}\right)^2 \Rightarrow L = \sqrt{H^2 + \left(\frac{D}{2}\right)^2}; \; L = Slant \; Height$$

$$Hence \; F_{11} = 1 - \frac{\pi/_4 D^2}{\frac{\pi D}{2}\sqrt{H^2 + \left(\frac{D}{2}\right)^2}} = 1 - \frac{D}{\sqrt{4H^2 + D^2}}$$

Solution:



c) $F_{11}+F_{12}=1$ as surfaces 1 & 2 form enclosure

Hence $F_{11}=1-F_{12}$

$$We\ also\ know\ that\ F_{12}A_1 = F_{21}A_2$$

$$Hence\ F_{12} = \frac{A_2}{A_1}F_{21} = \frac{A_2}{A_1}\ as\ F_{21} = 1$$

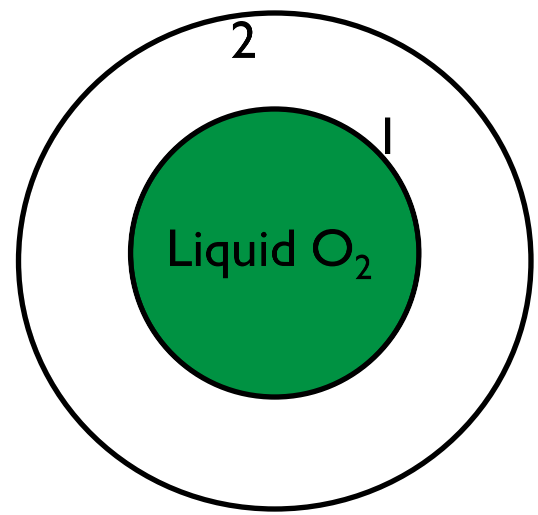
$$\therefore F_{11} = 1 - \frac{A_2}{A_1}\quad Now\ A_1 = \frac{4\pi\left(\frac{D}{2}\right)^2}{2} = \frac{\pi D^2}{2}\ \&\ A_2 = \frac{\pi}{4}D^2$$

$$\frac{A_2}{A_1} = \frac{\frac{\pi}{4}D^2}{\frac{\pi D^2}{2}} = 0.5\ Hence\ F_{11} = 1 - 0.5 = 0.5$$

Q7. Two concentric spheres 210mm and 300mm diameters with the space between evacuated are to be used to store liquid air (-153°C) in a room at 27°C. The surfaces of the spheres are flushed with aluminum (ε=0.03) and latent heat of vaporization of liquid air is 209.35 kJ/kg. Calculate the rate of evaporation of liquid air per hour. (Ans. 0.0217 kg/h)

<u>Solution</u>:

$$\textit{Heat Flow from inner surface 1 to outer surface 2 is given as:}$$



$$Q_{12} = \frac{A_1 \sigma \left(T_1^{\,4} - T_2^{\,4}\right)}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$$

Solution (Contd):

$$Q_{12} = \frac{4\pi(0.105)^2 x 5.67 x 10^{-8}\left[(-153+273)^4-(27+273)^4\right]}{\frac{1}{0.03}+\frac{4\pi(0.105)^2}{4\pi(0.150)^2}\left(\frac{1}{0.03}-1\right)}$$

$$\Rightarrow Q_{12} = -1.26\,W$$

Negative sign indicates heat being received by surface 1 (Oxygen)

Rate of evaporation:

$$= \frac{Q}{\lambda} = \frac{1.26x3600}{209.35x1000} = 0.0217\,kg/h$$

Q8: The filament of a 75W light bulb may be considered a black body radiating in to black enclosure at 70°C. The filament dia is 0.1mm and length is 5cm. Considering radiation, determine the filament temp. (Ans. 2756°C)

Solution:

$$Q_{12} = \frac{A_1 . \sigma \left(T_1^{\ 4} - T_2^{\ 4}\right)}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$$

Since small A_1 is enclosed in large A_2, hence A_1/A_2 » 0

$$\therefore Q_{12} = A_1 \varepsilon_1 \sigma \left(T_1^{\ 4} - T_2^{\ 4}\right)$$

$$5 \ = \pi x 0.1 x 10^{-3} x 0.05 x 1 x 5.67 x 10^{-8} \left[\left(T_1^{\ 4}\right) - (70+273)^4\right]$$

$$> T_1 = 3029 K \ (= 2756°C)$$

Q9: Determine the heat loss rate by radiation from a steel tube of outside dia 70mm and 3m long at a temp of 227°C, if the tube is located within a square brick conduit of 0.3m side and at 27°C. Take emissivity of steel=0.79 and that for brick=0.93. (Ans. 1589.7W)

<u>Solution:</u>

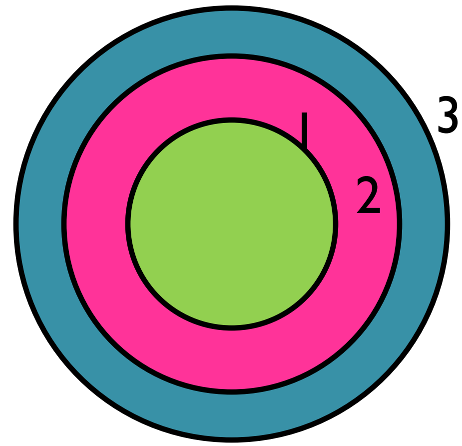
$$Q_{12} = \frac{A_1 \sigma \left(T_1^{\,4} - T_2^{\,4}\right)}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$$

$$Q_{12} = \frac{(\pi x 0.07 x 3) x 5.67 x 10^{-8}\left[(227+273)^4 - (27+273)^4\right]}{\frac{1}{0.79} + \frac{(\pi x 0.07 x 3)}{4 x 0.3 x 3}\left(\frac{1}{0.93} - 1\right)}$$

$589.7W$ *Answer*

Q10: Three hollow thin walled cylinders having dia 10cm, 20cm and 30cm are arranged concentrically. The temps of the innermost and outermost cylindrical surfaces are 100K and 300K respectively. Assuming vacuum in annular spaces, find the steady state temp attained by the surface having dia of 20cm. Take emissivities of all surfaces as 0.05.

(Ans. 269K)



<u>Solution:</u>

Under steady state:

$$Q_{12} = \frac{A_1 \sigma \left(T_1^{\,4} - T_2^{\,4}\right)}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)} = Q_{23} = \frac{A_2 \sigma \left(T_2^{\,4} - T_3^{\,4}\right)}{\frac{1}{\varepsilon_2} + \frac{A_2}{A_3}\left(\frac{1}{\varepsilon_3} - 1\right)}$$

Solution (Contd):

A_1=лDL=3.14x0.1x1=0.314

A_2=3.14x0.2x1=0.628

A_3=3.14x0.3x1=0.942

$$We\ have: \frac{A_1\sigma\left(T_1^{\,4}-T_2^{\,4}\right)}{\frac{1}{\varepsilon_1}+\frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2}-1\right)}=\frac{A_2\sigma\left(T_2^{\,4}-T_3^{\,4}\right)}{\frac{1}{\varepsilon_2}+\frac{A_2}{A_3}\left(\frac{1}{\varepsilon_3}-1\right)}$$

$$\frac{0.314\left(100^4-T_2^{\,4}\right)}{\frac{1}{0.05}+\frac{0.314}{0.628}\left(\frac{1}{0.05}-1\right)}=\frac{0.628\left(T_2^{\,4}-300^4\right)}{\frac{1}{0.05}+\frac{0.628}{0.942}\left(\frac{1}{0.05}-1\right)}$$

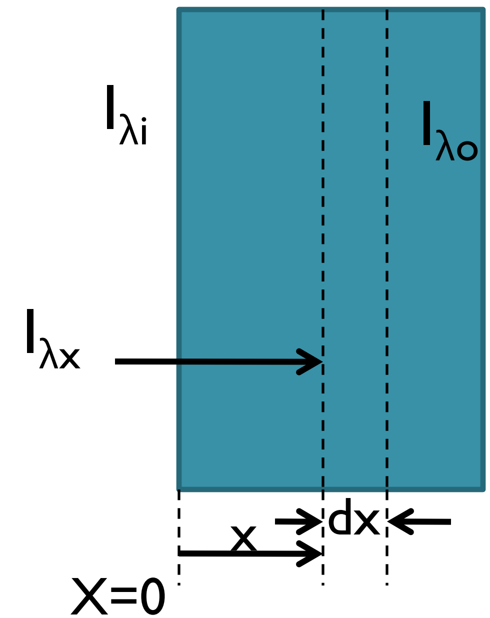
$$\Rightarrow T_2 = 269K\ \ Answer$$

Gas Radiation

- Gases in many cases are transparent to radiation
- When they absorb and emit radiation, they usually do so only in certain narrow wavelength bands.
- Some gases such as N_2, O_2 and other non-polar gases are essentially transparent to radiation and they do not emit radiation
- While polar gases like CO_2, H_2O and various hydrocarbon gases absorb and emit radiation to an appreciable extent in narrow wavelength bands.
- For solids and liquids, radiation occurs from thin ver (1μm to 1mm) of surface, hence it is surface ienomenon. However, for gases it is not surface it volumetric phenomenon.

Volumetric Absorption:

- Let a monochromatic beam of radiation having an Intensity $I_{\lambda i}$ impinges on the gas layer of thickness dx as shown in Fig.
- Decrease in intensity resulting from absorption in the layers is proportional to the thickness of layer and intensity of radiation at that point



- Thus;

$$dI_{\lambda} = -k_{\lambda} I_{\lambda} dx;$$

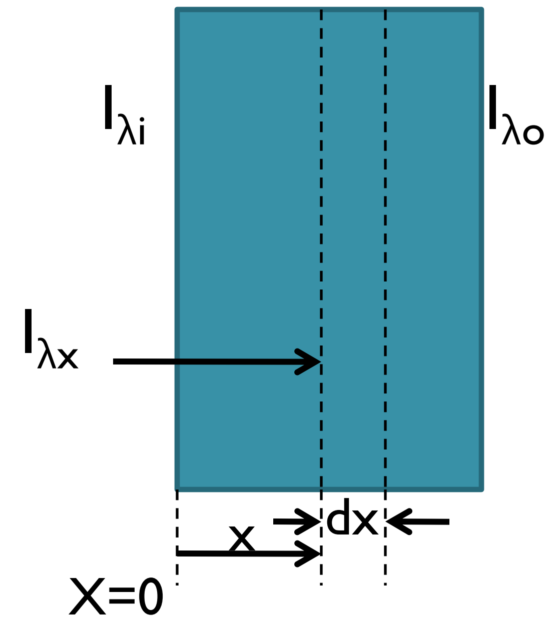
where k_{λ} is called monochromatic absorption coefficient

Volumetric Absorption:

- Integrating this equation gives;

$$\int_{I\lambda i}^{I\lambda x} \frac{dI_{\lambda}}{I_{\lambda}} = \int_{0}^{x} -k_{\lambda}.dx$$

$$or \; \frac{I_{\lambda x}}{I_{\lambda i}} = e^{-k_{\lambda} x}$$



- This is Beer's Law and represents xponential decay of radiation tensity

Gas Radiation

- we know that monochromatic transmissivity;

$$\tau_\lambda = e^{-k_\lambda x}$$

- If gas is non-reflecting, then;

$$\tau_\lambda + \alpha_\lambda = 1$$

$$and\ hence \quad \alpha_\lambda = 1 - \tau_\lambda$$

$$Therefore\ Absorptivity\ \alpha_\lambda = 1 - e^{-k_\lambda . x}$$

$$,\ for\ grey\ surface\ ,\ Emissivity \quad \varepsilon_\lambda = \alpha_\lambda = 1 - e^{-k_\lambda . x}$$

Emissivity of CO_2, H_2O Vapor & Gas

- Emissivity of a gas mixture is a function of total pressure (P), partial pressure of a gas(p), gas temperature (T) and characteristic dimension of the system; also known as beam length

$$\therefore \varepsilon = f\left(P, p, T, L\right)$$

- When the gas mixture is at 1 atm total pressure, the emissivity of CO2 and H2O vapors are given by following empirical relations;

$$\varepsilon_c = 3.5(p.L)^{0.33}\left[\frac{T}{100}\right]^{3.5}$$

$$= 3.5p^{0.8}L^{0.6}\left[\frac{T}{100}\right]^{3}$$

For most practical cases, Mean Beam Length is taken as

$$L = 3.6x\frac{Volume\ of\ Gas\ mixture}{Surface\ area\ of\ enclosure}$$

Heat Exchange between Gas Volume & its Enclosure

- Rate of radiant heat transfer from the gas to its enclosure is given by:

$$Q = \varepsilon_g . A_s . \sigma . T_g^{\ 4} ;$$

$$where\ \varepsilon_g\ emissivity\ of\ gase\ mixture$$

$$A_s\ Enclosure\ inside\ surface\ area$$

$$T_g\ Gase\ mixture\ temp$$

- If the enclosure surface is black, it will absorb all this radiation but it will also emit radiation. Hence net rate, at which the radiation is exchanged between the black enclosure surface at temp T_s and the gas mixture at temp T_g (T_g T_s) is given by:

$$Q = A_s . \sigma \left[\varepsilon_g . T_g^{\ 4} - \alpha_g . T_s^{\ 4} \right]$$

Heat Exchange between Gas Volume & its Enclosure

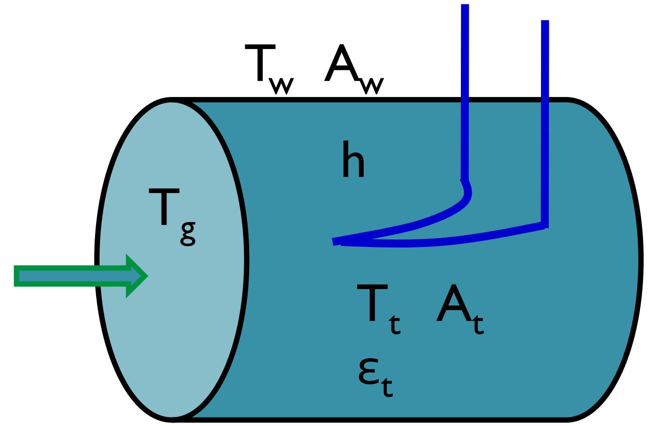
- If the enclosure surface is grey, the net heat transfer to grey enclosure having emissivity ε_{grey} is given by:

$$\frac{Q_{grey}}{Q_{black}} = \frac{\varepsilon_{grey} + 1}{2}$$

Error In Temp Measurement By Thermocouple

For measurement of temp of a hot gas flowing through a duct, a thermocouple (TC) is placed in direct contact with gas.

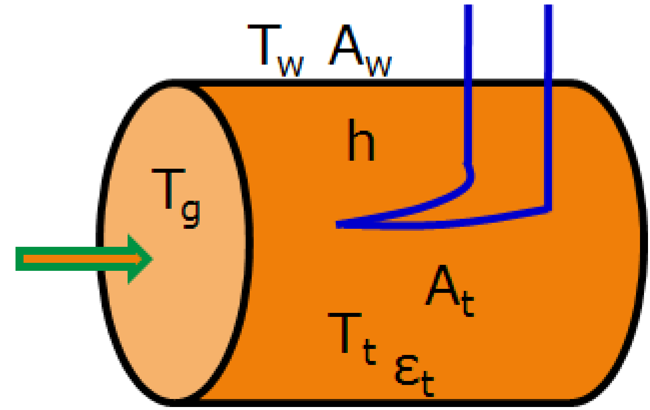
Thermocouple receives heat from gas by convection and tries to attain gas temp.



As the temp of thermocouple junction rises, it starts loosing energy by radiation to duct wall , which is at a lower temp.

Thus, temp of thermocouple (TC) reduces. Hence, temp recorded by thermocouple (T_t) is always less than gas temp (T_g)

Error In Temp Measurement By Thermocouple



When TC attains steady state temp T_t , this means heat energy being received by it by convection from hot gas at temp T_g is equal to heat being lost by it by radiation to duct which is at T_w

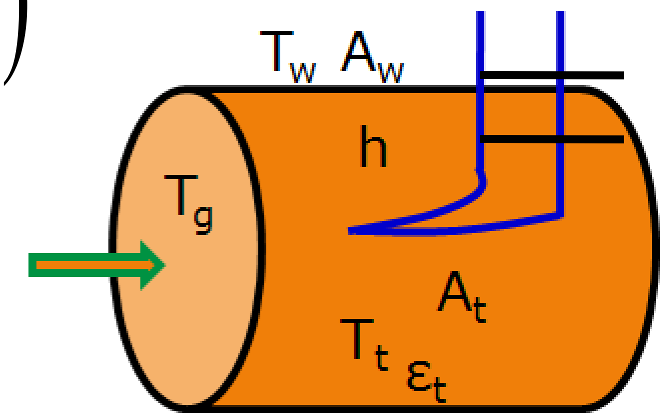
$$\therefore \; h.A_t\left(T_g - T_t\right) = \frac{\sigma.A_t\left(T_t^{\,4} - T_w^{\,4}\right)}{\dfrac{1}{\varepsilon_t} + \dfrac{A_t}{A_w}\left(\dfrac{1}{\varepsilon_w} - 1\right)}$$

$$Since \; \frac{A_t}{A_w} \rightarrow 0, \; hence \quad h\left(T_g - T_t\right) = \sigma.\varepsilon_t\left(T_t^{\,4} - T_w^{\,4}\right)$$

$$re\left(T_g - T_t\right) \; is \; error \, in \; measurement$$

Error In Temp Measurement By Thermocouple

$$h\left(T_g - T_t\right) = \sigma.\varepsilon_t\left(T_t^{\,4} - T_w^{\,4}\right)$$



Error $(T_g - T_t)$ in measurement can be reduced by;

1) Providing radiation shield around Thermocouple

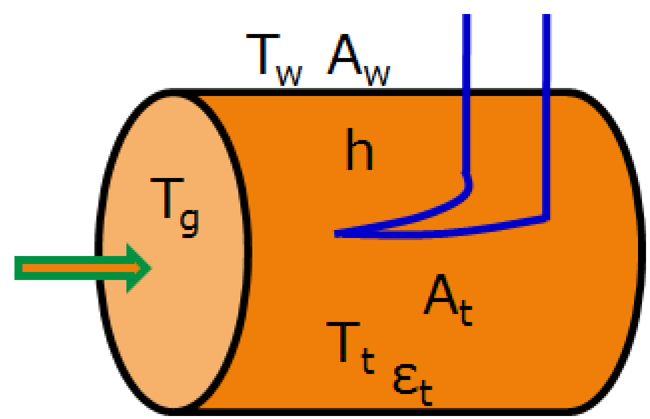
2) Reducing emissivity of thermocouple junction. For this, junction may be coated with some material having low emissivity like Aluminum, Zinc, Chromium, etc

Q6: A thermocouple (ε=0.6) is used to measure the temp of exhaust gases flowing through the duct. Temp of the duct is 20°C. If the thermocouple measures a temp of 500°C:

a) Calculate the error in temp measurement

b) If the thermocouple is enveloped by sufficiently long cylindrical shield (ε=0.3), find out the error in measurement of temp taking h=200 W/m^{2}K.

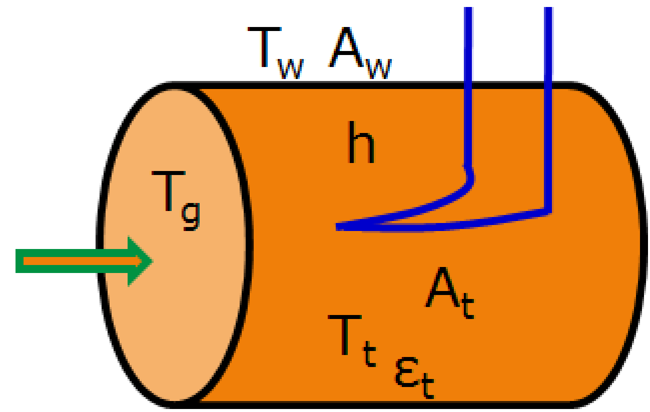
<u>Solution:</u>



Under steady state conditions, Heat being received by TC by Convection = Heat being lost TC by radiation to duct wall

Solution (Contd):

$$\therefore\ h.A_t\left(T_g - T_t\right) = \frac{\sigma.A_t\left(T_t^{\,4} - T_w^{\,4}\right)}{\dfrac{1}{\varepsilon_t} + \dfrac{A_t}{A_w}\left(\dfrac{1}{\varepsilon_w} - 1\right)}$$



Since A_t / A_w »0,

We have;

$$h.A_t\left(T_g - T_t\right) = \varepsilon_t\,\sigma.A_t\left(T_t^{\,4} - T_w^{\,4}\right)$$

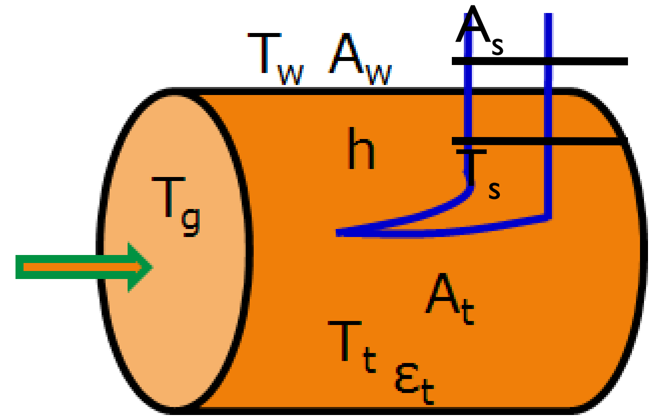
$$\therefore\ \left(T_g - T_t\right) = \frac{0.6\,x\,5.67x10^{-8}}{200}\left[\left(500+273\right)^4 - \left(20+273\right)^4\right]$$

ence, Error in temp measurement

$- T_t = 60\ K$

Solution (Contd):

T_g=500+273+60=833K



Assuming A_t<<A_s , as the shield is sufficiently long cylinder and it is receiving heat by convection from both surfaces; we have,

$$h.2A_s(T_g - T_s) = \varepsilon_s.A_s.\sigma .(T_s^4 - T_w^4)$$

Substituting;

$$200x2(833 - T_s) = 0.3x5.67x10^{-8}x(T_s^4 - 293^4)$$

Or $\quad 4.25x10^{-11}T_s^4 + T_s - 833.3=0$

T_s = 815 K

Solution (Contd):

Now taking up heat transfer from hot gases to TC by Convection and from TC to shield by radiation:

$$h.A_t.(T_g - T_t) = \sigma .\varepsilon_t.A_t.(T_t^4 - T_s^4)$$

Substituting:

$$200\ (833 - T_t) = 5.67\times10^{-8}\times0.6\times(T_t^4 - 815^4)$$

» T_t=829K

ence, error with shield now becomes:

$_g - T_t = 833 - 829 = 4$ K

End of Unit - V

Heat Transfer in Condensation & Boiling

(Special Cases of Convection)

Condensation:

Change of phase from vapor to liquid

Boiling:

Change of phase from liquid to vapor

Important Aspects:

- In both cases, Q=mλ; where λ is latent heat

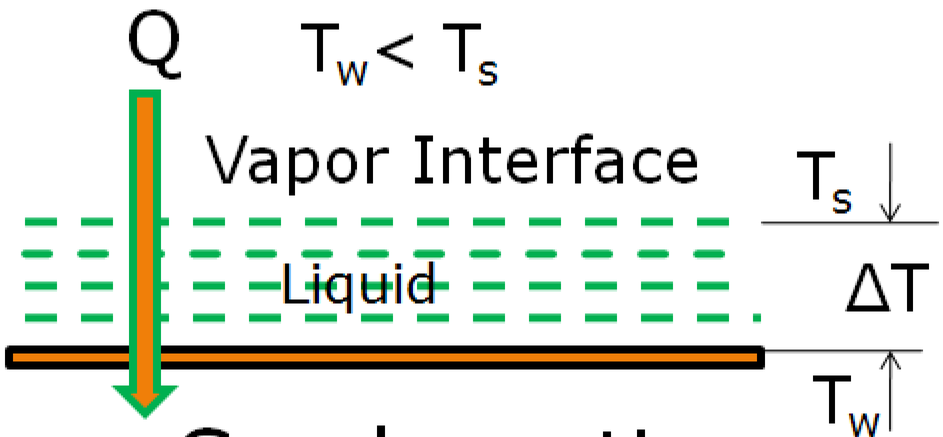
oth phenomena are two phase conversion

Heat Transfer in Condensation & Boiling

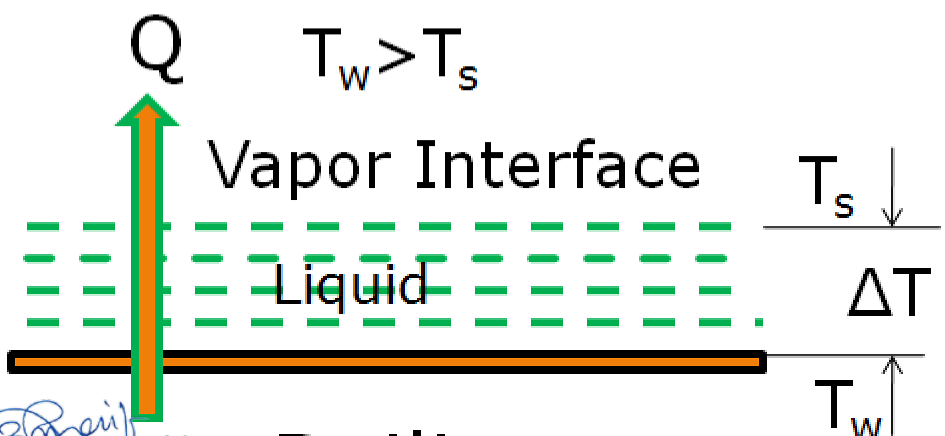
Important Aspects (contd):

- In both cases, temp at interface between the liquid and vapor phases is equal to saturation temp (T_s) of the matter
- In condensation, the condensate (liquid), while in boiling, the vapor , forms a film over the surface. Properties of this film governs the heat transfer process
- At moderate ΔT, very high heat transfer coefficient are obtained (5000 to 50,000). ence, where very high Q is needed, o phase convection is used.

Heat Transfer in Condensation & Boiling



Condensation



Boiling

Condensation

When phase change from vapor to liquid occurs by giving out latent heat to surface, on which it is condensing, which is at a temp lower than saturation temp, the process is called condensation.

Types of Condensation

1. Dropwise Condensation
2. Filmwise Condensation

Types of Condensation

1. Dropwise Condensation:

- When a saturated vapor comes in contact with a colder surface, it condenses giving out latent heat to surface and liquid droplets are formed on the surface.
- These droplets, if they do not have affinity with the surface, instead of getting deposited on the surface, these drop down under gravitational force, leaving the surface bare for successive droplets to form.
- Generally, steam has been found to condense in this manner.

Dropwise Condensation (Contd):

- Experimentally, it has been found that heat transfer rate is much higher than Filmwise condensation (5 to 8 times) as the surface remains in direct contact with vapor.

- Therefore, dropwise condensation is always desirable. However, it is not achievable for very long time, because once surface gets wet, it results in filmwise condensation.

- Some additives/ promoters can maintain and prolong dropwise condensation. Examples are Oliec Acid, highly polished surface etc

Filmwise Condensation:

- Due to affinity, droplets form a film of the condensate on the surface and due to gravitational force, it flows down the surface.
- Thickness of the film increases in the downwards direction
- Due to lower thermal conductivity of the condensate, liquid film offers high resistance to heat flow
- Due to the above reasons, heat transfer rate nd rate of condensation are lower than ropwise condensation.

Film Wise Condensation on Vertical Plate

(Nusselt Theory of Laminar Film Condensation)

According to Nusselt, the condensed liquid forms a continuous film on the surface and Heat flow rate is determined by the thermal resistance of this film.

Assumptions:

- Flow of condensate in the film is Laminar
- Fluid properties are constant
- Velocity & thermal BLs are same
- Heat transfer across film is due to pure conduction & temp distribution is linear
- Liquid-vapor interface is at saturation temp
- shear stress or thermal resistance at liquid-vapor erface

Film Wise Condensation on Vertical Plate

- Consider a thin section at a distance of z of thickness dz of vertical plate

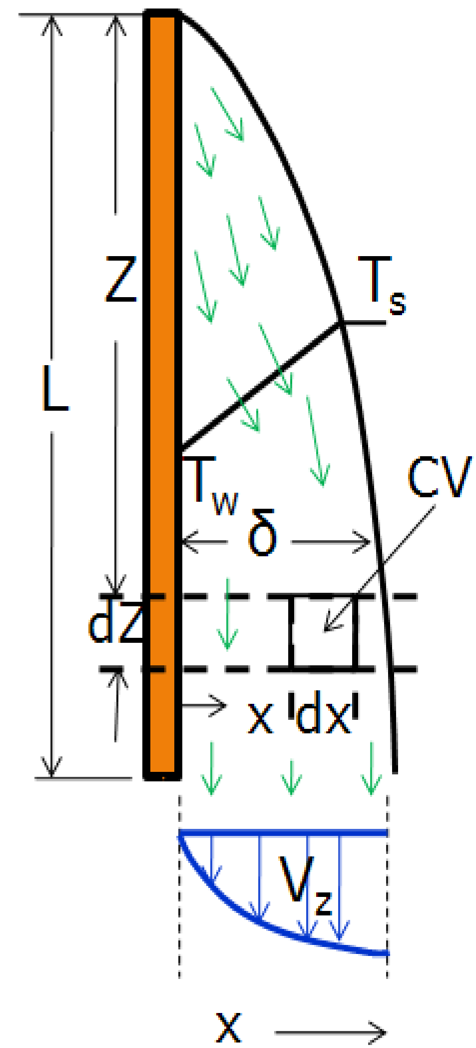
- From this section, consider a differential CV at a distance of x from the plate of thickness dx in the film

- Let the plate be of unit width

- T_w is plate temp & T_s interface temp

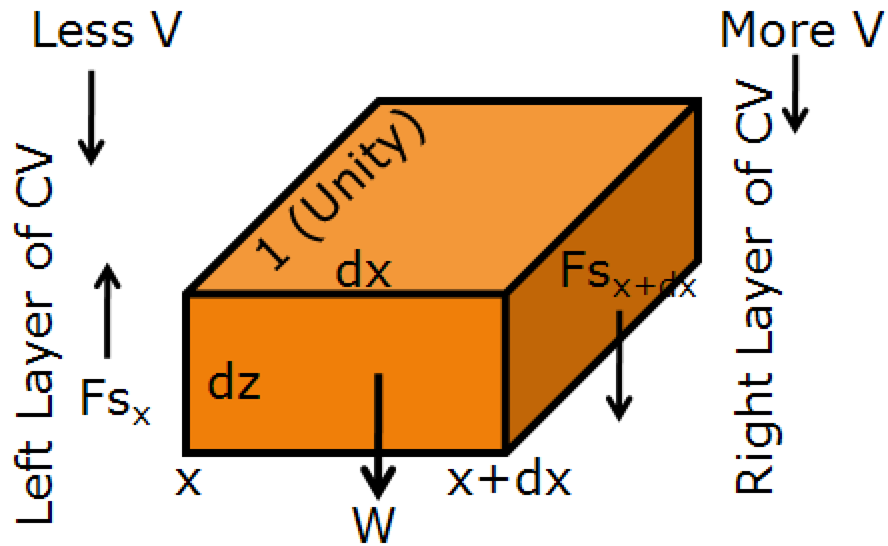
- Let V_z be the velocity of elemental volume in z direction

- Let δ be the film thickness at z



Film Wise Condensation on Vertical Plate

- Consider CV
- Direction of Shear Force Fs_x on left face at x will be in upwards dirn while Fs_{x+dx} on right face in downward dirn



- Weight of CV W=(dx.dz.1).ρ.g
- Shear Stress ∞ dV_z/dx

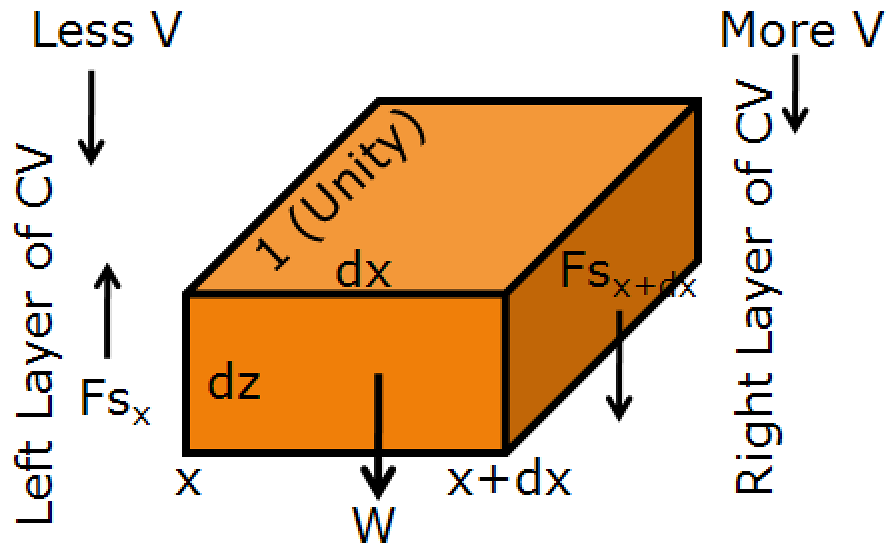
$$Hence, Shear\ Force\ on\ left\ face$$

$$upward\ dirn\ Fs_x = \mu \frac{dV_z}{dx}(dz.1)$$

Film Wise Condensation on Vertical Plate

- Weight of CV ie, W will act downward

- Similarly, Shear Force acting on right face at x+dx will act downward:



$$Fs_{x+dx} = \mu \frac{dV_z}{dx}.(dz.1) + \frac{d}{dx}\left(\mu \frac{dV_z}{dx}.(dz.1) \right).dx$$

Film Wise Condensation on Vertical Plate

Hence, writing force balance equation for CV:

$Fs_x = Fs_{x+dx} + W$

$$\Rightarrow \mu \frac{dV_z}{dx}.dz = \mu \frac{dV_z}{dx}.dz + \frac{d}{dx}\left(\mu \frac{dV_z}{dx}.dz \right) dx + \rho.g.dx.dz$$

$$OR \quad \mu \frac{d^2V_z}{dx^2}.dx.dz = -\rho.g.dx.dz$$

$$\frac{d^2V_z}{dx^2} = -\frac{\rho g}{\mu} \ldots\ldots\ldots\ldots(1)$$

Film Wise Condensation on Vertical Plate

$$\therefore \quad \frac{d^2V_z}{dx^2} = -\frac{\rho g}{\mu} \ldots\ldots\ldots\ldots(1)$$

Integrating eqn (1), *we have*;

$$\frac{dV_z}{dx} = -\frac{\rho g}{\mu}x + C_1 \ldots\ldots\ldots(2)$$

On further integrating *eqn* (2)

$$= -\frac{\rho g x^2}{2\mu} + C_1 x + C_2 \ldots\ldots\ldots(3)$$

Film Wise Condensation on Vertical Plate

Boundary Conditions:

BC 1) At $x=0; V_z=0$

BC 2) At $x=\delta$; $dV_z/dx=0$

From BC 1) and Eqn (3), we have $0=0+0+C_2 \rightarrow C2=0$

From BC 2) and eqn (2), we have,

$$0 = -\frac{\rho g \delta}{\mu} + C_1 \;\Rightarrow C_1 = \frac{\rho g \delta}{\mu}$$

$$Substituting\; C_1 \,\&\, C_2 \; in\, eqn \;\; (3)\; V_z = -\frac{\rho g x^2}{2\mu} + \frac{\rho g \delta}{\mu} x$$

$$V_z = \frac{\rho g}{\mu}\left(x\delta - \frac{x^2}{2} \right) ....(4)$$

Film Wise Condensation on Vertical Plate

Now, mass flow rate through CV;

$dm = \rho(dx.1).V_z$

$$Substituting\ V_z\ in\ dm\ equation,$$

$$dm = \rho.\left[\frac{\rho g}{\mu}\left(x\delta - \frac{x^2}{2}\right)\right]dx$$

$$Or\ \ dm = \frac{\rho^2 g}{\mu}\left(x\delta - \frac{x^2}{2}\right)dx........(5)$$

Film Wise Condensation on Vertical Plate

Now, mass flow rate of liquid through section at z can be obtained by integrating dm over film thickness δ:

$$\int dm = \int_0^{\delta} \frac{\rho^2 g}{\mu}\left(x\delta - \frac{x^2}{2} \right) dx$$

$$Or \quad m = \frac{\rho^2 g}{\mu}\left[\frac{\delta x^2}{2} - \frac{x^3}{6} \right]_0^{\delta} = \frac{\rho^2 g}{\mu}\left[\frac{\delta^3}{2} - \frac{\delta^3}{6} \right]$$

$$\frac{\rho^2 g}{\mu}\left[\frac{3\delta^3 - \delta^3}{6} \right] \qquad \Rightarrow m = \frac{\rho^2 g \delta^3}{3\mu} \quad \ldots\ldots\ldots(6)$$

Film Wise Condensation on Vertical Plate

Now, rate of condensation(rate of change of mass) at section Z of thickness δ will be dm/dδ

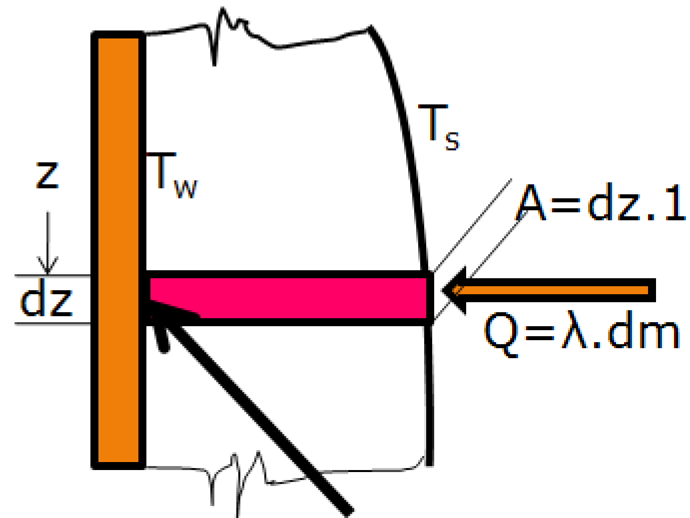
Hence, differentiating m wrt δ of eqn …(6);

$$\frac{dm}{d\delta} = \frac{\rho^2 g}{3\mu}.3\delta^2 = \frac{\rho^2 g\delta^2}{\mu}$$

$$r \quad dm = \frac{\rho^2 g\delta^2}{\mu} d\delta.........(7)$$

Film Wise Condensation on Vertical Plate

Assuming λ as latent heat; from energy balance at Interface between liquid & Vapor, we can write:



$$Q = \frac{k.(dz.1)(T_s - T_w)}{\delta}$$

Rate of heat released due to condensation of mass dm = Rate of heat conducted through film at z section

$$\therefore \lambda.dm == \frac{k.A.\Delta T}{\Delta x} = \frac{k.(dz.1)(T_s - T_w)}{\delta}$$

Film Wise Condensation on Vertical Plate

Substituting dm from eqn (7), we have,

$$\lambda.\frac{\rho^2 g\delta^2}{\mu}d\delta = \frac{k.(T_s - T_w)}{\delta}.dz$$

Separating variables, we have;

$$\delta^3.d\delta = \frac{k.(T_s - T_w)\mu}{\lambda\rho^2 g}.dz$$

Integrating we get for whole film of length z as:

$$\frac{\delta^4}{4} = \frac{k.(T_s - T_w).\mu}{\lambda\rho^2 g}.z$$

Film Wise Condensation on Vertical Plate

$$\Rightarrow \delta = \left[\frac{4k.(Ts - Tw).\mu.z}{\lambda \rho^2 g} \right]^{1/4} \text{...............}(8)$$

At elementary section of the wall, steady state heat transfer equation can be written as :

Rate of conduction through the film=

Rate of heat convection from the film to the wall

$$Or \quad \frac{k.A.(T_s - T_w)}{\delta} = h_z.A.(T_s - T_w)$$

$$h_z = \frac{k}{\delta}; \; where \; h_z \; is \; local \; heat \; transfer \; coeff$$

Film Wise Condensation on Vertical Plate

Substituting δ *from eqn* (8), *we have,*

$$h_z = k.\left[\frac{\lambda \rho^2 g}{4k(Ts - Tw).\mu.z}\right]^{1/4}$$

$$Or \quad h_z = \left[\frac{\lambda \rho^2 g k^3}{4(Ts - Tw).\mu.z}\right]^{\frac{1}{4}}$$

For obtaining h_{av} for entire length L:

$$h_{av} = \frac{1}{L}\int_0^L h_z .dz$$

Film Wise Condensation on Vertical Plate

$$h_{av} = \frac{1}{L}\int_0^L h_z \, .dz = \frac{1}{L}\left[\frac{\lambda \rho^2 g k^3}{4(Ts - Tw).\mu}\right]^{\frac{1}{4}} . \int_0^L z^{-1/4} dz$$

$$= \frac{1}{L}\left[\frac{\lambda \rho^2 g k^3}{4(Ts - Tw).\mu}\right]^{\frac{1}{4}} . \left[\frac{z^{\frac{3}{4}}}{3/4}\right]_0^L$$

$$\left[\frac{\lambda \rho^2 g k^3}{4(Ts - Tw).\mu}\right]^{\frac{1}{4}} \left[\frac{4.L^{3/4}}{3L}\right]$$

Film Wise Condensation on Vertical Plate

$$h_{av} = \frac{4}{3}\left[\frac{\lambda\rho^{2}gk^{3}}{4(Ts - Tw).\mu.L}\right]^{\frac{1}{4}} = \frac{4}{3}.h_{L}$$

$$h_{av} = 0.943\left[\frac{\lambda.\rho^{2}.g.k^{3}}{(Ts - Tw).\mu.L}\right]^{1/4}$$

This is h_{av} for Film wise Condensation on Vertical Plate

Laminar Flow of Condensate

For Condensate flow to be Laminar;

$$\mathrm{Re} = \frac{\rho_l . V . D_h}{\mu_l} \prec 1800$$

$$\mathrm{Re} = \frac{\rho_l . V . 4A}{\mu_l . P} = \frac{4m}{\mu_l . P}$$

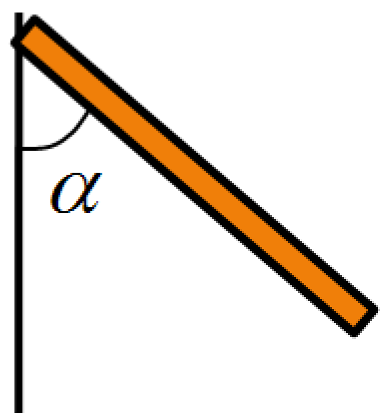
$$P = \pi D \text{ for vertical tube of outer dia}$$

$$= 2L \text{ for horizontal tube of length } L$$

$$= W \text{ for vertical or inclined plate of width } W$$

Condensation Over Inclined Plate

$$h_{av} = 0.943\left[\frac{\lambda \rho^{2} g k^{3}.Cos\,\alpha}{(Ts - Tw)\mu.L}\right]^{1/4}$$

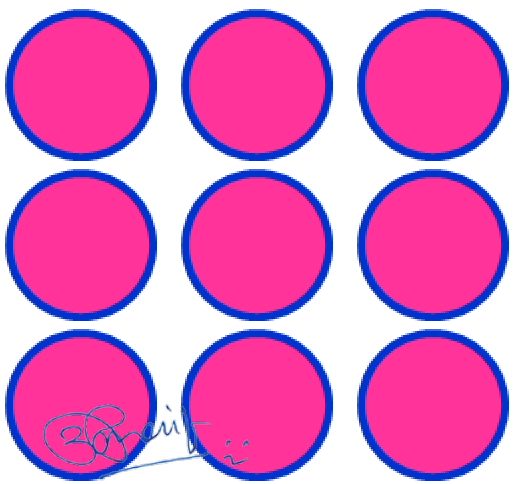


Condensation On Horizontal Tubes

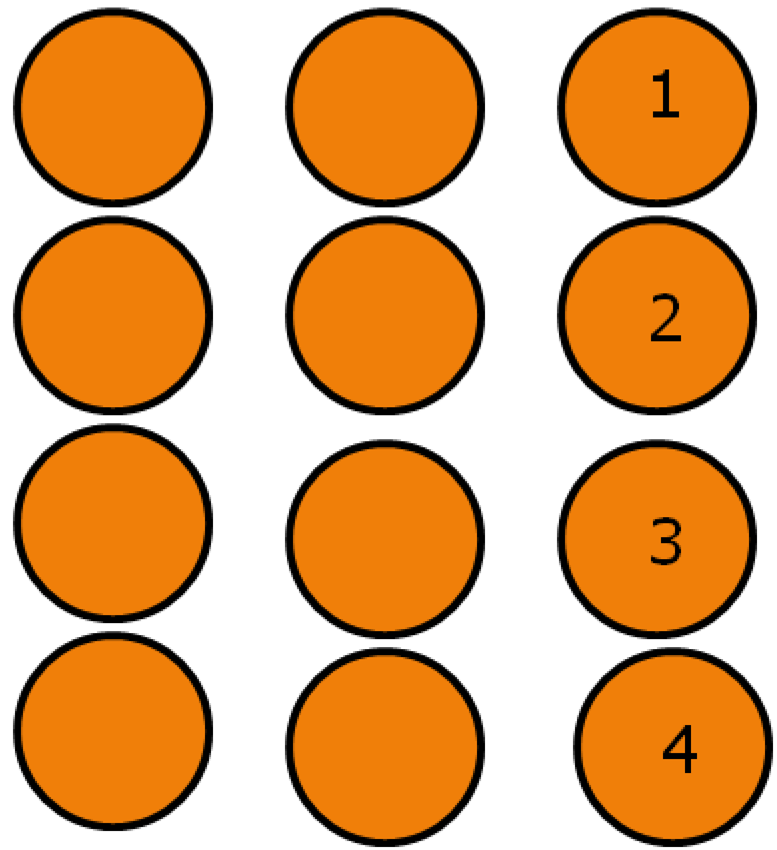
$$h_{av} = 0.725\left[\frac{\lambda\rho^{2} g k^{3}}{(Ts - Tw)\mu . D . N}\right]^{1/4}$$

N= No of horizontal
Tubes in vertical bank

Square Array



Rectangular Array



Condensation: Horizontal v/s Vertical Tubes

For max heat transfer, should the tubes in Condenser be kept horizontal or vertical?

$$\frac{h_v}{h_h} = \frac{0.943}{0.725}\left(\frac{D}{L}\right)^{1/4} = 1.3\left(\frac{D}{L}\right)^{1/4}$$

For $h_v = h_h$, we have $L/D = 2.86$ $(= h_h/h_v)$

- When L/D=2.86, immaterial whether tubes are kept horizontal or vertical
- When L/D<2.86; tubes should be kept vertical
- When L/D >2.86; tubes should be kept horizontal

Q1: A steam condenser consists of 16 tubes arranged in square array. Water flows through tubes at 65°C while steam at 75°C condenses over the tubes.

Find the rate of condensation if:

a) Tubes are kept horizontal

b) Tubes are kept vertical

Take latent heat of steam=2300kJ/kg and properties of water at 70°C as:

ρ=977.8 kg/m^3, C_p=4.187 kJ/kgK, k=0.668 W/mK, ν=0.415x10^{-6}, β=5.7x10^{-7}.

Length of the tubes=120cm, Dia of the tubes=25mm

ints: $Q=m\lambda=hA\Delta T$; m=?

For $Q=hA\Delta T$; h=?

Solution: a) **For Horizontal Tubes**

$$We\ know\ that\ \ h_h = 0.725\left[\frac{\lambda\,\rho^2 g\,k^3}{(T_s - T_w)\,\mu\,D\,N}\right]^{0.25}$$

$$= 0.725\left[\frac{2300x10^3\,x\,977.8^2\,x\,9.81\,x\,0.668^3}{(75-65)x\,977.8\,x\,0.415x10^{-6}\,x\,0.025\,x\,4}\right]^{0.25}$$

$$= 8134.3 W / m^2 K$$

$$Q = hA\Delta T$$

$$= 8134.3\,x\,\pi\,x\,0.025x1.2x16(75-65) = 122662\ .4W$$

$$= \frac{Q}{\lambda} = \frac{122662.4}{2300\,x10^3} = 0.0533\,kg/s = 192\,kg/h$$

Solution: b) For Vertical Tubes

$$We\ know\ that\ \ h_h = 0.943\left[\frac{\lambda \rho^2 g k^3}{(T_s - T_w)\mu L}\right]^{0.25}$$

$$= 0.943\left[\frac{2300x10^3 x 977.8^2 x 9.81 x 0.668^3}{(75-65) x 977.8 x 0.415x10^{-6} x1.2}\right]^{0.25}$$

$$= 5684.58\ W/m^2K$$

$$Q = hA\Delta T$$

$$= 5684.58 x \pi x 0.025x1.2x16(75-65) = 85723.47 W$$

$$= \frac{Q}{\lambda} = \frac{85723.47}{2300 x 10^3} = 0.03727 kg/s = 134.2 kg/h$$

Q2. A surface condenser was designed for a condensation rate of 50 kg of vapor per hour. It contained 100 tubes of 1cm OD, 1m long arranged in square array. The condenser was installed in vertical position (tubes Vertical) by mistake instead of horizontal, for which it was designed. Will there be any change in condensation rate? If yes, find out.

<u>Solution:</u>

$$h_h = 0.725\left[\frac{\lambda \rho^2 g k^3}{(T_s - T_w)\mu.D.N}\right]^{0.25} \quad \& \quad h_v = 0.943\left[\frac{\lambda \rho^2 g k^3}{(T_s - T_w)\mu.L}\right]^{0.25}$$

Since total area & ΔT are same in both cases;

Q ∞ h and Q=mλ hence Q ∞ m; therefore m ∞ h.

ce h values are different for horizontal and vertical outs, condensation rate will change

Solution (contd):

$$\frac{m_h}{m_v} = \frac{0.725}{0.943}\left[\frac{L}{ND}\right]^{0.25}$$

$$= \frac{0.725}{0.943}\left[\frac{1}{10x0.01}\right]^{0.25}$$

$$= 1.37$$

$$Hence\ m_v = \frac{m_h}{1.37} = \frac{50}{1.37}$$

$$= 36.57\ kg/hr$$

Q3: Saturated steam at 80°C condenses on the outside of a horizontal tube of 10cm dia maintained at temp of 70°C. λ for steam is 2300 kJ/kg. When the tube was kept vertical, it was observed that the rate of condensation was same as before. Find the length of the tube and rate of condensation per hour.

Take properties of condensate in the film at 75°C as:

ρ=977.8 kg/m^3 ; k=0.668 W/mK;

$=0.415 \times 10^{-6}$ m^2/s

Solution:

Since rate of condensation remains same, that means Heat Transfer Rate Q is same, which implies that h in both cases is same ie. $h_h = h_v$

$$Hence \quad 0.725\left[\frac{\lambda \rho^2 g k^3}{(T_s - T_w)\mu D}\right]^{0.25} = 0.943\left[\frac{\lambda \rho^2 g k^3}{(T_s - T_w)\mu L}\right]^{0.25}$$

$$Or \quad \frac{0.725}{D^{1/4}} = \frac{0.943}{L^{1/4}}$$

$$\Rightarrow \left(\frac{L}{D}\right)^{1/4} = 1.3 \quad or \quad \frac{L}{D} = 2.86$$

$$L = 2.86 x 0.10 = 0.286m = 28.6cm$$

Solution (contd):

$$h = 0.725\left[\frac{2300x10^3\,x\,977.8^2\,x\,9.81x\,0.668^3}{(80-70)977.8\,x\,0.415x10^{-6}\,x\,0.1}\right]^{0.25}$$

$$= 8134.3\ W/m^2K$$

$$Q = hA\Delta T$$

$$= 8134.3x\pi x0.1x0.286x10$$

$$= 7304.93W$$

$$m = \frac{Q}{\lambda} = 0.003176\,kg/s$$

$$= 11.43\,kg/hr$$

Q1. Two configurations are available for a condensing system for steam at 1 atm pressure, consisting of vertical plates maintained at 90C. The first configuration consists of single vertical plate of height=H and width=W. The second configuration consists of two vertical plates, each of height=H/2 and width=W. Which configuration will you choose for effective condensation?

Solution: Case-I: Height =H

$$h_1 = 0.943\left[\frac{\lambda\rho^2 g k^3}{(T_s - T_w)\mu}\right]^{1/4} \cdot \frac{1}{H^{1/4}}$$

$$Q_1 = h_1 A \Delta T$$

Solution (contd): Case-II: Height =H/2

$$h_2 = 0.943\left[\frac{\lambda \rho^2 g k^3}{(T_s - T_w)\mu}\right]^{1/4} \cdot \frac{1}{\left(\frac{H}{2}\right)^{1/4}}$$

$$Q_2 = h_2 A \Delta T$$

$$Hence \quad \frac{h_1}{h_2} = \frac{\frac{1}{H^{1/4}}}{\left(\frac{2}{H}\right)^{1/4}} = \left(\frac{1}{2}\right)^{1/4} = 0.84$$

$$\therefore h_1 = 0.84\, h_2 \Rightarrow h_2 \succ h_1 \quad \Rightarrow Q_2 \succ Q_1$$

ence second Configuration will be chosen.

Boiling

When a substance undergoes a phase change from liquid to vapor by taking latent heat from the heating surface, which is at temp higher than the saturation temp of the liquid, the process is called BOILING

Types of Boiling

1. Pool Boiling:

When heating surface/plate/wire is submerged in the pool of liquid to be boiled, process is called Pool Boiling

- Sub cooled Boiling
- Saturated Boiling

Example: Liquid boiling in a kettle or by wire heater

orced Boiling:

Boiling of water in water tubes of a boiler

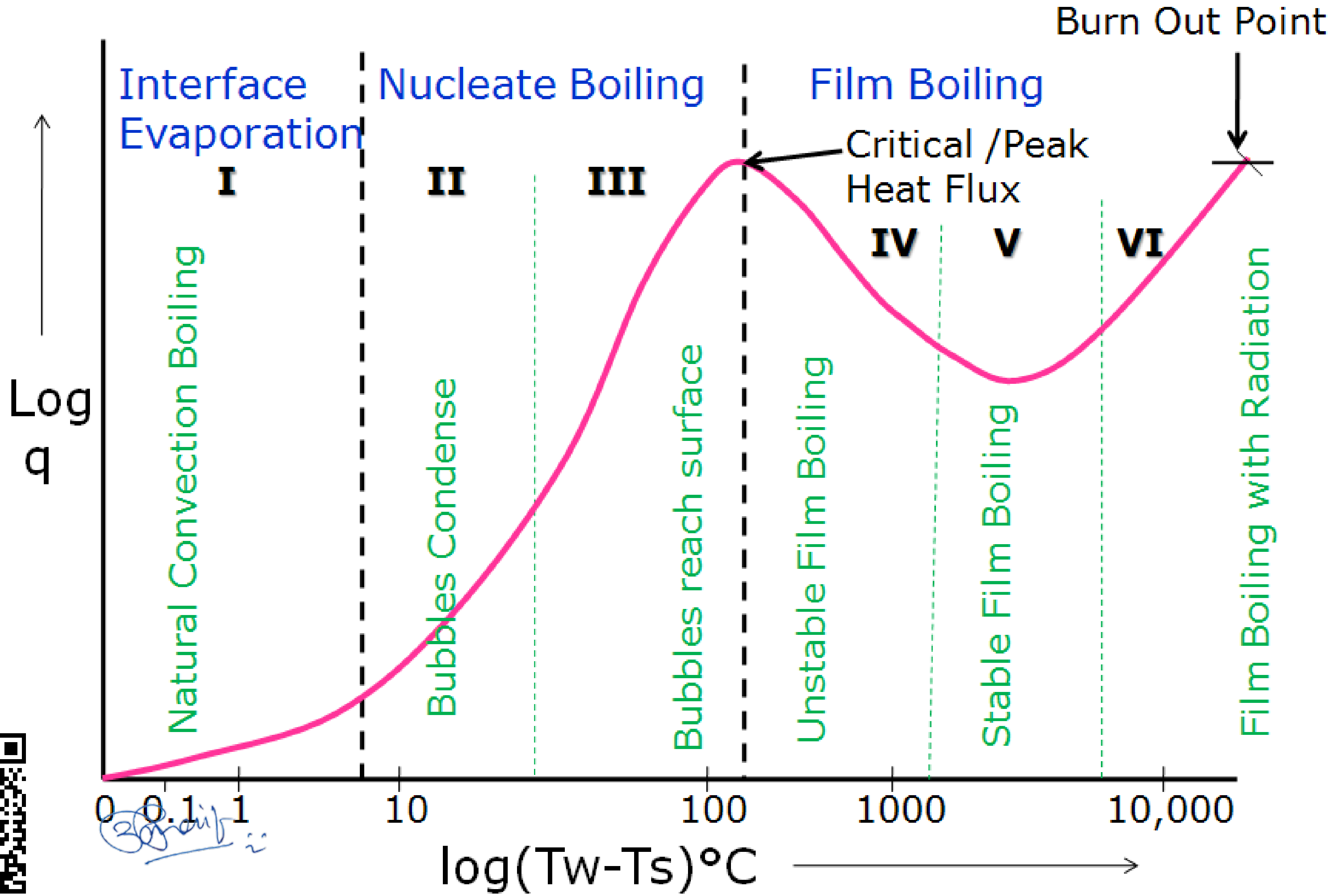
Six Regimes of Pool Boiling

- To study the phenomenon of pool boiling, a log-log plot of heat flux q and excess temp (T_w-T_s) is obtained by measuring heat input and temps on an electrically heated platinum resistance wire submerged in water/liquid.

- The Curve thus obtained is known as Pool Boiling Curve.

- Curve has three distinct regions, which are urther subdivided in to SIX REGIMES

Regimes of Pool Boiling



Regimes of Pool Boiling

Regime-I

- Heat flux increases gradually with increase in temperature difference
- Temp diff is of the order of 7-8°C
- Heat transfer takes place just like in natural convection
- Heated fluid particles at heating surface rise upwards, thus producing convection current in pool of liquid
- Vapor is produced at free surface of the liquid by evaporation, hence this regime is known as Interface Evaporation
- eat flux is proportional to $(T_w - T_s)^n$; where n is ghtly higher than 1 (≈1.3)

Regimes of Pool Boiling

2. Nucleate Boiling Region (Regime –II & III):

- Heat flux increases rapidly with increase in ΔT and reaches Peak value at the end of Regime - III
- Temp diff of the order of $(T_w-T_s)^n$; n≈3 (10 to 100°C)

Regime-II

- With increase in ΔT, bubbles start forming on heating surface at few places.
- Bubbles rise upwards but get condensed and do not reach the free surface of the liquid
- tense convection current due to rise of bubbles d hence flux increases rapidly

Regimes of Pool Boiling

Regime-III

- With further increase in ΔT, large number of bubbles are formed on heating surface at almost all places.
- Bubbles grow in size and rise to free surface of liquid, where vapor is released
- As the bubbles on forming, leave the heating surface almost immediately, the heating surface becomes available for further bubble formation, hence heat transfer rate continuously increases in this regime and attains max value.
- ...e max value attained at the end of the regime ... called Critical Heat Flux (shown in the diagram)

Regimes of Pool Boiling

3. Film Boiling Region :

Regime-IV: Unstable Film Regime

- With increase in ΔT, the curve starts coming down
- Rate of bubble formation becomes very high
- These large no of bubbles form a film/blanket over the heating surface
- Thermal conductivity of vapor being very small, the film of bubbles acts as shield for heat transfer, hence, Q reduces and curve starts coming down

this regime , bubble film is unstable, however, does offer resistance to Q, thus reducing it

Regimes of Pool Boiling

Regime-V: Stable Film Region

- With further increase in ΔT, the bubbles formation is so high, that the film becomes stable and thus offers more resistance to heat flow

- Heat flux thus reduces to minimum

- The lowest point on the curve achieved in this regime is known as Leidenfrost Point.

Regimes of Pool Boiling

Regime-VI: Radiation Dominant Regime

- With further increase in ΔT, the heat flux curve starts rising as the heat transfer by radiation becomes dominant, although stable bubble film remains and does offer same resistance to heat transfer

- Temp in this regime is very high, of the order of 10^4 °C

- With slight increase in heat flux above Critical Heat Flux, the temp of heating surface becomes so high that no heating body/surface/wire will be able to thstand that temp and will melt away.

his point on the curve is called Burn Out Point

Importance of Critical Heat Flux & Burn Out Points

- From the curve, it can be seen that heat transfer rate decreases beyond Peak Flux, even on increasing the heating temperature.

- Now, by obtaining even slight increase in heat flux above peak value, it will give rise to very high temp.

- At this high temp, most of the metals will melt/fail/ burn out called Burn Out Point

- So, there is no point in going above the Peak Flux at transfer rate, while designing equipment for ax possible heat transfer rate.

Correlations in Pool Boiling

1. Natural Convection Boiling:

$$\frac{Q}{A}=\frac{k}{D}\left(T_w-T_s\right)\left[0.36+\frac{0.158\left(G_r.P_r\right)^{1/4}}{\left\{1+\left(\frac{0.6}{P_r}\right)^{9/16}\right\}}\right] \quad for\ 10^6 \prec G_r.P_r \prec 10^9$$

$$\frac{Q}{A}=\frac{k}{D}\left(T_w-T_s\right)\left[0.6+\frac{0.387\left(G_r.P_r\right)^{1/6}}{\left\{1+\left(\frac{0.6}{P_r}\right)^{8/27}\right\}}\right]^2 \quad for\ 10^9 \prec G_r.P_r \prec 10^{12}$$

l properties of fluid to be taken at $(T_w+T_s)/2$

Correlations in Pool Boiling

2. Nucleate Boiling (Rohsenhow's Relation):

$$C_p \frac{(T_w - T_s)}{\lambda} = k_{sf}\left[\frac{Q}{A\mu\lambda}\cdot\sqrt{\frac{\sigma}{g(\rho_l - \rho_v)}}\right]^{1/3} . P_r^{\,n}$$

All properties of fluid to be taken at T_{sat}

Correlations in Pool Boiling

3. Critical Heat Flux (Zuber Relation):

$$\frac{Q}{A} = \frac{\pi}{24}.\lambda.\rho_v.\left[\frac{\sigma g(\rho_l - \rho_v)}{\rho_v{}^2}\right]^{1/4}\left[\frac{\rho_l - \rho_v}{\rho_l}\right]^{1/2}$$

for horizontal plate facing up

All properties of fluid to be taken at T_{sat}

Forced (Convection) Boiling

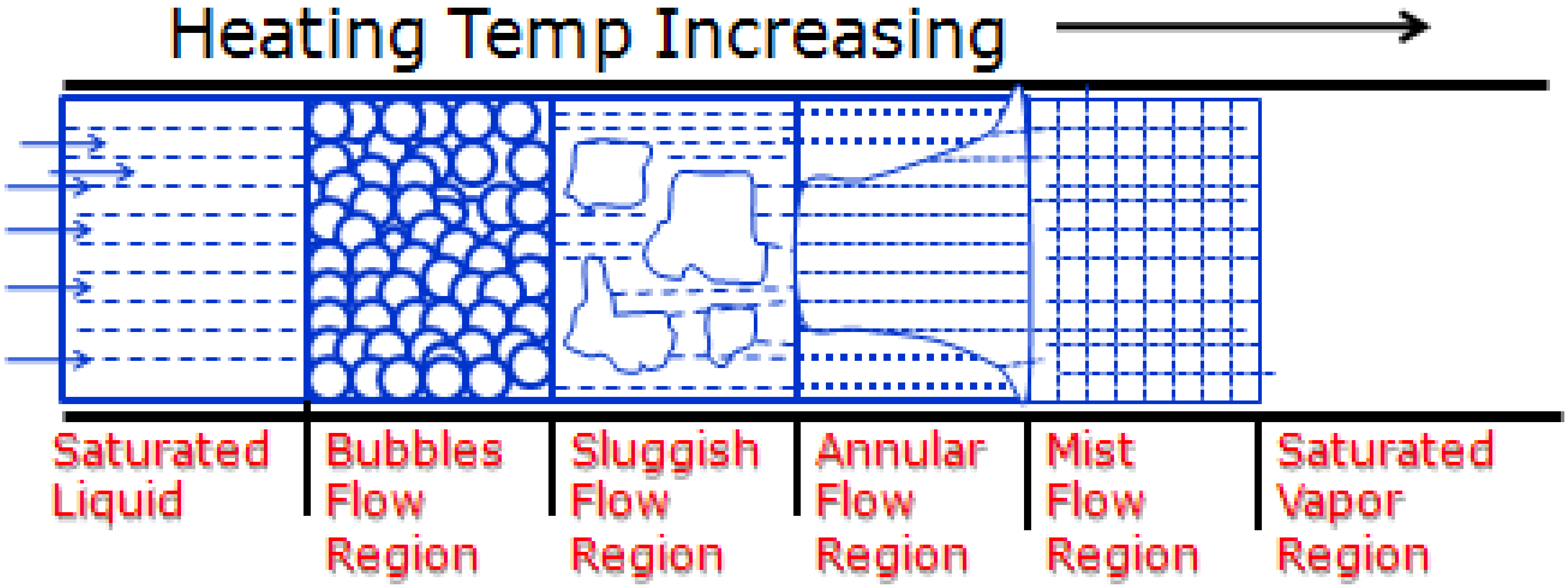
- When a liquid is forced to flow through a tube, which is being heated continuously from outside surface, the process of boiling is known as Forced Boiling. Example : Production of steam in a boiler tube

- When saturated liquid is forced to flow, the following regions are observed along the length of pipe, when saturated fluid is getting converted to vapor:

1. Bubble Flow Region:
   Vapor bubbles form here and these can be observed with saturated fluid in this region

2. Sluggish Flow Region:
   Large number of vapor bubbles coagulate to form large vapors and they flow along with saturated liquid

Forced (Convection) Boiling



Forced Boiling (Continued)

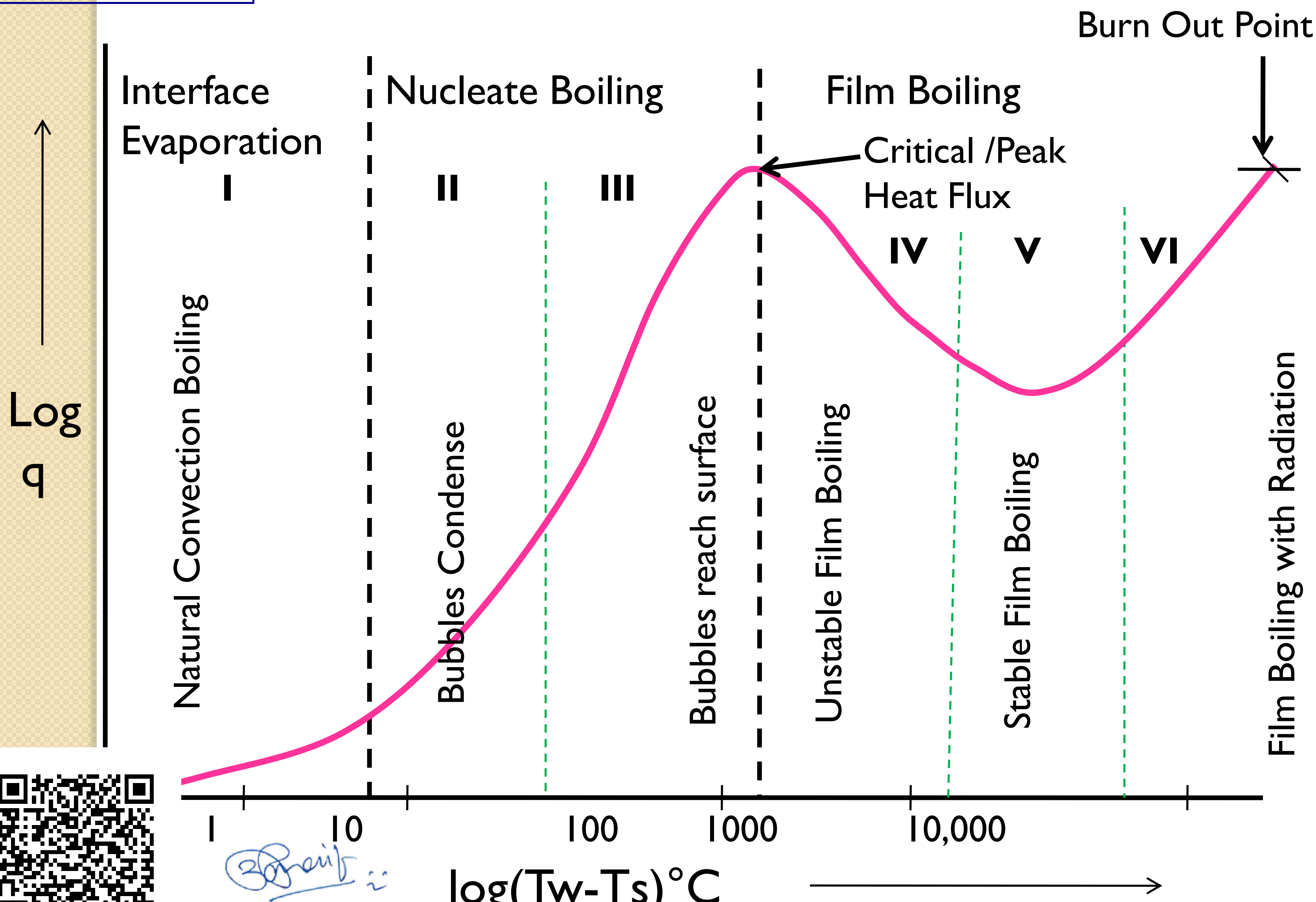
3. Annular Flow Region:

Here large vapor masses formed in the earlier region combine and flow through the central region of the tube, while the liquid flows in annular passage around the vapor core

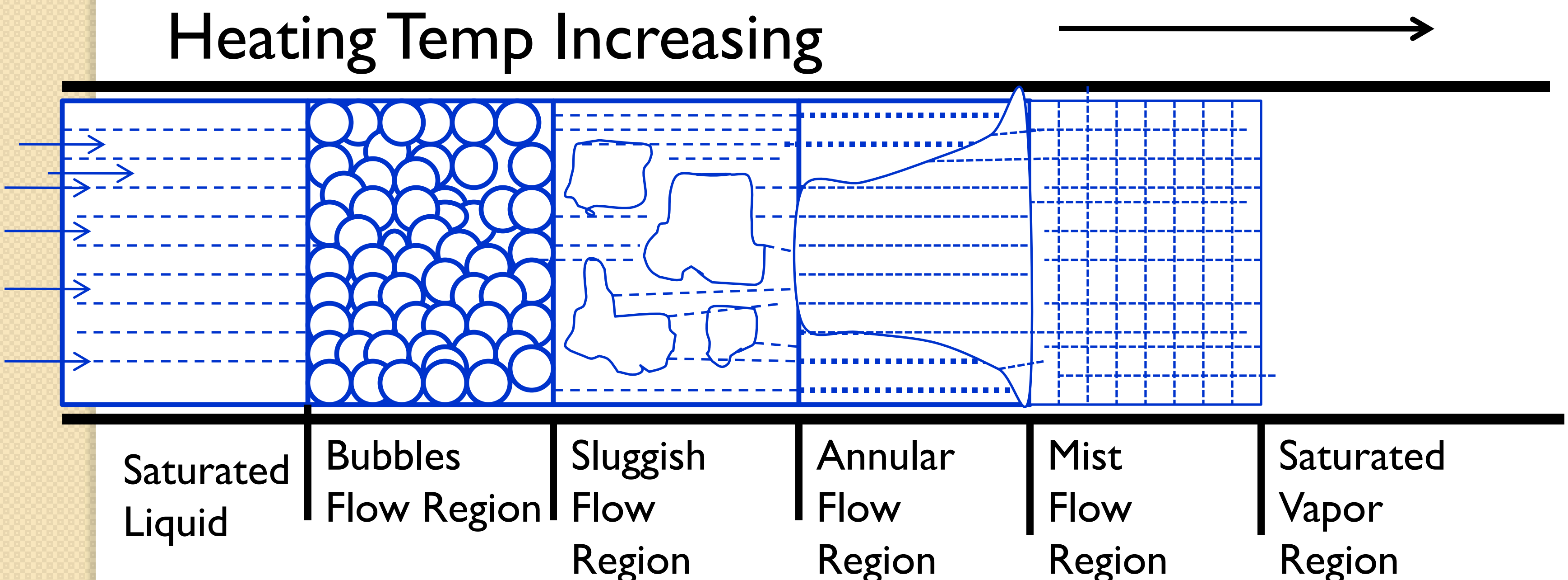
4. Mist Flow Region:

In the previous region, thickness of annular ring of liquid goes on decreasing and then vanishes at the end the region. Now there are vapor with suspended liquid particles and is known as Mist Flow. Finally, suspended liquid particles also get evaporated at the end of this region to give saturated vapor, which become invisible to our eyes

Regimes of Pool Boiling



Forced (Convection) Boiling



Heat Exchangers

- Heat exchanger is an equipment, in which transfer of heat energy takes place from hot fluid to cold fluid.

- Examples are:

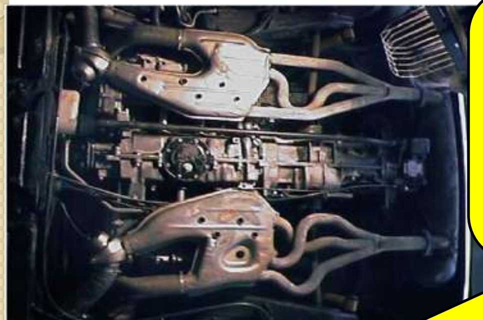
  Automobile Radiators
  Preheaters
  Intercoolers
  Boilers
  Condensers
  Oil coolers
  Cooling Towers

ome manufactures:

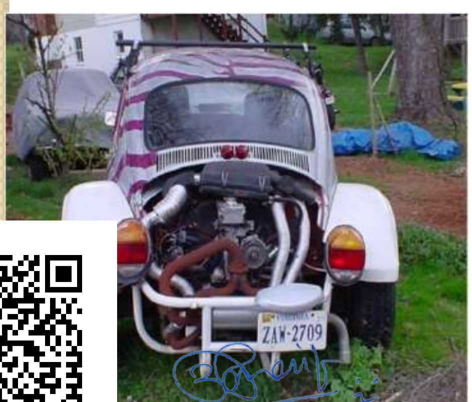
ermax, Forbes Marshall, TATA, Behr, Alfa Laval,

arpur.

Applications of Heat Exchangers



Heat Exchangers prevent car engine overheating and increase efficiency



Heat exchangers are used in AC

Heat exchangers are used in chemical Industry for heat transfer

Types of Heat Exchangers

Direct Transfer type (Recuperator):

Automobile Radiators, Oil Coolers, Air preheaters, Super heaters, Condensers, Evaporators etc.

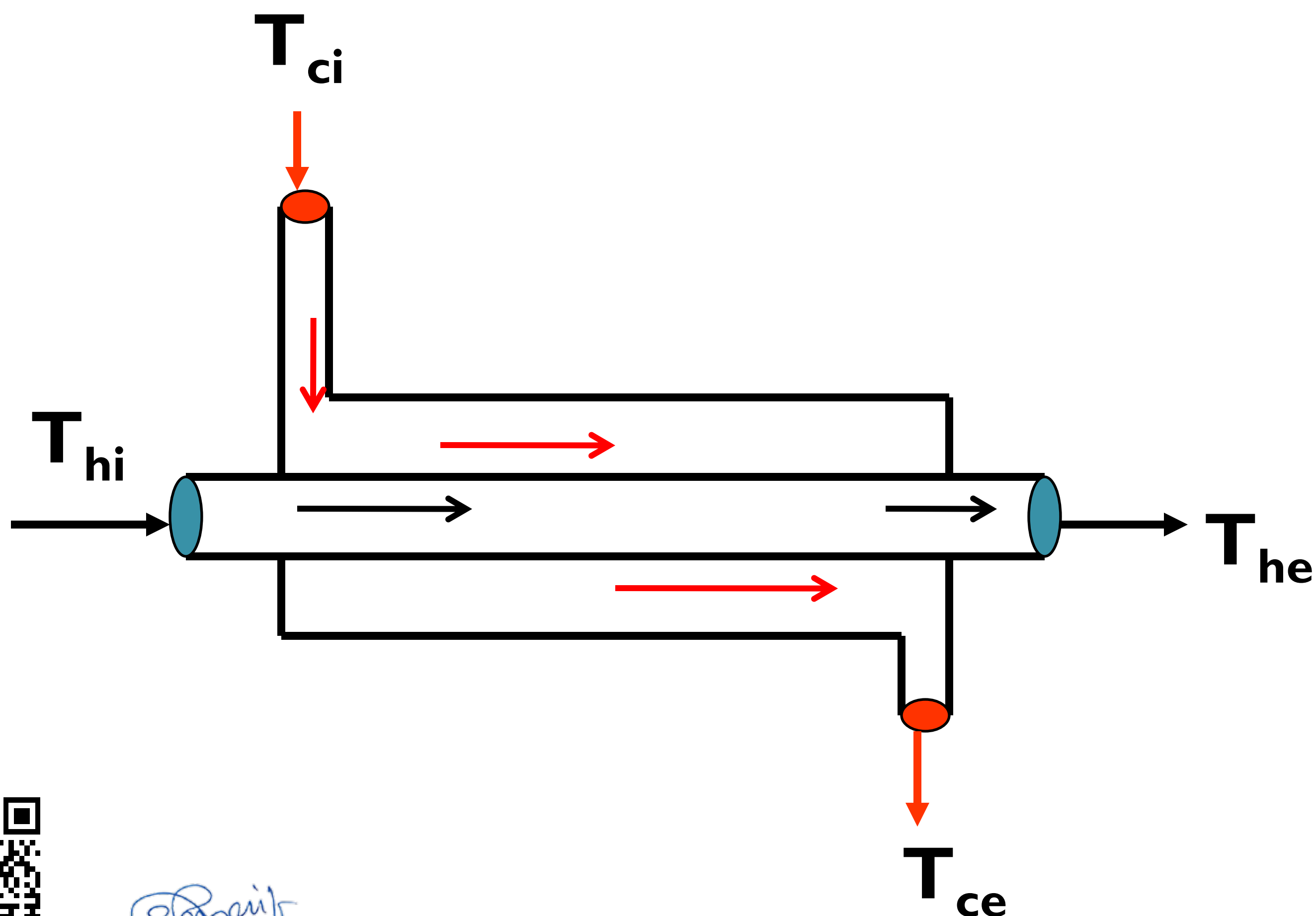
Storage Type (Regenerator):

Open hearth and glass melting furnaces, Air heaters of Blast furnaces

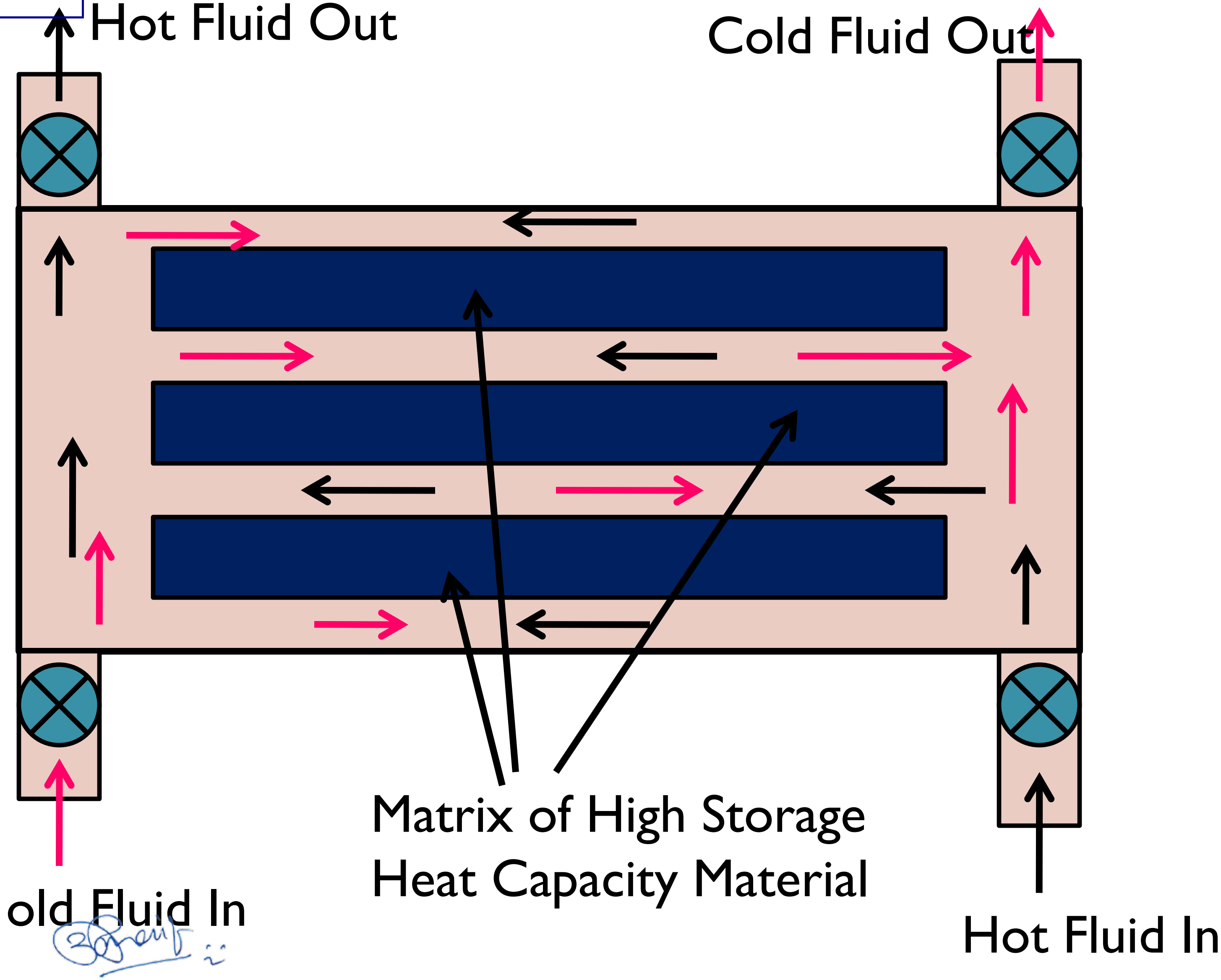
Direct Contact Types:

Cooling Towers, Jet condensers

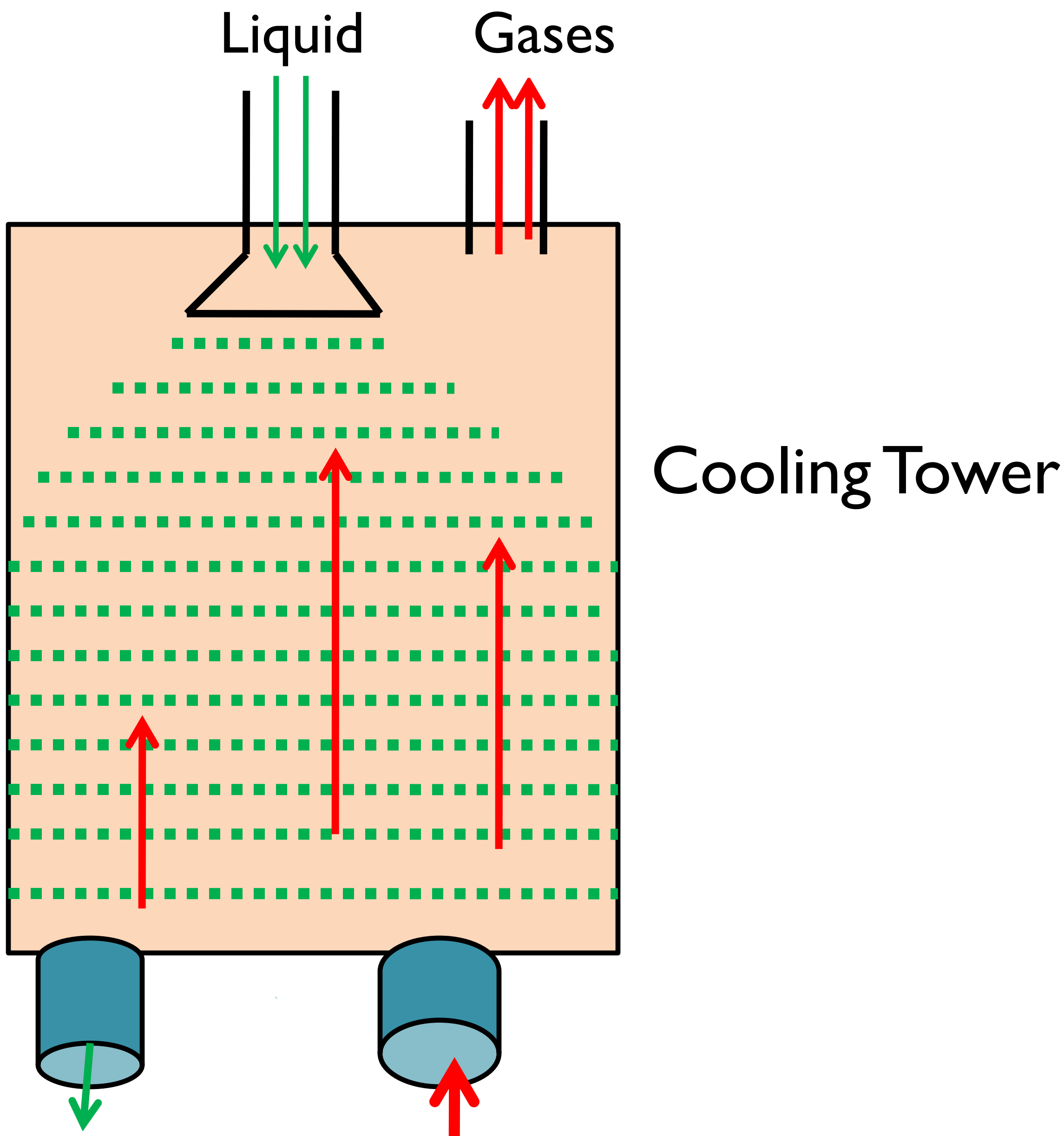
Direct Transfer Type Heat Exchanger



Storage Type Heat Exchanger



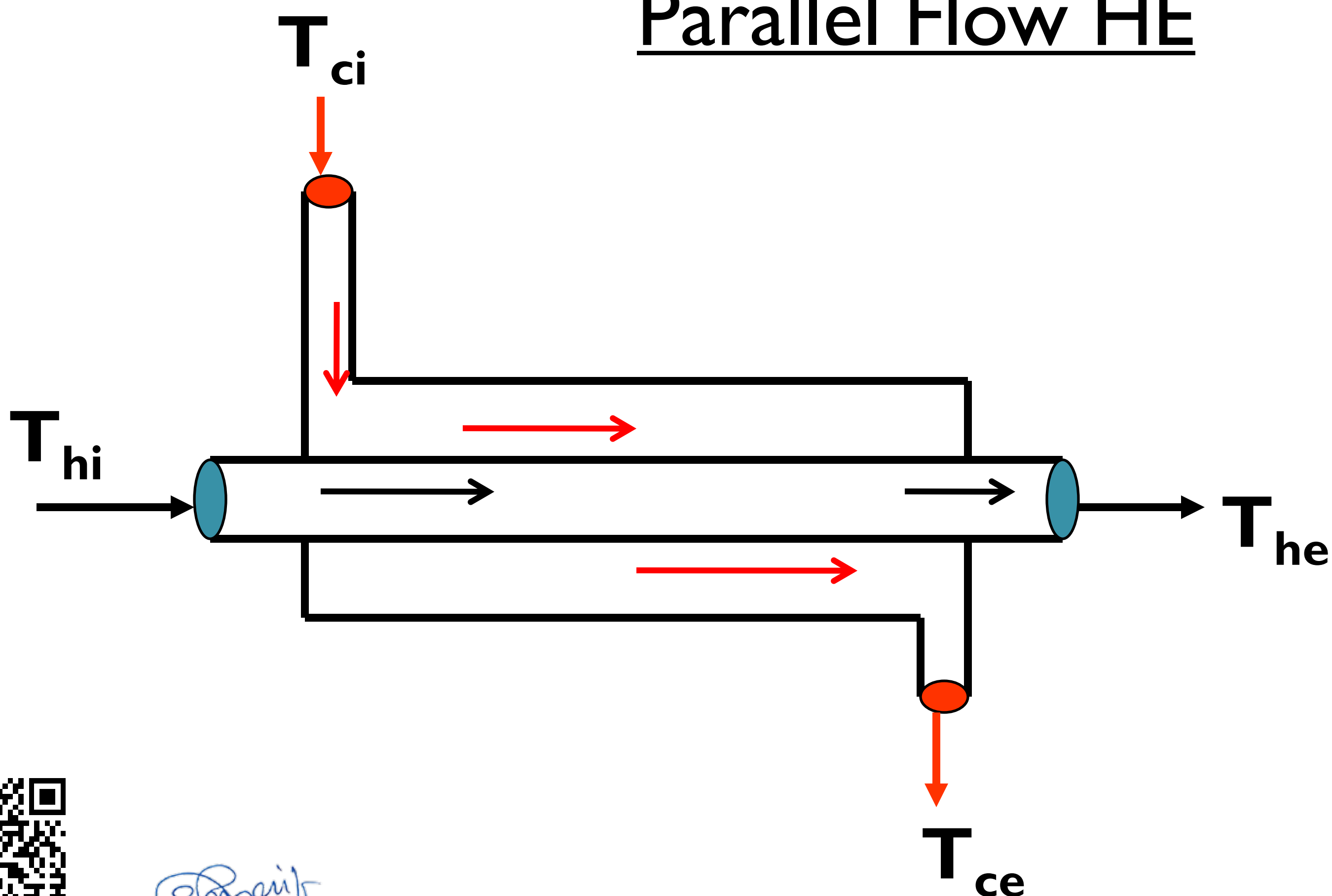
Direct Contact Type Heat Exchanger



Direct Transfer Type Heat Exchanger

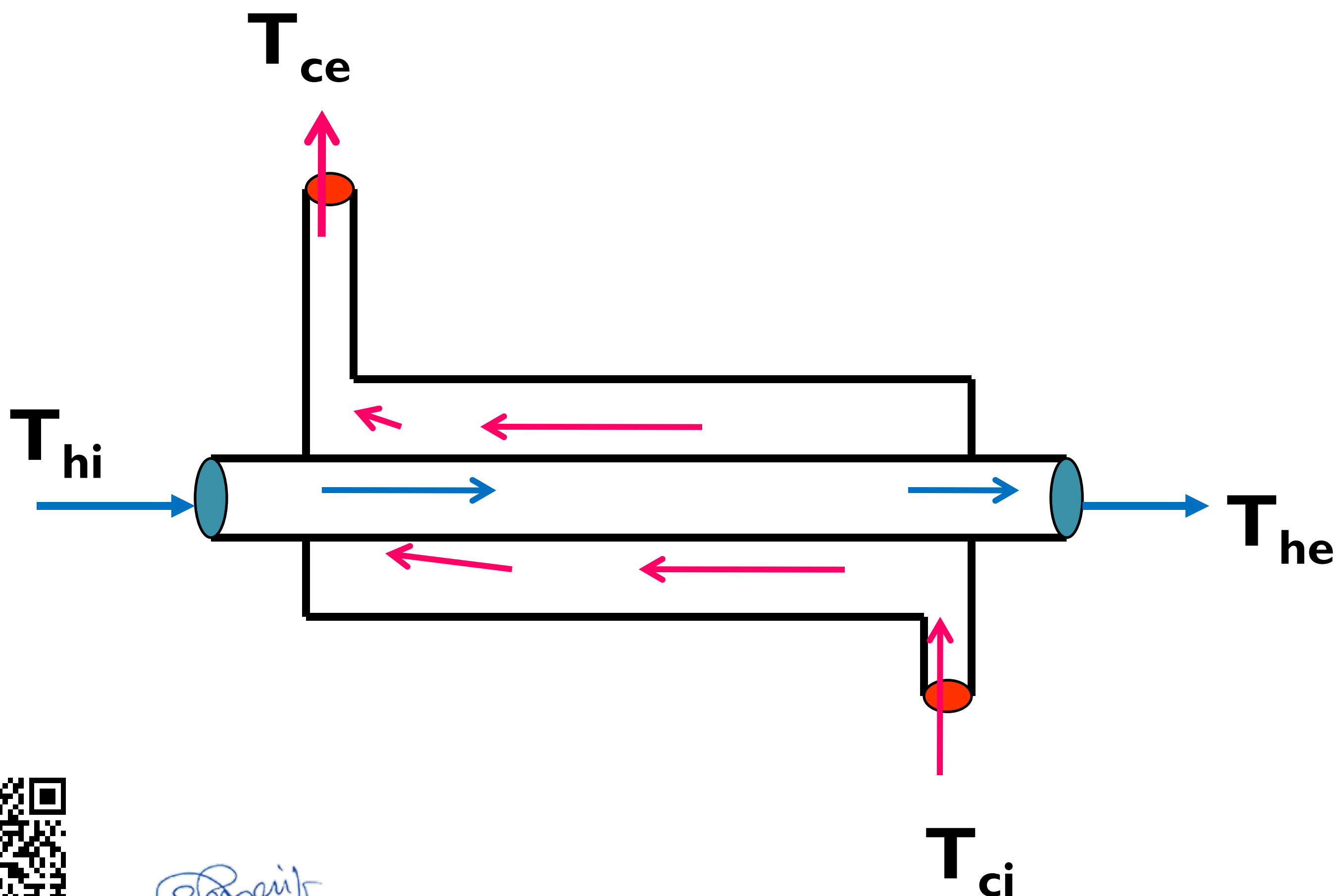
Tubular Heat Exchanger (Concentric Tubes)

Parallel Flow HE



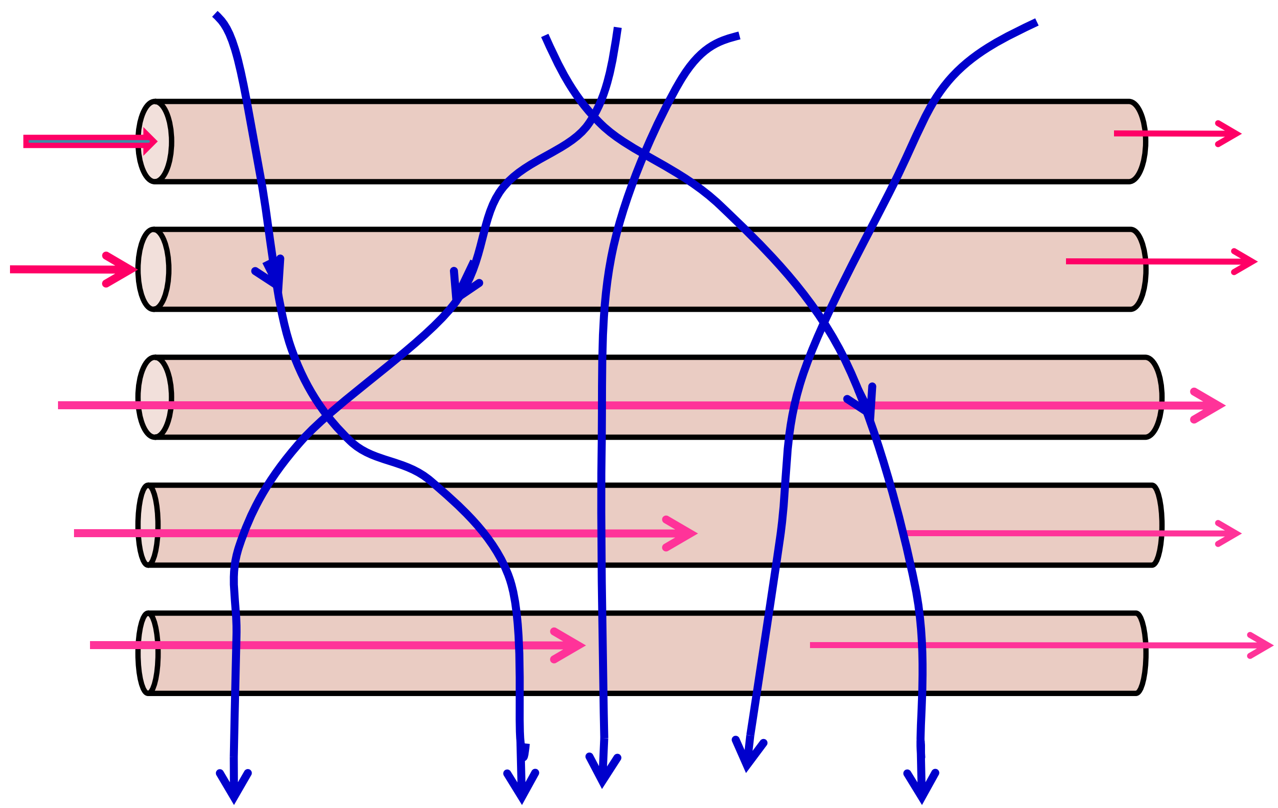
Direct Transfer Type Heat Exchanger

Counter Flow HE



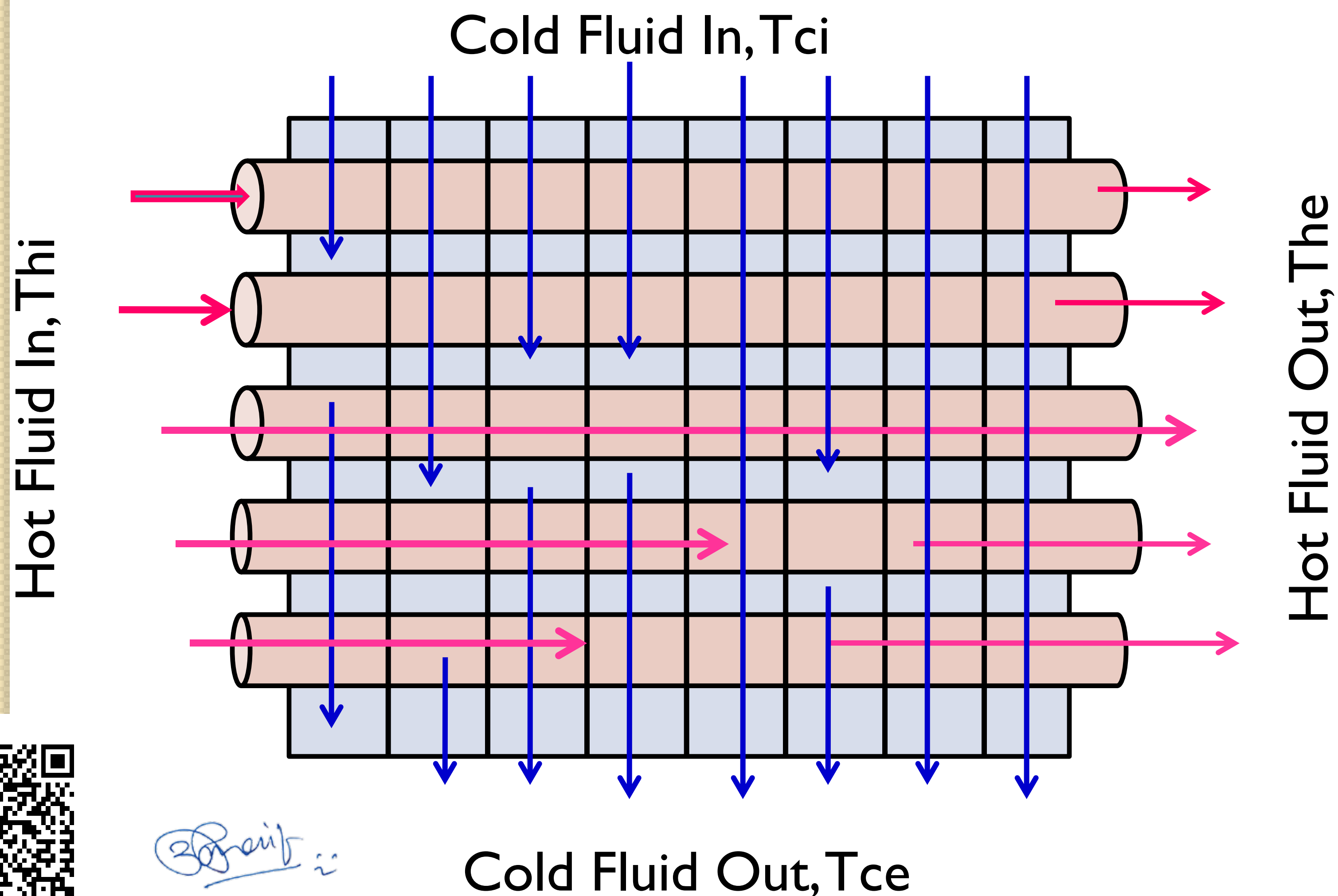
Direct Transfer Type Heat Exchanger

Cross Flow HE (One Fluid Mixed, One Unmixed):

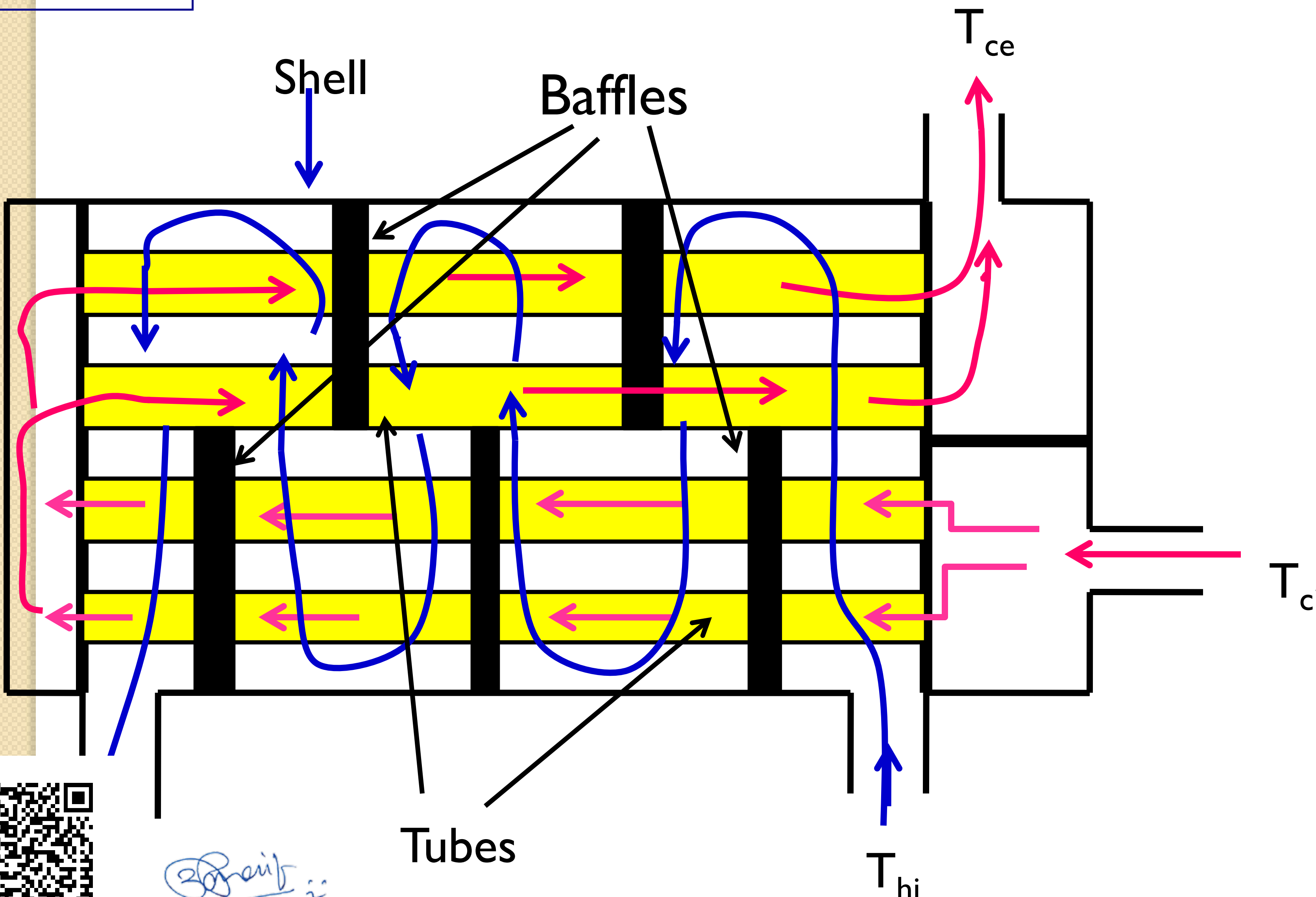


Direct Transfer Type Heat Exchanger

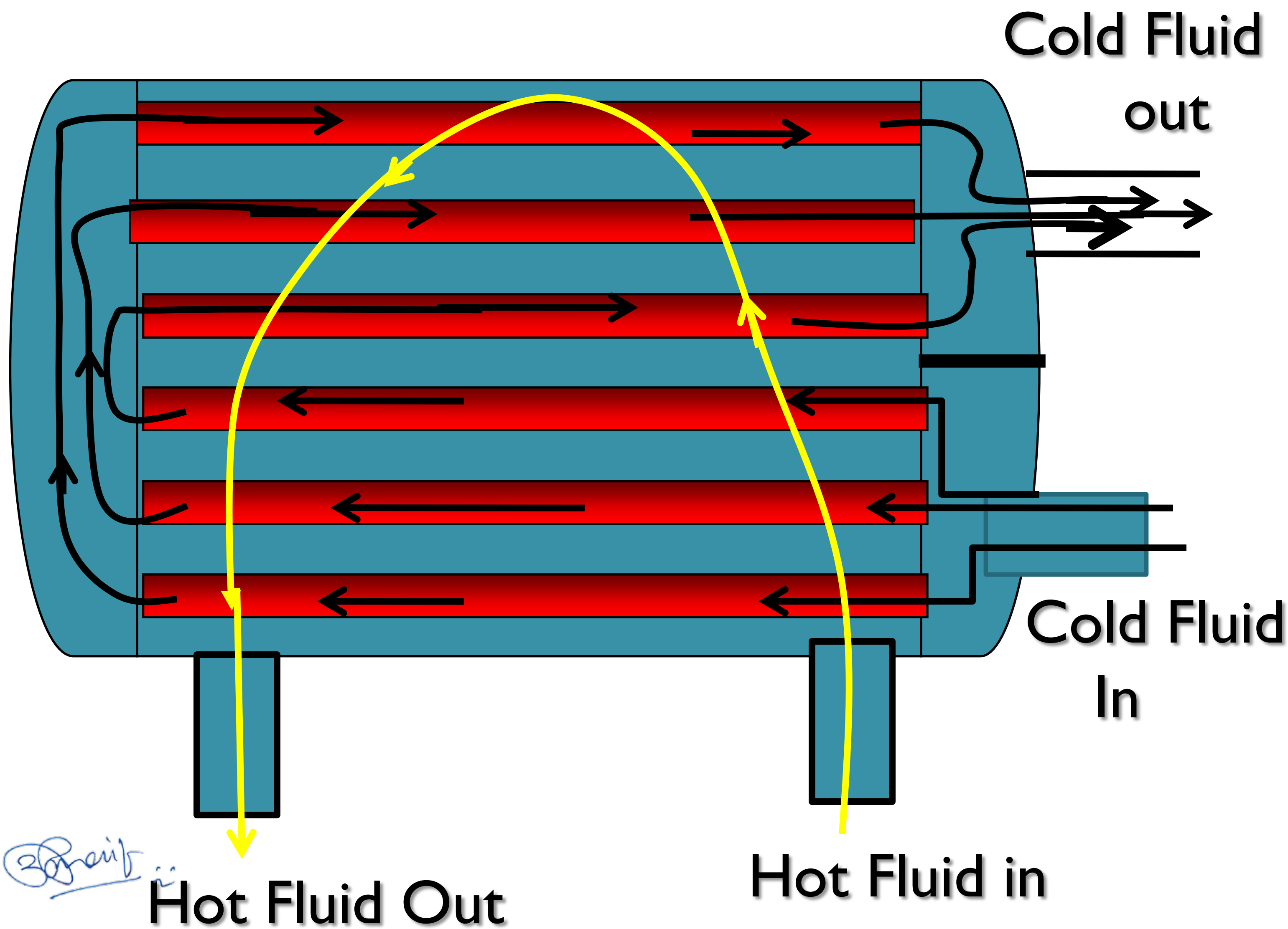
Cross Flow HE (Both Fluids Unmixed):



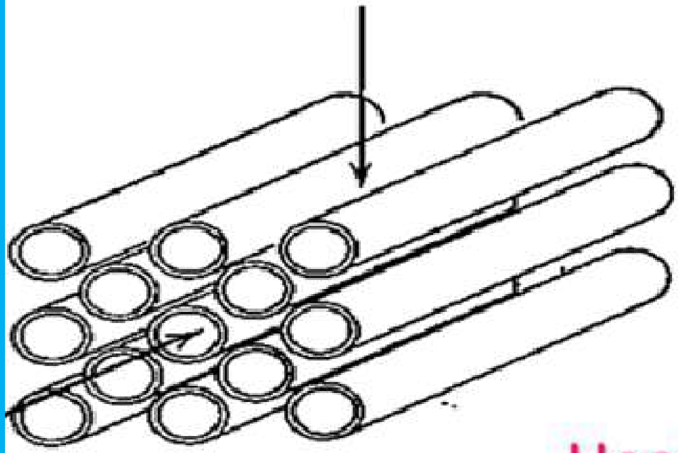
Direct Transfer Type HE (Shell-Tube Type)

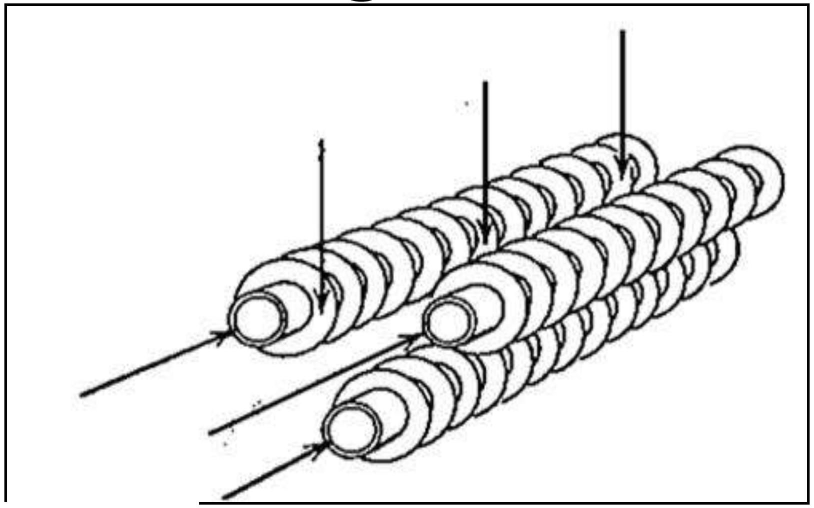


Heat Exchanger

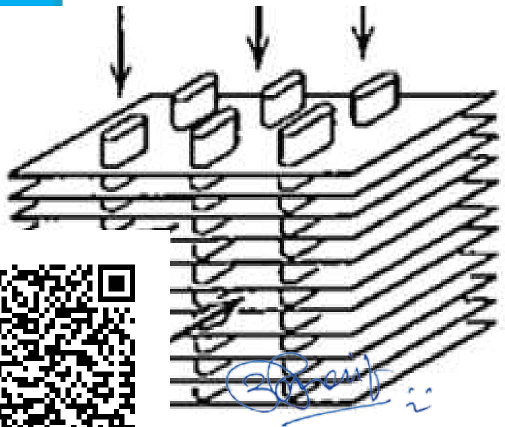


Compact Heat Exchangers

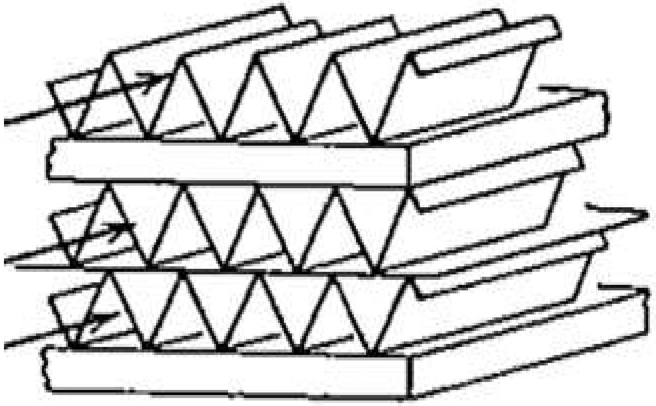




Heat Transfer Surface Area >700m²/m³ on either or both sides

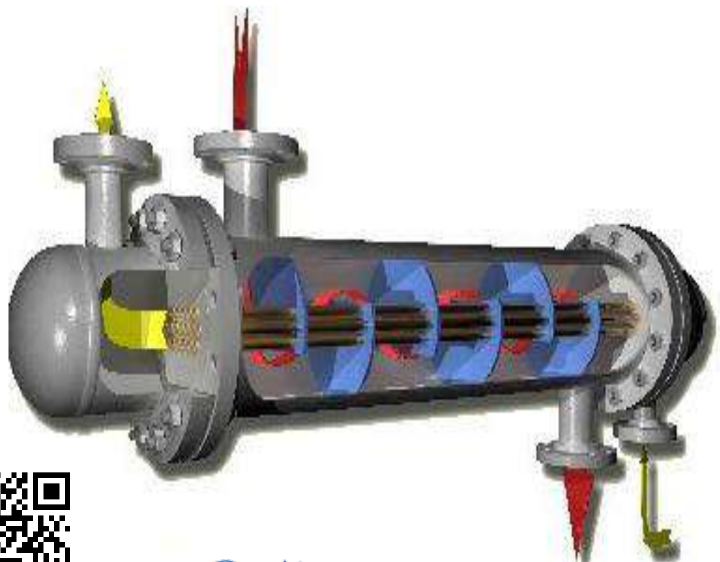


D_h < 6mm



Heat Flow in Fluids

Typical equipment consists of a bundle of parallel tube encased in a cylindrical shell



Some Important Aspects of HEs

- Heat energy given by hot fluid = Energy gained by cold fluid $(m.Cp.\Delta T)_{hot\ fluid} = (m.Cp.\Delta T)_{cold\ fluid}$

- In direct transfer type HEs, transfer of energy takes place across the wall of metal and rate of heat flow can be estimated using the term ' Over All Heat Transfer' as: $Q = U.A.\Delta T$

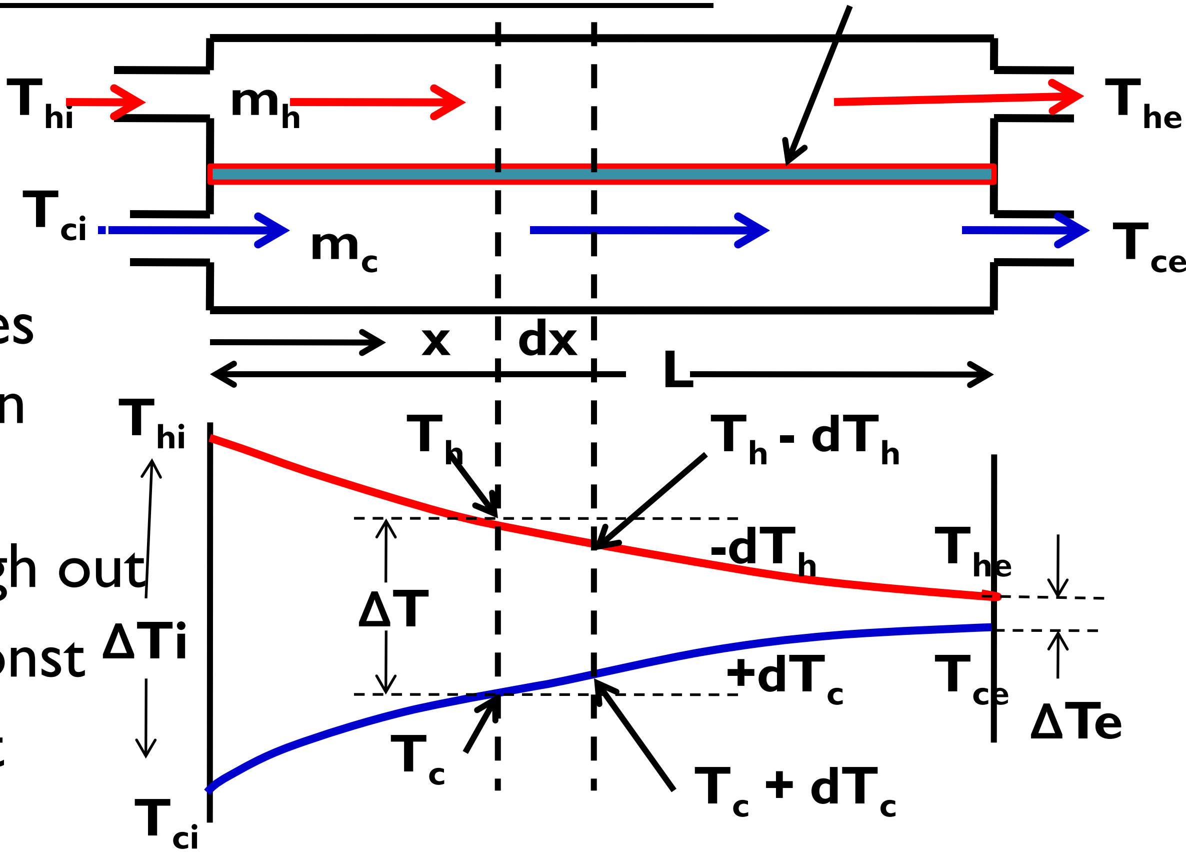
- In HEs, ΔT varies across the length of HE, therefore, while applying above formula, some mean temp difference has to be used.

- Surfaces of HEs get coated with deposits with passage of time resulting in deterioration of performance. The effect deposits/scales is represented by FOULING FACTOR, ich has to be added to other thermal resistances for luation of over all heat transfer coefficient.

Representative Values of Fouling Factors

Fluid	Fouling Factor, m^2-K/W
Seawater and treated boiler feedwater (<50°C)	0.0001
Seawater and treated boiler feedwater (>50°)	0.0002
River water (<50°C)	0.0002-0.001
Fuel oil	0.0009
Refrigerating liquids	0.0002
Steam	0.0001

Analysis of Parallel Flow HE

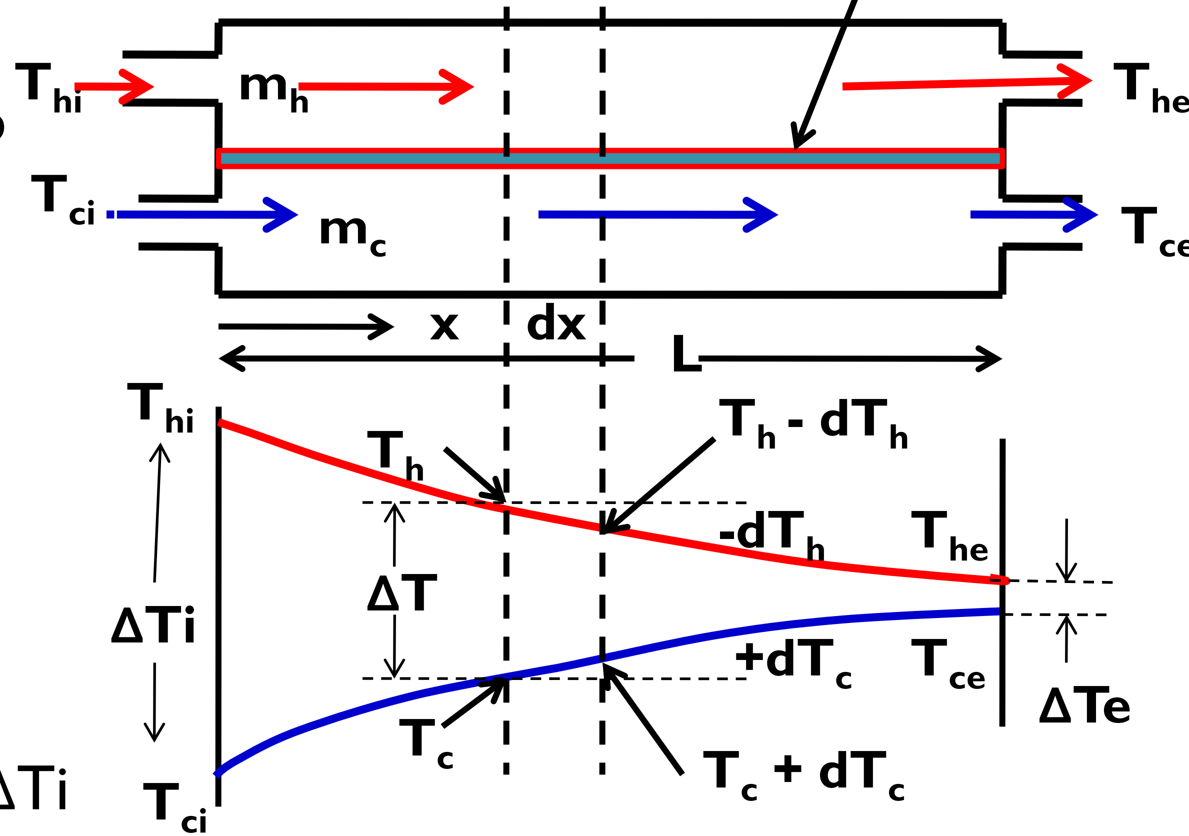


Assumptions:

- Heat transfer takes place only between two fluids
- U is const through out
- C_p of fluids are const
- No temp gradient across the wall
- No change in KE & PE of the fluids

- Consider HE, in which heat is transferred across an area A of width B and length L.
- Let flow rates on hot and cold sides be m_h & m_c respectively

Analysis of Parallel Flow HE



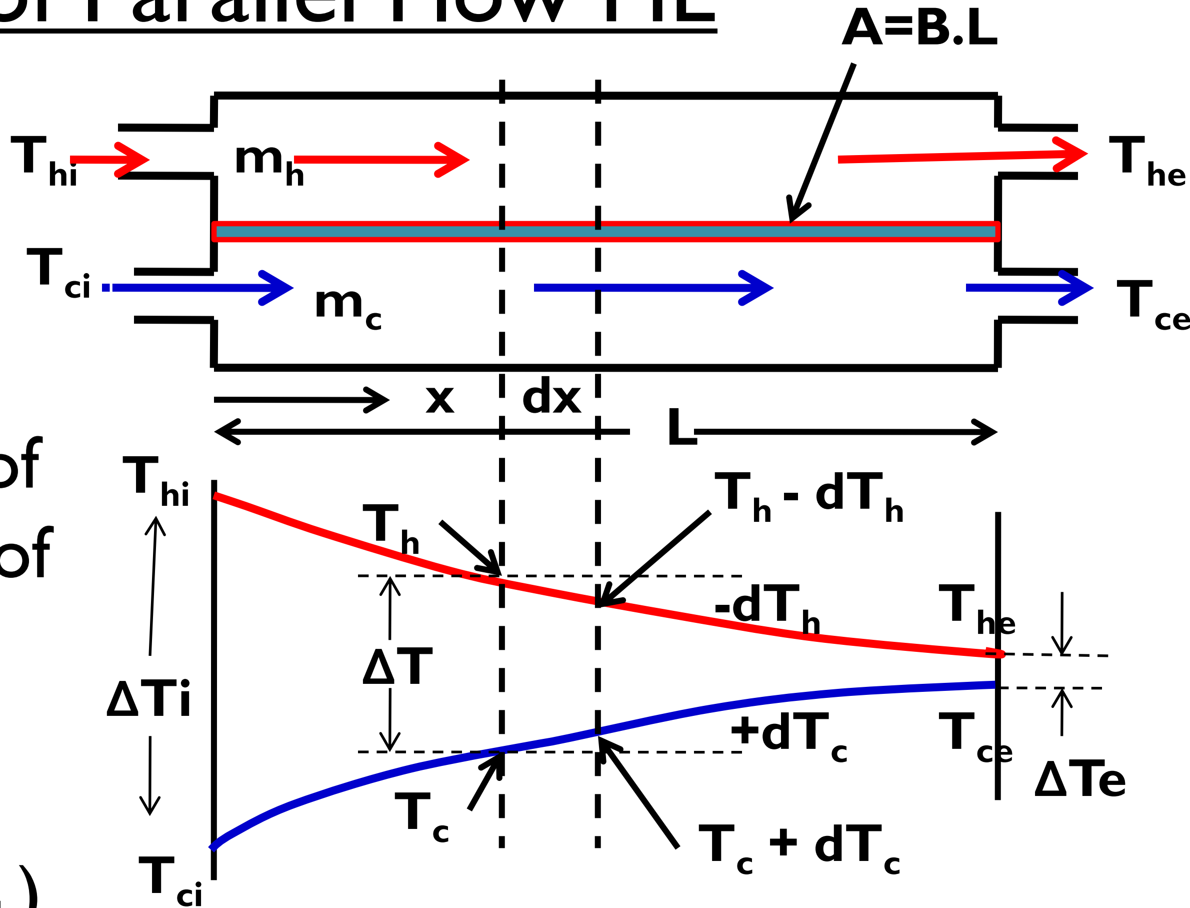
Thi- hot fluid temp at inlet. The- hot fluid temp at exit
Tci- cold fluid temp at inlet
Tce- cold fluid temp at exit

From the Fig, temp diff at inlet is max ΔTi and min at exit ΔTe.

Consider an elemental area dA at distance x of length dx.

the temp at the beginning of elemental area be T_h and T_c and the change in temps while they flow over area dA be dT_h and as shown in Fig.

Analysis of Parallel Flow HE



For steady state conditions,
Rate of Heat Transfer =
Rate of change of Internal energy of the fluid

Therefore,

$Q = U.dA.\Delta T$

$= m_h.C_{ph}.(-dT_h)$

$= m_c.C_{pc}.(+dT_c)$.......(1);

where $dA = B.dx$

the beginning of the elemental area dA, we can write;

$\Delta T = T_h - T_c$ ....(2)

Differentiating (2), We have $d(\Delta T) = dT_h - dT_c$ .....(3)

Analysis of Parallel Flow HE

Substituting values of dT_h & dT_c from eqn..(1) in (3), we have;

$$d(\Delta T) = \frac{U.dA.\Delta T}{-m_h.C_{ph}} - \frac{U.dA.\Delta T}{m_c.C_{pc}} \quad OR$$

$$\frac{d(\Delta T)}{\Delta T} = -\left(\frac{1}{m_h C_{ph}} + \frac{1}{m_c C_{pc}}\right).U.B.dx$$

$$Integrating \int_{\Delta Ti}^{\Delta Te} \frac{d(\Delta T)}{\Delta T} = -\left(\frac{1}{m_h C_{ph}} + \frac{1}{m_c C_{pc}}\right).U.B\int_0^L dx$$

OR

$$\left[\ln \Delta T\right]_{\Delta Ti}^{\Delta Te} = -\left(\frac{1}{m_h C_{ph}} + \frac{1}{m_c C_{pc}}\right).U.B.L$$

Analysis of Parallel Flow HE

$$OR \quad [\ln \Delta T]_{\Delta Ti}^{\Delta Te} = -\left(\frac{1}{m_h C_{ph}} + \frac{1}{m_c C_{pc}}\right).U.B.L$$

$$OR \quad \ln \Delta Te - \ln \Delta Ti = -\left(\frac{1}{m_h C_{ph}} + \frac{1}{m_c C_{pc}}\right).U.A$$

$$Or \ \ln\left(\frac{\Delta Te}{\Delta Ti}\right) = -\left(\frac{1}{m_h C_{ph}} + \frac{1}{m_c C_{pc}}\right)U.A \ldots\ldots\ldots(4)$$

Analysis of Parallel Flow HE

$$Also, \quad Q = m_h C_{ph}(T_{hi} - T_{he}) = m_c C_{pc}(T_{ce} - T_{ci})$$

$$\Rightarrow \frac{1}{m_h C_{ph}} = \frac{T_{hi} - T_{he}}{Q} \; \& \; \frac{1}{m_c C_{pc}} = \frac{T_{ce} - T_{ci}}{Q}$$

Substituting in eqn.......(4), We have;

$$\ln\left(\frac{\Delta Te}{\Delta Ti}\right) = -\left(\frac{T_{hi} - T_{he}}{Q} + \frac{T_{ce} - T_{ci}}{Q}\right) U.A$$

$$OR \quad Q = \frac{-U.A}{\ln\left(\frac{\Delta Te}{\Delta Ti}\right)}\left[(T_{hi} - T_{ci}) - (T_{he} - T_{ce})\right]$$

Analysis of Parallel Flow HE

$$Q = U.A.\frac{\Delta Ti - \Delta Te}{\ln\left(\frac{\Delta Ti}{\Delta Te}\right)}$$

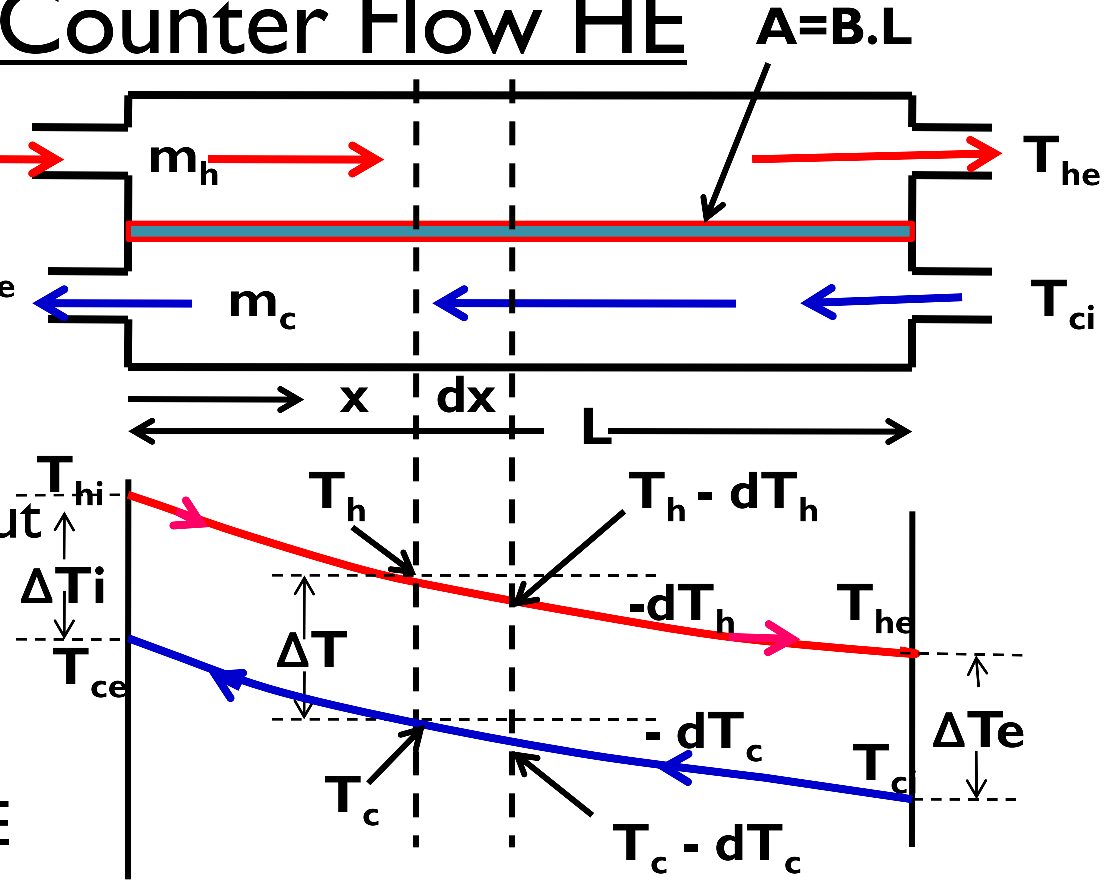
$$Q = U.A.\Delta T_m$$

From this expression Q can now be calculated

Comparing we have; $\Delta T_m = \dfrac{\Delta Ti - \Delta Te}{\ln\left(\frac{\Delta Ti}{\Delta Te}\right)}$

nce ΔT_m contains log term, it is called
garithmic Mean Temp Difference (LMTD)

Analysis of Counter Flow HE



Assumptions:

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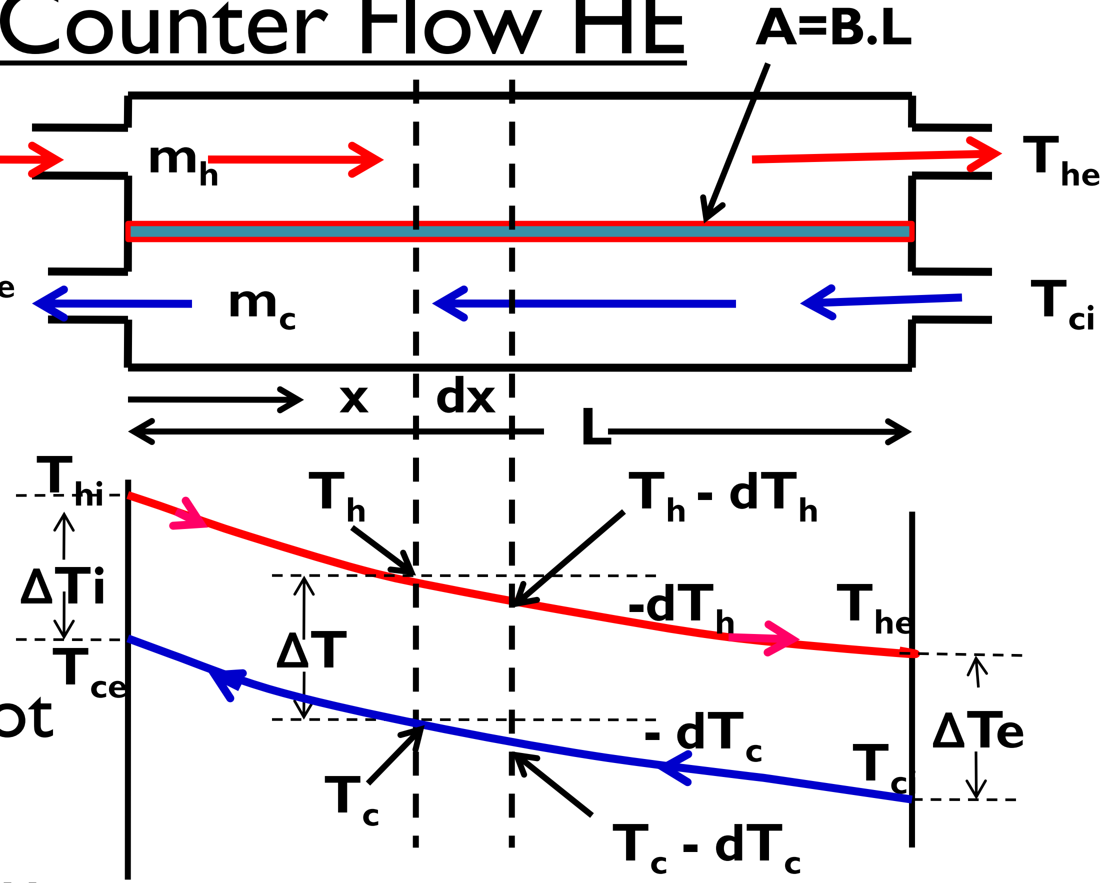
Analysis of Counter Flow HE



Consider HE, in which heat is transferred across an area A of width B and length L.

Let flow rates on hot and cold sides be m_h & m_c respectively

For steady state conditions

$Q=U.dA.\Delta T$

$=m_h.C_{ph}.(-dT_h)$

$=m_c.C_{pc}.(-dT_c)$.......(1);

where dA=B.dx

Analysis of Counter Flow HE

$$\frac{d(\Delta T)}{\Delta T} = -\left(\frac{1}{m_h C_{ph}} - \frac{1}{m_c C_{pc}}\right).U.B.dx$$

$$Integrating \int_{\Delta Ti}^{\Delta Te} \frac{d(\Delta T)}{\Delta T} = -\left(\frac{1}{m_h C_{ph}} - \frac{1}{m_c C_{pc}}\right).U.B\int_0^L dx$$

$$OR \quad [\ln \Delta T]_{\Delta Ti}^{\Delta Te} = -\left(\frac{1}{m_h C_{ph}} - \frac{1}{m_c C_{pc}}\right).U.B.L$$

$$\left(\frac{\Delta Te}{\Delta Ti}\right) = -\left(\frac{1}{m_h C_{ph}} - \frac{1}{m_c C_{pc}}\right).U.A......(4)$$

Analysis of Counter Flow HE

$$Also,\ Q = m_h C_{ph}(T_{hi} - T_{he}) = m_c C_{pc}(T_{ce} - T_{ci})$$

$$\Rightarrow \frac{1}{m_h C_{ph}} = \frac{T_{hi} - T_{he}}{Q} \ \& \ \frac{1}{m_c C_{pc}} = \frac{T_{ce} - T_{ci}}{Q}$$

Substituting in eqn.......(4), We have;

$$\ln\left(\frac{\Delta T_i}{\Delta T_e}\right) = \left(\frac{T_{hi} - T_{he}}{Q} - \frac{T_{ce} - T_{ci}}{Q}\right) U.A$$

$$OR \quad Q = \frac{U.A}{\ln\left(\frac{\Delta T_i}{\Delta T_e}\right)}\left[T_{hi} - T_{he} - T_{ce} + T_{ci}\right]$$

Analysis of Counter Flow HE

$$Q = \frac{U.A}{\ln\left(\frac{\Delta Ti}{\Delta Te}\right)}\left[\left(T_{hi} - T_{ce}\right) - \left(T_{he} - T_{ci}\right)\right]$$

$$Q = U.A.\frac{\Delta Ti - \Delta Te}{\ln\left(\frac{\Delta Ti}{\Delta Te}\right)} = U.A.\Delta T_m$$

$$Comparing\ we\ have;\ \Delta T_m = \frac{\Delta Ti - \Delta Te}{\ln\left(\frac{\Delta Ti}{\Delta Te}\right)}$$

now can be calculated from the expression:

Q=U.A.ΔT

Heat Exchangers

Parallel Flow	Counter Flow
$\Delta Ti = T_{hi} - T_{ci}$	$\Delta Ti = T_{hi} - T_{ce}$
$\Delta Te = T_{he} - T_{ce}$	$\Delta Te = T_{he} - T_{ci}$

Counter Flow HEs

A special case of Counter Flow HE occurs when the Capacity Rates on the two sides are equal:

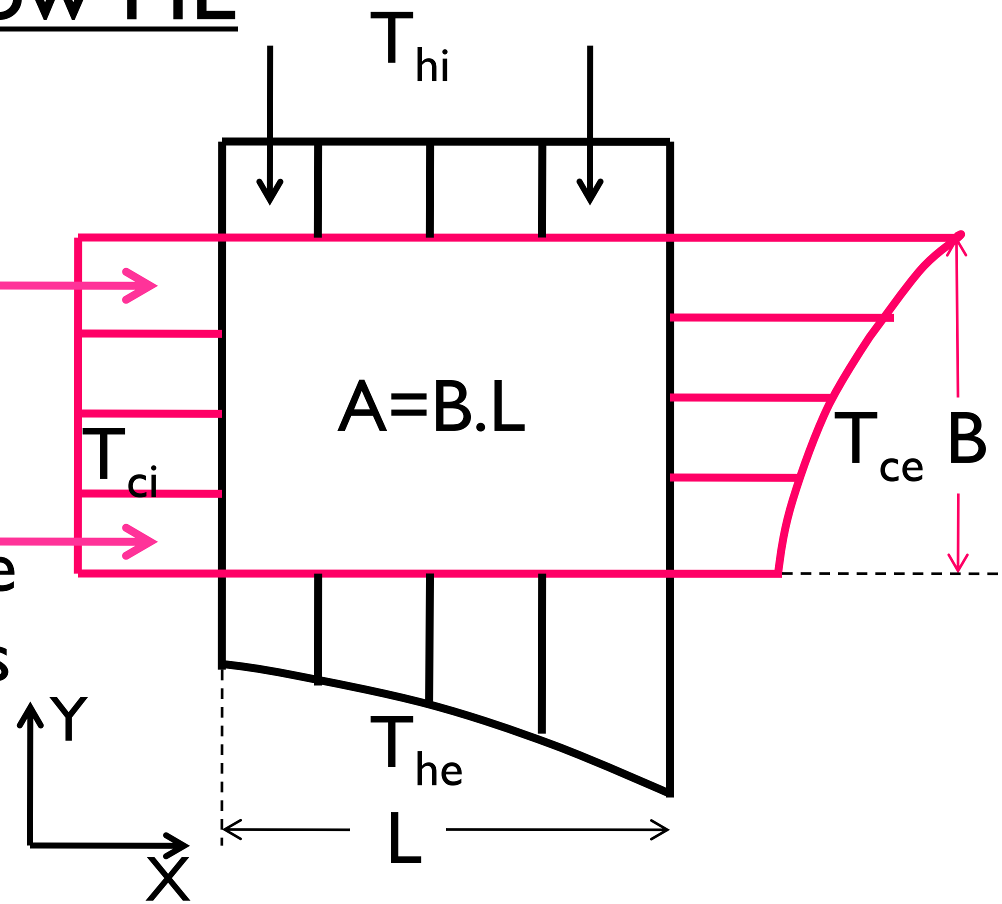
$m_h C_{ph} = m_c C_{pc} \rightarrow (T_{hi} - T_{ce}) = (T_{he} - T_{ci}) \rightarrow \Delta Ti = \Delta Te$

$$Hence\ \Delta T_m = \frac{\Delta Ti - \Delta Te}{\ln \frac{\Delta Ti}{\Delta Te}}\ will\ become\ Indetermine$$

This can be solved by applying L' hospitals rule and it can be shown that ΔTm = ΔTe = ΔTi

Cross Flow HE

- In both parallel and counter flow HEs, temps on both sides vary only along the length of HEs and are function of single variable, say x. This is not so in case of cross flow HE.



- It is obvious that T_h & T_c are now function of x and y … exit temp profiles are not uniform. Determination … LMTD involves double integration and becomes complicated

Cross Flow HE

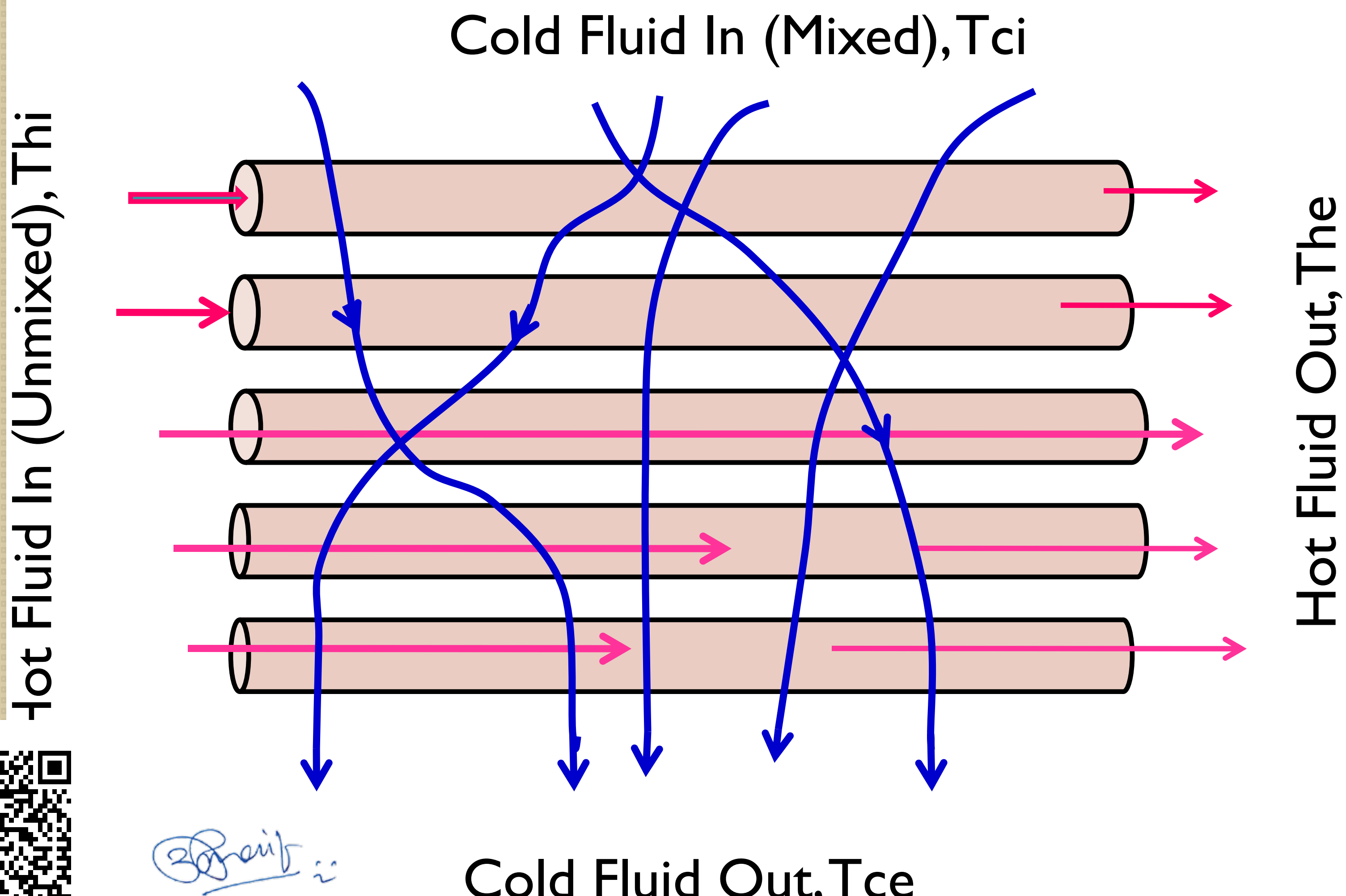
- Study and development of relations for cross flow and for many other types multi-pass flow arrangements was carried out by Bowman, Mueller and Nagle.

- Some of these are:
  - Both fluids unmixed
  - Both fluids mixed
  - One fluid mixed, one unmixed
  - One Shell & Two tube passes (and multiples of 2)
  - Two Shell passes and multiple tube passes

- Under these conditions, heat transfer rate is calculated as: $Q=U.A.F.(\Delta T_{m})_{counter\ flow}$, where F is correction factor, which graphically determined with the help of two parameters R and S

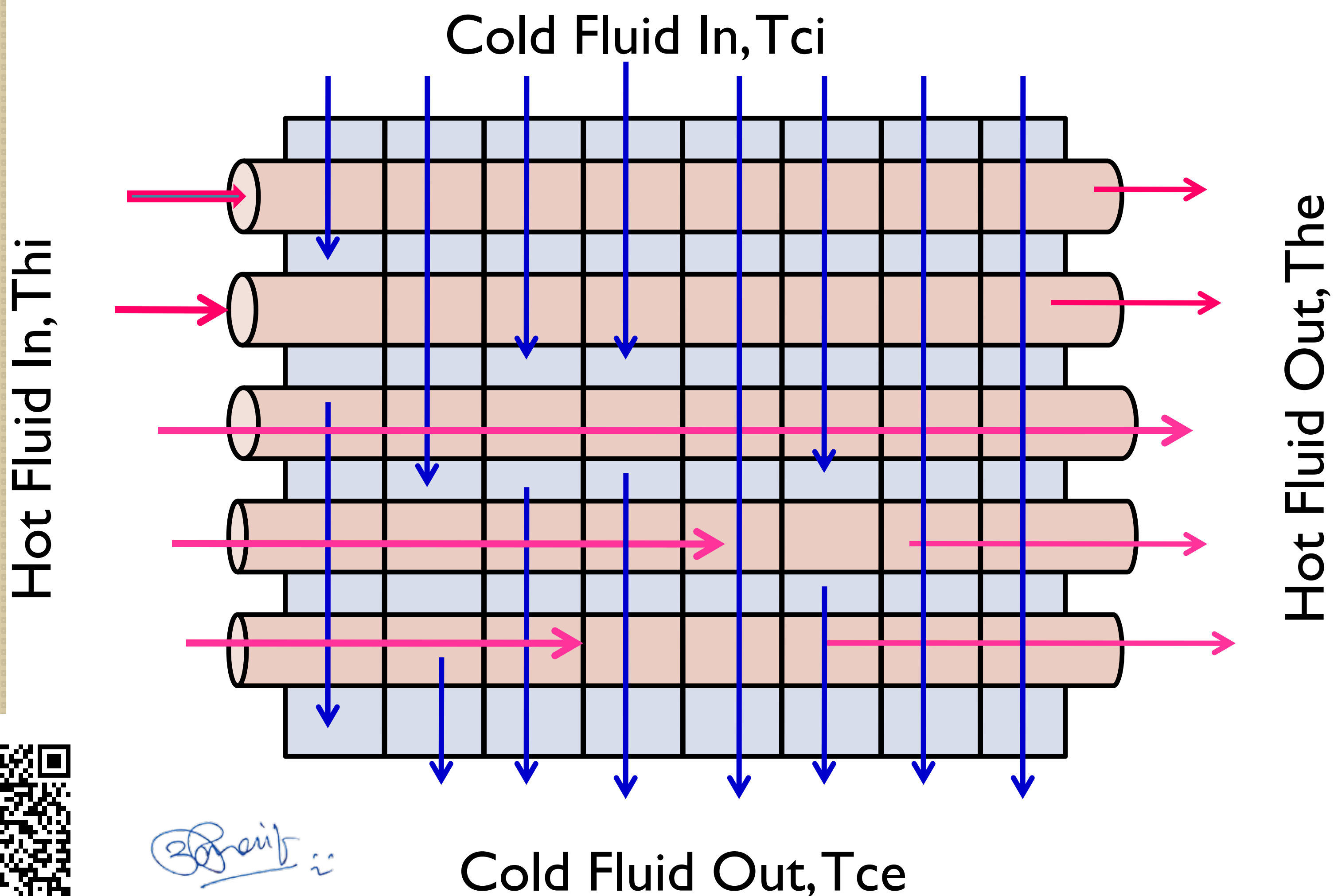
Direct Transfer Type Heat Exchanger

Cross Flow HE (One Fluid Mixed, One Unmixed):



Direct Transfer Type Heat Exchanger

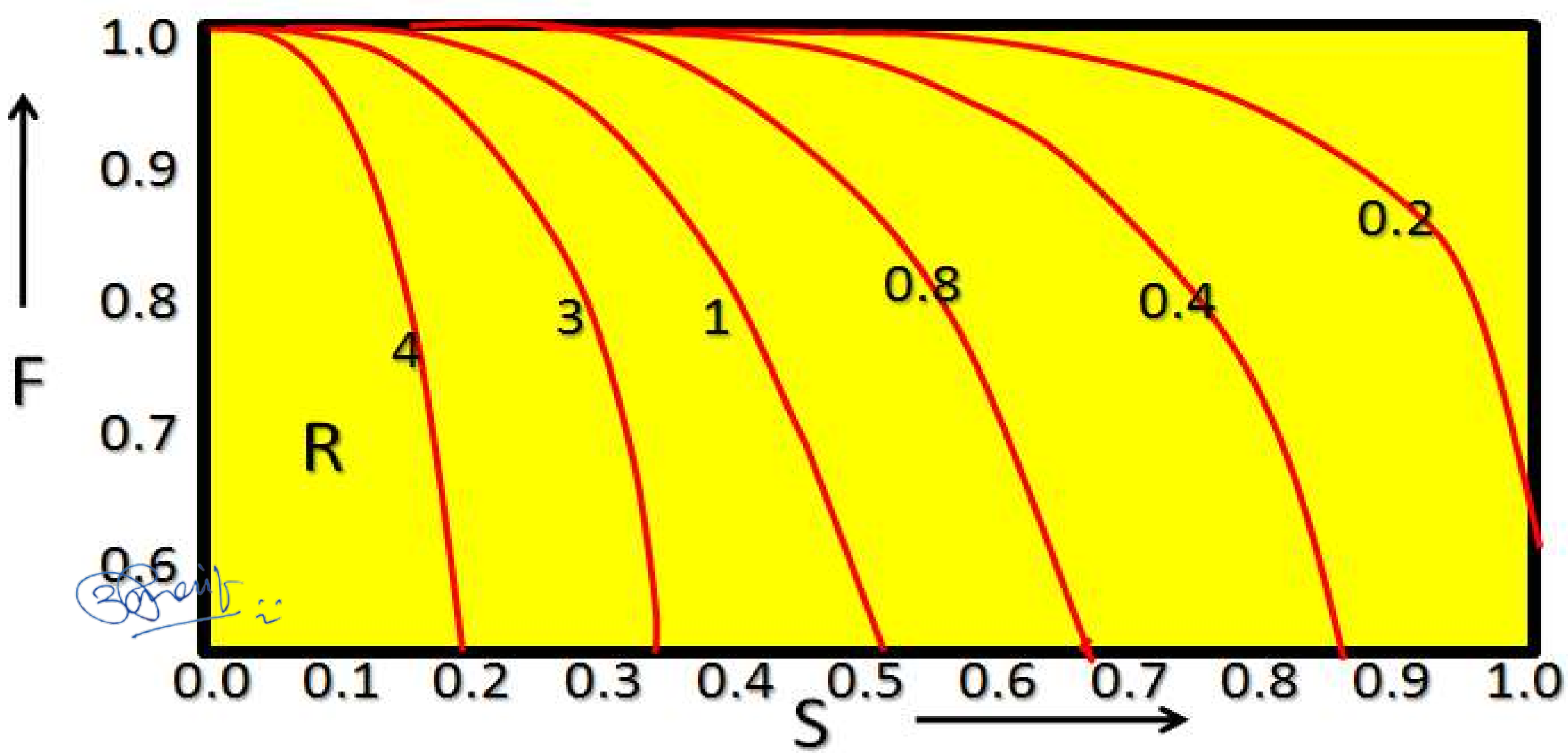
Cross Flow HE (Both Fluids Unmixed):



Cross Flow HE

$$R = \frac{T_{1i} - T_{1e}}{T_{2e} - T_{2i}} \; ; \qquad S = \frac{T_{2e} - T_{2i}}{T_{1i} - T_{2i}}$$

Subscript 1 must be assigned to MIXED fluid, in case one fluid is mixed and other is UNMIXED. In other cases, any subscript can be assigned to any fluid.



Special Cases of HEs

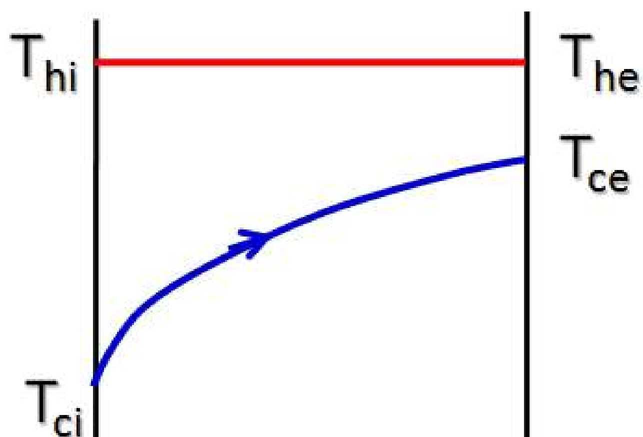
- Two special cases occur, when the product of the flow rate and specific heat (mC_p) is INFINITE either on hot or cold side.
- This happens when one of the fluids changes phase (Boiling and Condensation) and Q is calculated as

$Q = m\lambda$.

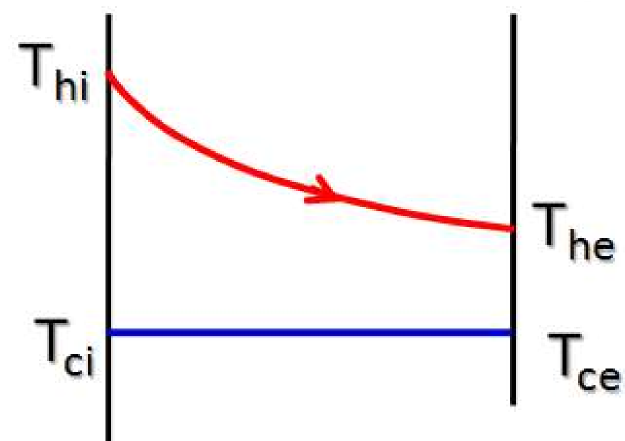
- $m_h C_{ph}(T_{hi} - T_{he}) = m_c C_{pc}(T_{ce} - T_{ci})$
- For Condensation: $T_{hi} - T_{he} = 0$ as $T_{hi} = T_{he}$

Since LHS = RHS and $T_{hi} - T_{he} = 0$, Therefore

$m_h C_{ph} = \infty$

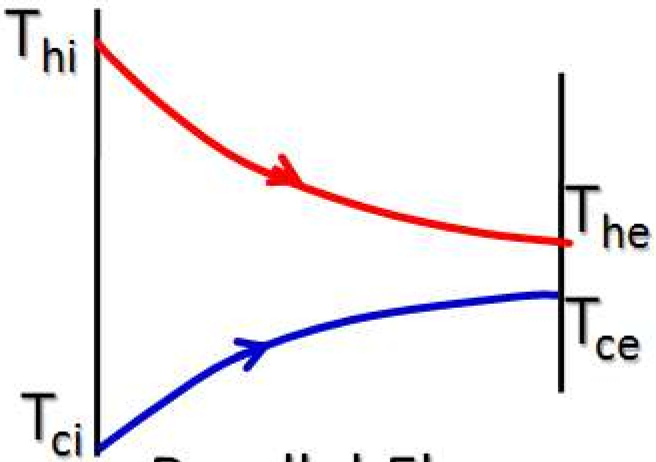


Condensation

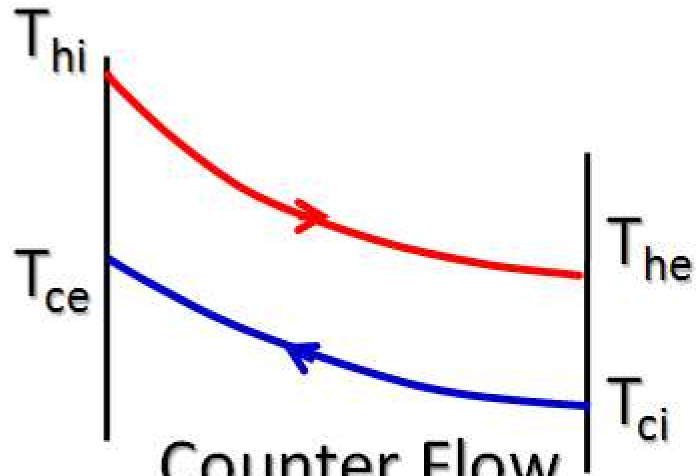


Boiling

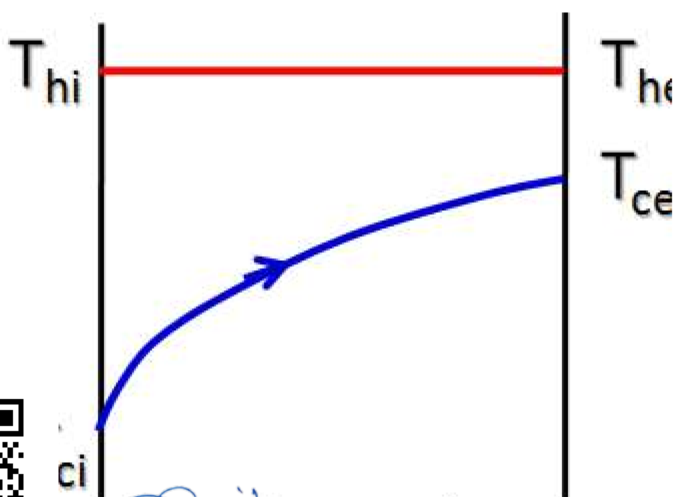
Temperature Profiles



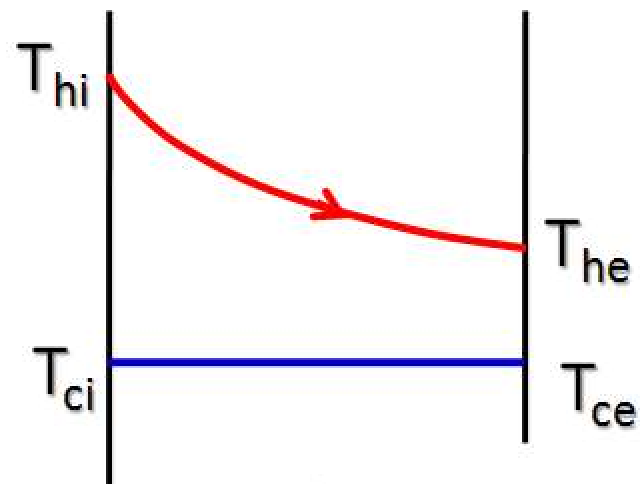
Parallel Flow



Counter Flow



Condensation



Boiling

NTU-Effectiveness Method

- While designing and testing, two types of problems are required to be tackled:

1. For the two given fluids, mass flow rates and inlet & exit temps are specified, and size of HE is required to designed for specified performance.

2. For given HE, only inlet temps and mass flow rates of the two fluids are specified and exit temps are required to be found out. This type of problem is basically evaluation of performance of a given heat exchanger.

- First kind of problems can be solved by LMTD method but second type of problems can not be solved by LMTD method.

o solve the second type of problems, NTU-Effectiveness s used. However, first type of problems can also be solved y this method.

NTU-Effectiveness Method

- For this method, 3 parameters will be defined:

1. Capacity Ratio (C): It is defined as the ratio of heat capacities of the two fluids and is given as:

$$C = \frac{(mC_p)_{small}}{(mC_p)_{large}}; \quad 0 \le C \le 1$$

2. Number of (Heat) Transfer Units (NTU):

$$NTU = \frac{U.A}{(mC_p)_{Small}}$$

r specified value of U/mC_p, NTU is the measure of actual ...at transfer area A or size of the HE.

NTU-Effectiveness Method

3. Effectiveness (ε):

$$Effectiveness(\varepsilon) = \frac{Actual\ Heat\ Transfer\ Rate\ (in\ HE)}{Max\ Possible\ Heat\ Transfer\ Rate\ (in\ HE)}$$

- Max heat transfer will occur when one of the fluids undergoes max temp change.

- When $Q = m_h C_{ph}(T_{hi} - T_{he}) = m_c C_{pc}(T_{ce} - T_{ci})$, obviously the fluid having smaller heat capacity will only undergo max temp in HE.

$$Hence\ \varepsilon = \frac{(mC_p \Delta T)_{hot\ or\ cold}}{(mC_p)_{small}(T_{hi} - T_{ci})}$$

NTU-Effectiveness Method

3. Effectiveness (ε):

$$\varepsilon = \frac{\left(mC_p \Delta T\right)_{hot\,or\,cold}}{\left(mC_p\right)_{small}\left(T_{hi} - T_{ci}\right)}$$

- For $(mC_p)_h < (mC_p)_c$:

$$\varepsilon = \frac{\left(mC_p\right)_h\left(T_{hi} - T_{he}\right)}{\left(mC_p\right)_h\left(T_{hi} - T_{ci}\right)} = \frac{T_{hi} - T_{he}}{T_{hi} - T_{ci}}$$

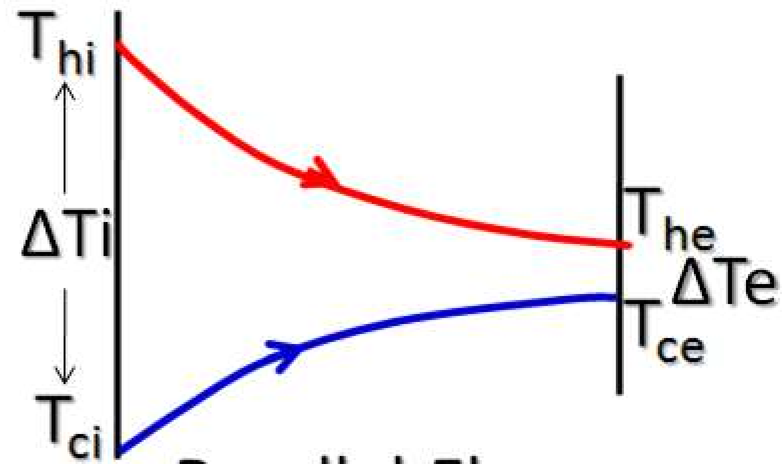
- For $(mC_p)_c < (mC_p)_h$:

$$\varepsilon = \frac{\left(mC_p\right)_c\left(T_{ce} - T_{ci}\right)}{\left(mC_p\right)_c\left(T_{hi} - T_{ci}\right)} = \frac{T_{ce} - T_{ci}}{T_{hi} - T_{ci}}$$

NTU Effectiveness Method For Parallel Flow HE

- Let $(mC_p)_h < (mC_p)_c$

$$Hence\ C = \frac{(mC_p)_h}{(mC_p)_c}$$



Parallel Flow

- Since $(mC_p)_h(T_{hi}-T_{he})=(mC_p)_c(T_{ce}-T_{ci})$

$$\therefore \frac{(mC_p)_h}{(mC_p)_c} = \frac{T_{ce}-T_{ci}}{T_{hi}-T_{he}} = C.......(1)$$

$$d\ \varepsilon = \frac{(mC_p)_h(T_{hi}-T_{he})}{(mC_p)_h(T_{hi}-T_{ci})} = \frac{T_{hi}-T_{he}}{T_{hi}-T_{ci}}........(2)$$

NTU-Effectiveness Method for Parallel Flow HE

Let us obtain T_{ci} & T_{ce} in terms of T_{hi} & T_{he} :

From eqn …(2), we have, $$T_{hi} - T_{ci} = \frac{T_{hi}}{\varepsilon} - \frac{T_{he}}{\varepsilon}$$

$$\Rightarrow T_{ci} = T_{hi} - \frac{T_{hi}}{\varepsilon} + \frac{T_{he}}{\varepsilon} \ldots\ldots\ldots(3)$$

And from eqn…(1), we have, $$T_{ce} - T_{ci} = C.T_{hi} - C.T_{he}$$

$$\therefore \quad T_{ce} = T_{ci} + C.T_{hi} - C.T_{he}$$

$$Putting\ \ value\ of\ \ T_{ci}\ \ from\ eqn...(3)$$

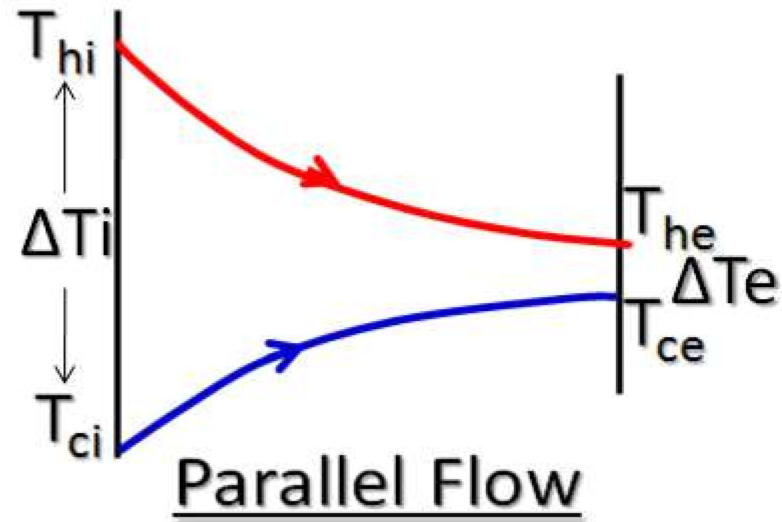
$$T_{ce} = T_{hi} - \frac{T_{hi}}{\varepsilon} + \frac{T_{he}}{\varepsilon} + C.T_{hi} - C.T_{he} \ldots\ldots(4)$$

NTU-Effectiveness Method for Parallel Flow HE

Here $\Delta T_i = T_{hi} - T_{ci}$

$$\Delta Ti = T_{hi} - (T_{hi} - \frac{T_{hi}}{\varepsilon} + \frac{T_{he}}{\varepsilon}$$

$$\Delta Ti = \frac{T_{hi} - T_{he}}{\varepsilon}$$



And $\Delta Te = T_{he} - T_{ce}$

Substituting T_{ce} *from eqn...(4), we have,*

$$\Delta Te = T_{he} - \left(T_{hi} - \frac{T_{hi}}{\varepsilon} + \frac{T_{he}}{\varepsilon} + C.T_{hi} - C.T_{he} \right)$$

$$Te = T_{he} - T_{hi} + \frac{T_{hi}}{\varepsilon} - \frac{T_{he}}{\varepsilon} - C.T_{hi} + C.T_{he}$$

NTU-Effectiveness Method for Parallel Flow HE

$$\Delta Te = T_{he} - T_{hi} + \frac{T_{hi}}{\varepsilon} - \frac{T_{he}}{\varepsilon} - C.T_{hi} + C.T_{he}$$

$$\Delta Te = \frac{1}{\varepsilon}\left(T_{hi} - T_{he}\right) - C\left(T_{hi} - T_{he}\right) - \left(T_{hi} - T_{he}\right)$$

$$= \left(T_{hi} - T_{he}\right)\left(\frac{1}{\varepsilon} - C - 1\right)$$

Heat balance eqn for hot fluid,

$$\left(mC_p \Delta T\right)_h = U.A.\Delta T_m = Q$$

$$C_p\Big)_h \left(T_{hi} - T_{he}\right) = U.A.\frac{\Delta Ti - \Delta Te}{\ln\left(\frac{\Delta Ti}{\Delta Te}\right)}$$

NTU-Effectiveness Method for Parallel Flow HE

$$\left(mC_p\right)_h \left(T_{hi} - T_{he}\right) = U.A.\frac{\Delta Ti - \Delta Te}{\ln\left(\dfrac{\Delta Ti}{\Delta Te}\right)}$$

$$\left(T_{hi} - T_{he}\right)\frac{\ln\left(\dfrac{\Delta Ti}{\Delta Te}\right)}{\Delta Ti - \Delta Te} = \frac{U.A}{\left(mC_p\right)_h} = NTU$$

Substituting values of ΔTi & ΔTe

above equation, we have;

NTU-Effectiveness Method for Parallel Flow HE

$$NTU = \frac{\left(T_{hi} - T_{he}\right)\ln\left[\dfrac{\dfrac{\left(T_{hi} - T_{he}\right)}{\varepsilon}}{\left(T_{hi} - T_{he}\right)\left(\dfrac{1}{\varepsilon} - C - 1\right)}\right]}{\dfrac{T_{hi} - T_{he}}{\varepsilon} - \left(T_{hi} - T_{he}\right)\left(\dfrac{1}{\varepsilon} - C - 1\right)}$$

$$TU = \frac{\left(T_{hi} - T_{he}\right)\ln\left[\dfrac{1/\varepsilon}{1/\varepsilon - C - 1}\right]}{\left(T_{hi} - T_{he}\right)\left[\dfrac{1}{\varepsilon} - \left(1/\varepsilon - C - 1\right)\right]}$$

NTU-Effectiveness Method for Parallel Flow HE

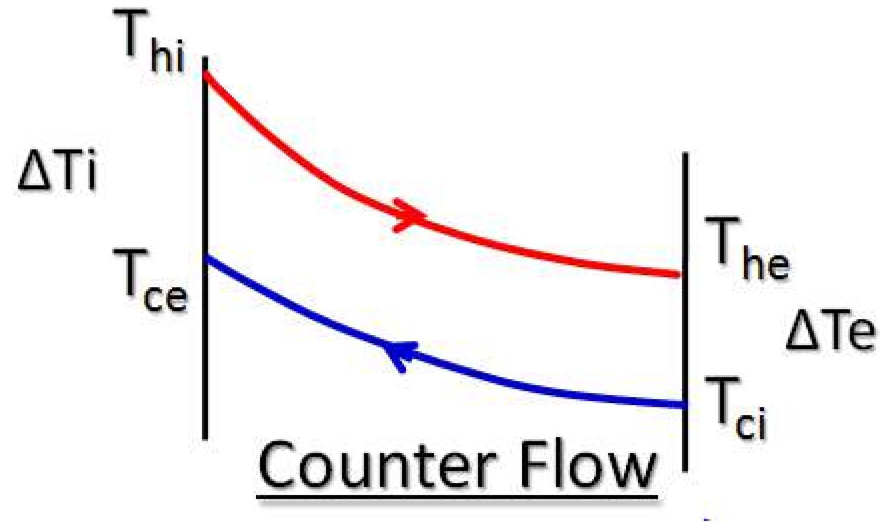
$$NTU = \frac{\ln\left(\frac{1}{1 - C\varepsilon - \varepsilon}\right)}{\frac{1}{\varepsilon} - \frac{1}{\varepsilon} + C + 1} \qquad OR$$

$$NTU(1+C) = \ln\left(\frac{1}{1 - C\varepsilon - \varepsilon}\right) \Rightarrow \frac{1}{1 - C\varepsilon - \varepsilon} = e^{NTU(1+C)}$$

$$\therefore \quad 1 - C\varepsilon - \varepsilon = e^{-(1+C)NTU} \Rightarrow \varepsilon(1+C) = 1 - e^{-(1+C)NTU}$$

$$\therefore Hence \quad \varepsilon = \frac{1 - e^{-(1+C)NTU}}{1+C}$$

NTU-Effectiveness Method For Counter Flow HE



- Let $(mC_p)_h < (mC_p)_c$

$$Hence\ C = \frac{(mC_p)_h}{(mC_p)_c}$$

- Since $(mC_p)_h(T_{hi}-T_{he})=(mC_p)_c(T_{ce}-T_{ci})$

$$\therefore \frac{(mC_p)_h}{(mC_p)_c} = \frac{T_{ce}-T_{ci}}{T_{hi}-T_{he}} = C.......(1)$$

$$d\ \varepsilon = \frac{(mC_p)_h(T_{hi}-T_{he})}{(mC_p)_h(T_{hi}-T_{ci})} = \frac{T_{hi}-T_{he}}{T_{hi}-T_{ci}}........(2)$$

NTU-Effectiveness Method for Counter Flow HE

Let us obtain T_{ci} & T_{ce} in terms of T_{hi} & T_{he} :

From eqn …(2), we have,

$$T_{hi} - T_{ci} = \frac{T_{hi}}{\varepsilon} - \frac{T_{he}}{\varepsilon}$$

$$\Rightarrow T_{ci} = T_{hi} - \frac{T_{hi}}{\varepsilon} + \frac{T_{he}}{\varepsilon} \quad ....(3)$$

And from eqn…(1), we have,

$$T_{ce} - T_{ci} = C.T_{hi} - C.T_{he}$$

$$\therefore \quad T_{ce} = T_{ci} + C.T_{hi} - C.T_{he}$$

Putting value of T_{ci} *from eqn…(3)*

$$T_{ce} = T_{hi} - \frac{T_{hi}}{\varepsilon} + \frac{T_{he}}{\varepsilon} + C.T_{hi} - C.T_{he} \quad ....(4)$$

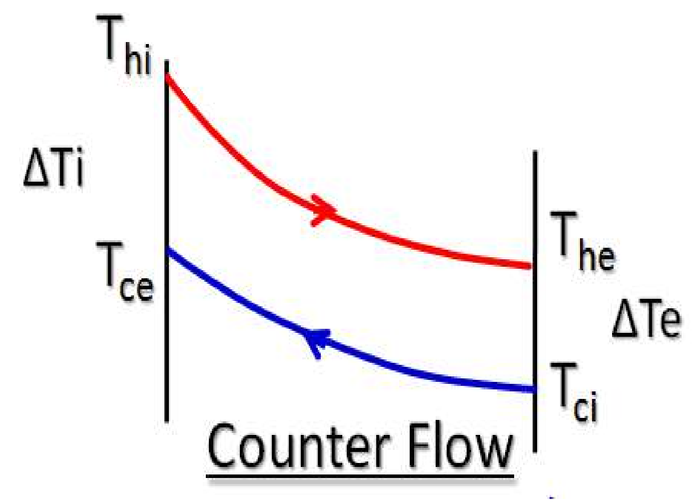
NTU-Effectiveness Method for Counter Flow HE

$$Here \quad \Delta T_i = T_{hi} - T_{ce}\,;\ On\ substituting\ from\ eqn\,(4),\ we\ have$$

$$\Delta Ti = T_{hi} - T_{hi} + \frac{T_{hi}}{\varepsilon} - \frac{T_{he}}{\varepsilon} - CT_{hi} + CT_{he}$$

$$= \frac{(T_{hi} - T_{he})}{\varepsilon} - C(T_{hi} - T_{he})$$

$$= (T_{hi} - T_{he})\left(\frac{1}{\varepsilon} - C\right) \text{.............}(5)$$



$$And \quad \Delta Te = T_{he} - T_{ci}$$

$$Substituting\ T_{ci}\ \ from\ eqn...(3),\ we\ have,$$

$$\Delta Te = T_{he} - T_{hi} + \frac{T_{hi}}{\varepsilon} - \frac{T_{he}}{\varepsilon} = \frac{T_{hi} - T_{he}}{\varepsilon} - (T_{hi} - T_{he}$$

$$Te = (T_{hi} - T_{he})\left(\frac{1}{\varepsilon} - 1\right) \text{.......}(6)$$

NTU-Effectiveness Method for Counter Flow HE

Heat balance eqn for hot fluid ,

$$\left(mC_p\Delta T\right)_h = U.A.\Delta T_m = Q$$

$$\left(mC_p\right)_h\left(T_{hi} - T_{he}\right) = U.A.\frac{\Delta Ti - \Delta Te}{\ln\left(\frac{\Delta Ti}{\Delta Te}\right)}$$

$$\ldots_{hi} - T_{he}\Big)\frac{\ln\left(\frac{\Delta Ti}{\Delta Te}\right)}{\Delta Ti - \Delta Te} = \frac{U.A}{\left(mC_p\right)_h} = NTU$$

NTU-Effectiveness Method for Parallel Flow HE

Substituting values of ΔTi & ΔTe *in above equation, we have;*

$$NTU = \frac{(T_{hi} - T_{he}) \ln \left[\dfrac{(T_{hi} - T_{he})\left(\dfrac{1}{\varepsilon} - C\right)}{(T_{hi} - T_{he})\left(\dfrac{1}{\varepsilon} - 1\right)} \right]}{(T_{hi} - T_{he})\left(\dfrac{1}{\varepsilon} - C\right) - (T_{hi} - T_{he})\left(\dfrac{1}{\varepsilon} - 1\right)}$$

NTU-Effectiveness Method for Parallel Flow HE

$$NTU=\frac{\ln\left[\dfrac{1/\varepsilon - C}{1/\varepsilon - 1}\right]}{\left(\dfrac{1}{\varepsilon}-C-\dfrac{1}{\varepsilon}+1\right)}=\frac{\ln\left[\dfrac{1-\varepsilon C}{1-\varepsilon}\right]}{1-C}$$

$$OR \qquad \ln\left[\frac{1-\varepsilon C}{1-\varepsilon}\right]=(1-C)NTU$$

$$R \quad \frac{1-\varepsilon C}{1-\varepsilon}=e^{(1-C)NTU}$$

NTU-Effectiveness Method for Counter Flow HE

$$OR \quad \frac{1-\varepsilon C}{1-\varepsilon} = e^{(1-C)NTU} \Rightarrow \frac{1-\varepsilon}{1-\varepsilon C} = e^{-(1-C)NTU}$$

$$OR \quad 1-\varepsilon = (1-\varepsilon C).e^{-(1-C)NTU}$$

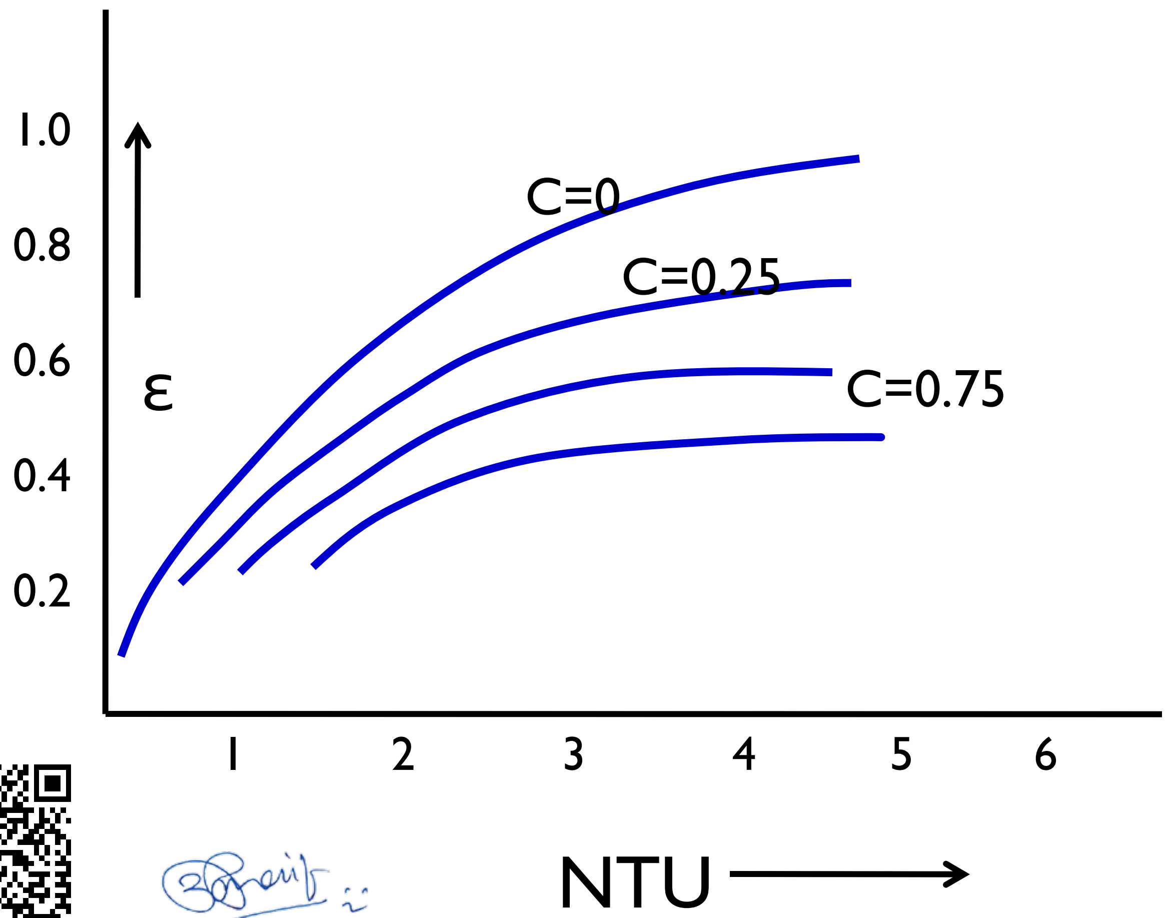
$$= e^{-(1-C)NTU} - \varepsilon C e^{-(1-C)NTU}$$

$$OR \quad 1-e^{-(1-C)NTU} = \varepsilon - \varepsilon.C.e^{-(1-C)NTU}$$

$$= \varepsilon\left[1-C.e^{-(1-C)NTU}\right]$$

$$\therefore \varepsilon = \frac{1-e^{-(1-C)NTU}}{1-Ce^{-(1-C)NTU}}$$

Cross Flow Heat Exchangers



Special Cases

Case-I: C=1 & Counter Flow Arrangement ;

i.e heat capacity is same for both fluids

$$Putting\ \ C = 1\ in\ equation$$

$$\varepsilon = \frac{1 - e^{-(1-C)NTU}}{1 - C.e^{-(1-C)NTU}},\ we\ get\quad \frac{0}{0} \Rightarrow Indeterminate$$

$$Hence\ by\ applying\ \ L'hospital's\ Rule,$$

$$We\ get\quad \varepsilon = \frac{NTU}{1 + NTU}$$

Special Cases

Case-II: C=0 (Condensers & Evaporators)

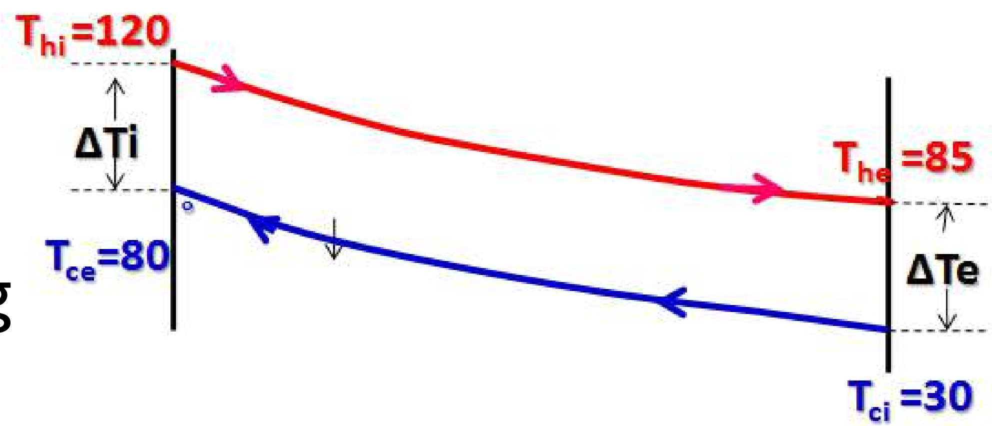
- When any of the two fluids changes its phase, its temp remains same; hence its heat capacity can be assumed as ∞.
- So, in case of condensers……..$(mC_p)_h = \infty$; i,e $(T_{hi} - T_{he}) = 0$
- And, In case of Evaporator…….$(mC_p)_c = \infty$; i.e $(T_{ce} - T_{ci}) = 0$
- Therefore, in both the cases, $$\varepsilon = 1 - e^{-NTU}$$
- In such cases, flow direction of fluids is immaterial

or Condenser…..$(m\lambda)_h = (mC_p)_c$

r Evaporator…..$(m\lambda)_c = (mCp)_h$

Numerical Problems on Heat Exchangers

Q1 : A counter flow shell and tube type heat exchanger used to heat water at a rate of 0.8 kg/sec from 30°C to 80°C with hot oil entering at 120°C and leaving at 85°C Over all heat transfer coefficient is 125 $W/m^2 °C$. Calculate the size of heat exchanger required. Take specific heat for Water as 4180 J/kg°C.



Solution:

In design, size means area of HE transferring heat ⟹ **A**

It is not the physical size of HE

rea A can be found out from the expression

$Q = U.A.\Delta T_m$

Solution (Contd):

Q can be found out from $Q=m.C_p.(T_e-T_i)$ for water

$Q=(mCp)_w.(T_e - T_i)=0.8x4180x(80-30)=167200\ W$

$$Q = UA\frac{\Delta Ti - \Delta Te}{\ln\frac{(\Delta Ti)}{(\Delta Te)}} = 167200$$

$$Q = 125xAx\frac{(120-80)-(85-30)}{\ln\frac{(120-80)}{(85-30)}} = 167200$$

$$\Rightarrow A = 28.4m^2$$

Q2 : Engine oil is to be cooled from 80 to 50°C by using a single pass counter flow, concentric tube HE with cooling water available at 20°C. Water flows inside the tube of 2.5cm ID @ 288 kg/hr and oil flows through annulus @ 576 kg/hr. Heat transfer coeffs on water and oil sides are 1000 and 80 W/m^{2}K respectively, fouling factors on both sides are 0.00018 m^{2}K/W and tube wall resistance is negligible. Take Cp_w=4180 J/kgK, Cp_{oil}=2090J/kgK. Calculate the tube length required.

Solution:

$Q=U.A.\Delta T_m$ $A=\pi DL$ L = ? (=59m) Ans

To find ΔTm, all 4 temps are required; since only 3 temps are given, 4th temp can be found as follows:

$(m.C_p)_{oil}(T_{hi}-T_{he}) = (m.C_p)_{water}(T_{ce}-T_{ci})$

Solution (Contd):

U can be found out from given data as under:

$$\frac{1}{UA} = \frac{1}{h_w A_i} + \frac{F_w}{A_i} + \frac{\ln \frac{r_2}{r_1}}{2\pi kL} + \frac{F_o}{A_o} + \frac{1}{h_o A_o}$$

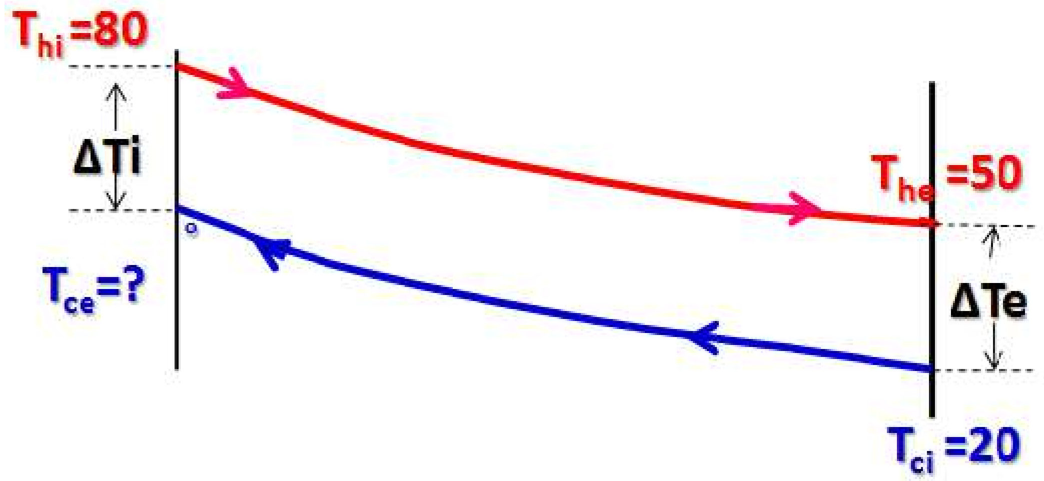
Since only ID has been given, conductive resistance term $\ln(r_2/r_1)/2\pi kL$ has to be neglected.

Also, A_i is required to be taken equal to A_o and hence A; Hence above expression of UA simplifies to:

$$\frac{1}{U} = \frac{1}{h_w} + F_w + F_o + \frac{1}{h_{oil}}$$

Solution (Contd):

To find T_{ce} ?



$$(m.C_p)_{oil}\ (T_{hi}-T_{he}) = (m.C_p)_{water}(T_{ce}-T_{ci})$$

$$\frac{288}{3600}x2090x(80-50)=\frac{576}{3600}x4180x(Tce-20)$$

$$\Rightarrow T_{ce} = 50°C$$

Now, Q = (288/3600)x2090x(80-50)=10,032 W

Solution (Contd):

$$\frac{1}{U}=\frac{1}{h_w}+F_w+F_o+\frac{1}{h_{oil}}=\frac{1}{1000}+0.00018+0.00018+\frac{1}{80}$$

Therefore, U=72.15 W/m^{2}K

Now, ΔTi=(80-50)=30 & ΔTe=(50-20)=30

$$\Delta T_m=\frac{\Delta T_i-\Delta T_e}{\ln\frac{\Delta T_i}{\Delta T_e}}=\frac{30-30}{\ln\frac{30}{30}}=\frac{0}{0}\ Indeterminate$$

Hence, applying L'Hospitals Rule, ΔT_m=30°C

$$Q=UA\Delta T_m\Rightarrow A=\frac{Q}{U.\Delta T_m}=\frac{10032}{72.15x30}=4.63m^2$$

=лDL→L=4.63/(лx0.025)=59m ANSWER

Q3 :A HE has mean over all heat transfer coefficient of 400 W/m^2K based on the side whose surface area is 100m^2. Find outlet temps of both the fluids for parallel and counter Flow arrangements for the following data given:

	Hot Fluid	Cold Fluid
Inlet Temp	750°C	100°C
Sp Heat kJ/kgK	3.6	4.2
Flow kg/min	1000	1500

<u>Solution:</u>

Since only two temps are given, LMTD method can not be plied.

ence NTU-Effectiveness Method shall be used

Solution:

Temps can be found out from ε expression as follows:

$$\varepsilon = \frac{T_{hi} - T_{he}}{T_{hi} - T_{ci}} \quad OR \quad \varepsilon = \frac{T_{ce} - T_{ci}}{T_{hi} - T_{ci}}$$

So, let us find out $m_c C_{pc}$ =(1500/60)x4200=1,05,000

And $m_h C_{ph}$ =(1000/60)x3600=60,000

$$\therefore C = \frac{60,000}{1.05,000} = 0.571$$

$$NTU = \frac{UA}{(mC_p)_{small}} = \frac{400x100}{60,000} = 0.667$$

Solution: Parallel Flow Arrangement

$$\varepsilon = \frac{1-e^{-(1+C)NTU}}{1+C} = \frac{1-e^{-(1+0.571)0.667}}{1+0.571} = 0.413$$

$$\varepsilon = 0.413 = \frac{T_{hi}-T_{he}}{T_{hi}-T_{ci}} = \frac{750-T_{he}}{750-100}$$

$$\Rightarrow T_{he} = 481.55°C$$

Now $(mC_p)_c(T_{ce}-T_{ci}) = (mC_p)_h(T_{hi}-T_{he})$

$$T_{ce} = T_{ci} + \frac{(mC_p)_h(T_{hi}-T_{he})}{(mC_p)_c}$$

$$T_{ce} = 100 + \frac{60000}{105000} x(750-481.55)$$

$$\Rightarrow T_{ce} = 253.4°C$$

Solution: Counter Flow Arrangement (NTU & C will not change)

$$\varepsilon = \frac{1-e^{-(1-C)NTU}}{1-Ce^{-(1-C)NTU}} = \frac{1-e^{-(1-0.571)0.667}}{1-0.571e^{-(1-0.571)0.667}} = 0.4356$$

$$\varepsilon = 0.4356 = \frac{T_{hi}-T_{he}}{T_{hi}-T_{ci}} = \frac{750-T_{he}}{750-100}$$

$$\Rightarrow T_{he} = 466.85°C$$

Now $(mC_p)_c(T_{ce}-T_{ci}) = (mC_p)_h(T_{hi}-T_{he})$

$$T_{ce} = T_{ci} + \frac{(mC_p)_h(T_{hi}-T_{he})}{(mC_p)_c}$$

$$T_{ce} = 100 + \frac{60000}{105000}x(750-466.85)$$

$$\Rightarrow T_{ce} = 261.8°C$$

Q1: Determine size and length for a HE fabricated from 25.4mm OD tube to cool 6.93 kg/min of a 90% ethyl Alcohol solution (Cp=3810 J/kgK) from 65.6°C to 39.4°C. Water at 10°C is available as coolant at a flow rate of 6.3 kg/min. Take over all heat transfer coeff based on outer tube surface as 568 W/m^{2}K.

Consider the following arrangements:

(a) Parallel flow tube and shell
(b) Counter flow tube and shell
(c) Reverse current HE with 2 shell passes and 72 tube passes, alcohol flowing through shell and water flowing through tubes
(d) Cross flow with one tube and one shell pass, shell side fluid mixed

Solution:

$(mC_p)_{alcohol}(T_{hi}-T_{he})=(mC_p)_{water}(T_{ce}-T_{ci})$

$$\frac{6.93}{60}x3810x(65.6-39.4)=\frac{6.3}{60}x4178x(T_{ce}-10)$$

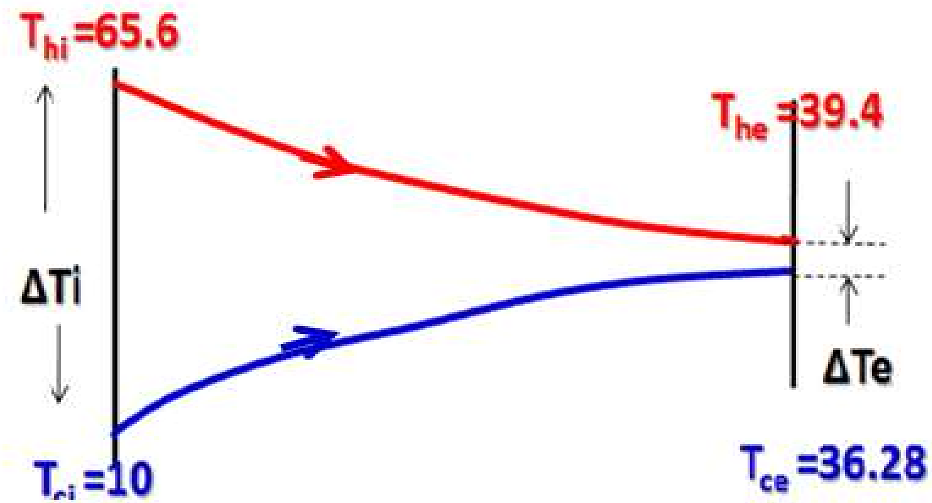
$\rightarrow$ Tce=36.28°C

Q=(6.93/60)x3810x(65.6-39.4)

$\rightarrow$ Q=11529.44 W

Solution (contd): For Parallel Flow

$Q = U_o A_o \Delta T_m$



$A_o = \pi D L$ $\quad\quad$ L = ?

$$LMTD = \frac{(65.6-10)-(39.4-36.28)}{\ln\dfrac{(65.6-10)}{(39.4-36.28)}} = 18.22$$

$$A_o = \frac{Q}{U_o \Delta T_m} = \frac{11529.44}{568x18.22} = 1.114\ m^2$$

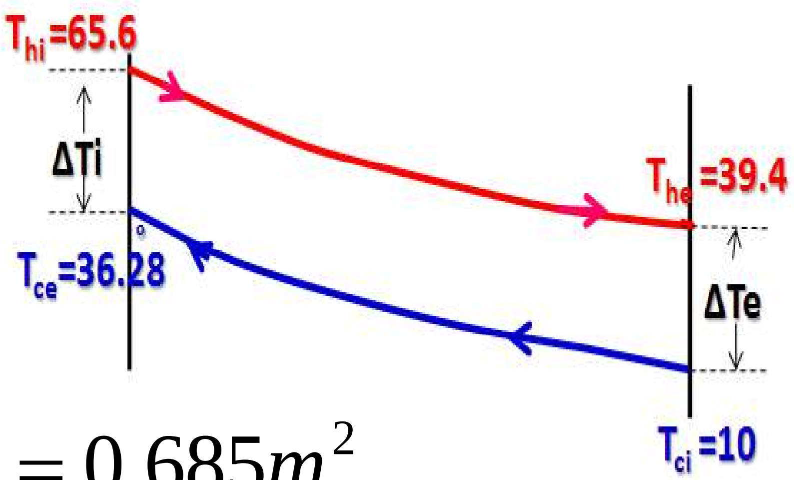
$$\pi D_o L = 1.114$$

$$\Rightarrow L = 13.97m$$

Solution (contd): For Counter Flow

$$\Delta T_m = \frac{(65.6-36.28)-(39.4-10)}{\ln\frac{(65.6-36.28)}{(39.4-10)}}$$

$$= 29.63$$



$$A_o = \frac{Q}{U_o \Delta T_m} = \frac{11529.44}{568x29.63} = 0.685m^2$$

$$A_o = \pi D_o L = 0.685\ m^2$$

$$\Rightarrow L = 8.59m$$

Solution (contd): For 2 Shell Passes & 72 Tube Passes

$\Delta T_m = F \times \Delta T_{m\,C/F}$

$$R = \frac{T_{hi} - T_{he}}{T_{ce} - T_{ci}} = \frac{65.6 - 39.4}{36.28 - 10} = 1; \quad S = \frac{T_{ce} - T_{ci}}{T_{hi} - T_{ci}} = \frac{36.28 - 10}{65.6 - 10} = 0.47$$

Hence, F=0.97 from plot

Now, $Q = U.A_o.F.\Delta T_m$

$$\Rightarrow A_o = \frac{Q}{U.F.\Delta T_m} = \frac{11529.44}{568 x 0.97 x 29.63} = 0.706 m^2$$

$$= \pi D_o L = 0.706 m^2 \Rightarrow L = 8.86 m$$

Solution (contd): For Cross Flow HE

$\Delta T_m = F \times \Delta T_{m\ C/F}$

$$R = \frac{T_{1i} - T_{1e}}{T_{2e} - T_{2i}} = \frac{65.6 - 39.4}{36.28 - 10} = 1; \quad S = \frac{T_{2e} - T_{2i}}{T_{1i} - T_{2i}} = \frac{36.28 - 10}{65.6 - 10} = 0.47$$

Hence, F=0.88 from plot

Now, $Q = U.A_o.F.\Delta T_m$

$$\Rightarrow A_o = \frac{Q}{U.F.\Delta T_m} = \frac{11529.44}{568x0.88x29.63} = 0.778m^2$$

te: Counter Flow arrangement requires minimum a.Parallel Flow arrangement requires maximum area

Q5 : In a parallel flow HE, cold water at 15°C flows through the inner tube of a tube-in-tube-type HE. Water exit temp is 50°C. This water is heated by hot oil entering at 130°C and leaving at 60°C.

Find the exit temps of the two fluids, if the Parallel Flow arrangement is switched over to Counter Flow one. Assume same NTU in both cases.

Solution:

$Q=(mC_p)_{hot}(T_{hi}-T_{he})=(mC_p)_{cold}(T_{ce}-T_{ci})$

$(mC_p)_{hot}(130-60)=(mC_p)_{cold}(50-15)$

$$\frac{(mC_p)_{hot}}{(mC_p)_{cold}}=\frac{35}{70}=0.5=C \;(as\; C\; is\; always\; less\; than\; 1)$$

Solution (contd):

$$\varepsilon = \frac{T_{hi} - T_{he}}{T_{hi} - T_{ci}} = \frac{130-60}{130-15} = 0.609$$

$$For\ \ Parallel\ \ Flow\ \ \varepsilon = \frac{1-e^{-(1+C)NTU}}{1+C}$$

$$0.609(1+0.5) = 1-e^{-(1+0.5)NTU} \Rightarrow 0.913 = 1-e^{-1.5NTU}$$

$$\Rightarrow NTU = 1.63$$

When arrangement is switched over to Counter Flow, NTU=1.63 will remain same [as such, for given HE, NTU=UA/$(mCp)_s$ can not change by change in flow arng]

so, C = $(mC_p)_s/(mC_p)_L$ =0.5 can not change with change flow arrangement

Solution (contd):

For counter Flow, exit temps will change.

$$\varepsilon = \frac{1-e^{-(1-C)NTU}}{1-Ce^{-(1-C)NTU}} = \frac{T_{hi}-T_{he}}{T_{hi}-T_{ci}}$$

$$\frac{1-e^{-(1-0.5)1.63}}{1-0.5.e^{-(1-0.5)1.63}} = \frac{130-T_{he}}{130-15}$$

$$\Rightarrow T_{he} = 47.74°C$$

Now, $(mC_p)_{hot}(T_{hi}-T_{he})=(mC_p)_{cold}(T_{ce}-T_{ci})$

$$T_{ce} = T_{ci} + \frac{(mC_p)_h}{(mC_p)_c}(T_{hi}-T_{he}) = 15+0.5(130-47.74)$$

$$\Rightarrow T_{ce} = 56.13°C$$

Design Aspects of HEs

1. Heat Transfer Rate Q:
    - Q requirement to be met
    - Requires selection of many parameters
2. Flow Rates:
    - Flow rates decide velocities of fluids
    - Lower velocity, lower the 'h' and higher velocity causes noise, vibrations, higher frictional losses & pressure drops (Generally 5-6 m/s)
3. Fouling Factor :
    - Fluid properties, tendency of scale formation
    - Selection of HE material, expected life span, maintenance requirement

Design Aspects of HEs

4. Outer Shape & Over all Dimensions:
   - Space & shape of space available in main equipment
   - Based on above, tube length, layout etc to be decided

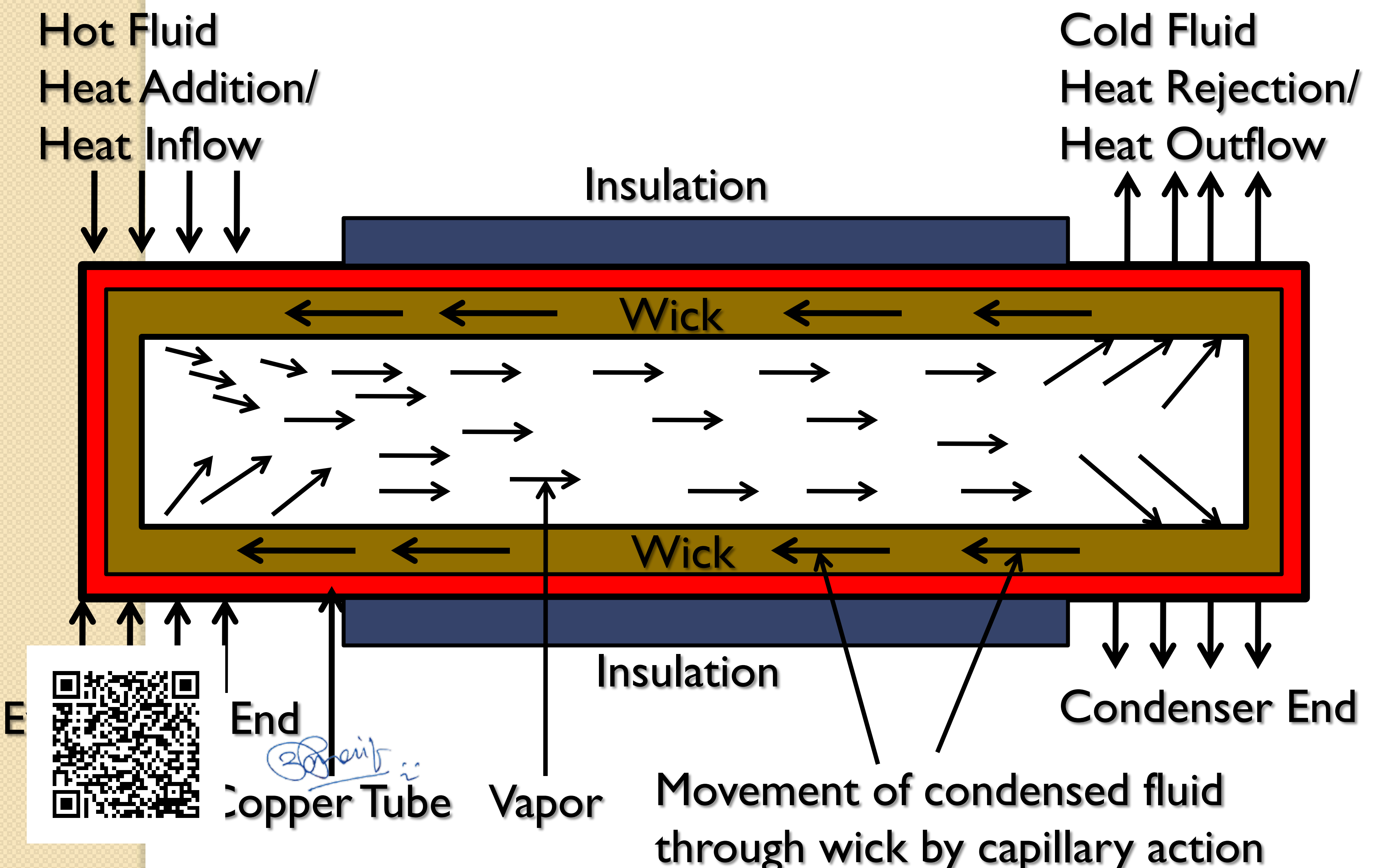
5. Strength Factor:
   - HE to be designed for high strength keeping temp, operating pressure, vibrations etc, in mind
   - Lowest weight and size

6. Pressur Drop & Pumping Power Requirement :
   - $\Delta P \propto V^2$; $m \propto V$; Power $\propto$ m.ΔP; $P \propto V^3$ & $P \propto 1/D_h$
   - Hence compromise reqd among Q, P & ΔP

Cost:

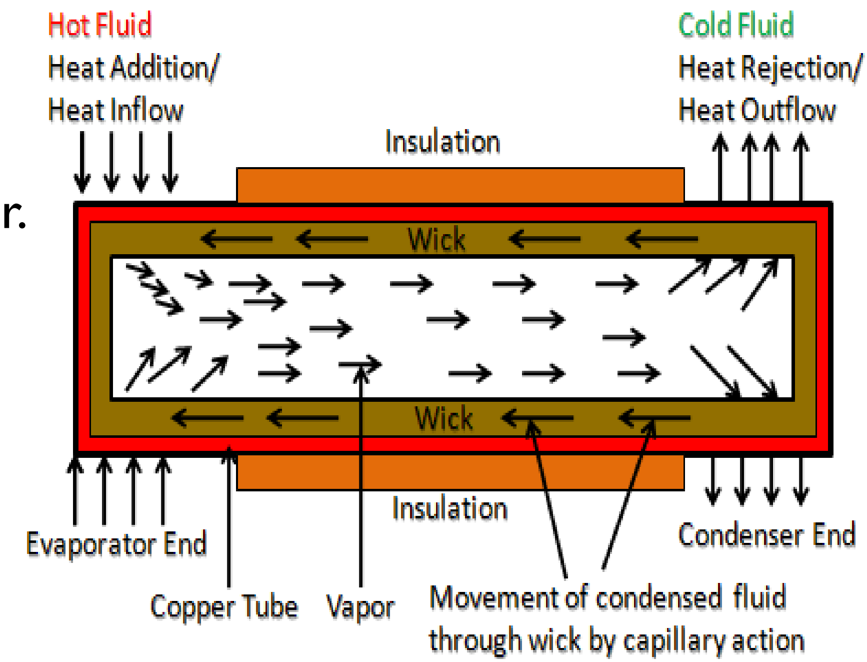
.owest initial and maintenance /operating cost

Heat Pipe


Hot Fluid
Heat Addition/
Heat Inflow
Cold Fluid
Heat Rejection/
Heat Outflow
Insulation
Wick
Wick
Insulation
End
Condenser End
opper Tube
Vapor
Movement of condensed fluid
through wick by capillary action

Heat Pipe

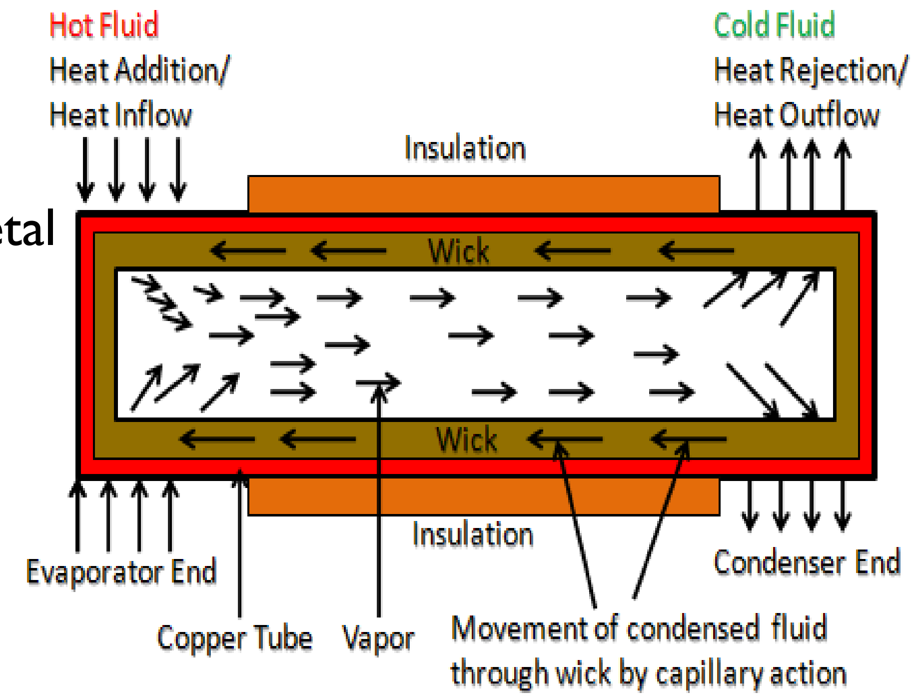
Heat pipe is a device (HE), which is utilized to obtain very high rate of heat flow from the surfaces having smaller area for heat transfer. Heat transfer is at constant temp.



Construction:

- Heat pipe consists of a hollow circular pipe (Copper) with its ends sealed
- Heat pipe is filled with a condensable working fluid like water
- Pipe is covered on the inside by a layer of wicking material
- Central portion of the pipe is heavily insulated on the outside surface
- ne end is known as "Heating End (Evaporator)" and it receives heat om the outside hot fluid, from which heat is to be removed
- ther end is known as "Cooling End (Condenser)" and it gives out heat o outside cold fluid.

Heat Pipe



Working:

- Heating end receives heat from outside hot fluid (to be cooled) and this heat moves through metal pipe to working fluid
- This working fluid in the wick material gets vaporized and this vapor moves towards empty central portion of the pipe
- Vapor, on reaching cooling end, gives out its heat to outside cold fluid through the pipe (condenser) and condenses in the wicking material
- The condensate now flows through the wicking layer back to heating end by capillary action and the cycle repeats. Working fluid acts as carrier of heat energy due to change of its phase
- Commonly used working fluids are water, mercury, sodium, lithium, methanol, liquid ammonia, etc

Applications of Heat Pipe

- Heating and cooling of space vehicles

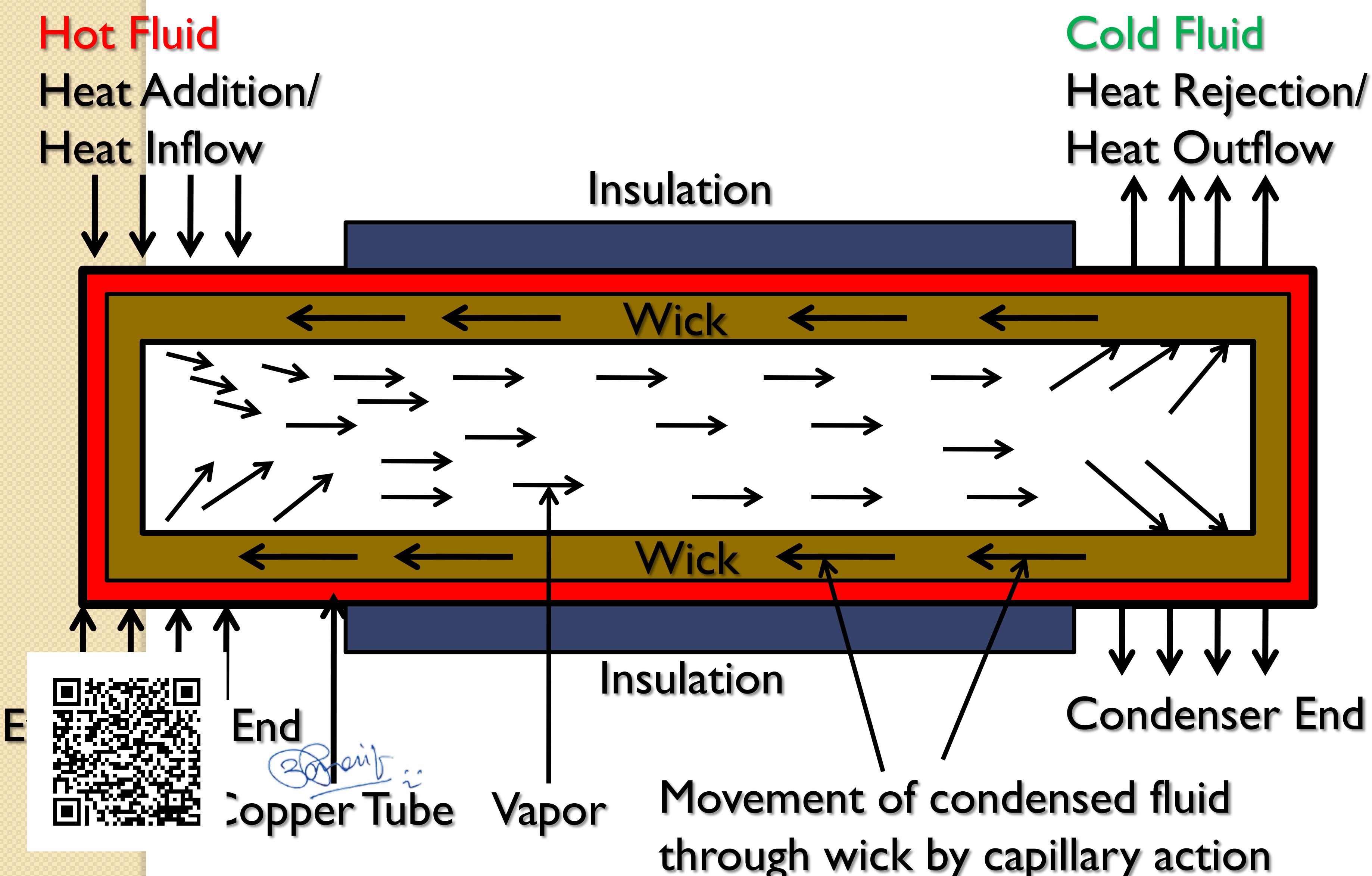
- Heat dissipation/removal from small micro-electronic circuits/ small electronic circuits generating large amount of heat

- Temperature control device to maintain temperature of a system to desired/specified value

Heat pipe of copper using water
as working fluid
can
transport an axial heat flux
of 6500 kW/m^2 at 200°C

Heat Transfer Augmentation Techniques

- Use of twisted tape
- Use of internal extended surfaces
- Use of dimple surface


Hot Fluid
Heat Addition/
Heat Inflow
Cold Fluid
Heat Rejection/
Heat Outflow
Insulation
Wick
Wick
Insulation
E
End
opper Tube
Vapor
Condenser End
Movement of condensed fluid
through wick by capillary action

End of Unit - VI

Mass Transfer

- Mass transfer is defined as, "movement *of mass due to concentration difference in a mixture*".
- The concentration difference is the driving potential for the mass transfer.
- Mass transfer occurs in many processes, such as absorption, evaporation, adsorption, desorption, solvent extraction, humidification and drying.
- In many practical applications, heat transfer processes occur simultaneously with mass transfer processes and the principles of mass transfer are very similar to those of heat transfer, therefore, the analogy between heat and mass transfer can easily be established.

MODES OF MASS TRANSFER

The mechanism of mass transfer process can be classified as :

(i) Mass transfer by diffusion,

(ii) Mass transfer by convection,

(i) <u>Mass transfer by diffusion</u>:

- The *molecular diffusion is the* transfer of mass on a microscopic level as a result of concentration gradient of one or more constituents (species) in the system.
- The diffusion of mass from a constituent occurs through a layer of stagnant fluid, and it may be occurred due to concentration gradients, temperature gradient, or pressure gradient.

The various **examples of molecular diffusion** can be taken from our day to day experiences. Some of these are :

(1) Dissolution of sugar or salt in the hot water and its uniform diffusion throughout, although their molecules are much heavier than water molecules.

(2) Evaporation of water from a pond to increase the humidity of passing air stream.

(3) Evaporation of petrol in engine carburetor.

(4) Evaporation of moisture during drying of clothes or wood.

(5) Humidification of air in air coolers, cooling towers etc.

(6) Spread of fragrance of perfume or flowers in surroundings.

(7) Diffusion of smoke through a tall chimney into atmosphere.

(ii) Mass transfer by convection:

- The mass transfer between a surface and a moving fluid or between two immisible moving fluids is referred as *convective mass transfer.*

- *The convective mass transfer* depends on
(1) transport properties of fluid, &
(2) dynamic characteristics (laminar or turbulent flow) of flowing fluid.

- The evaporation of water by moving air in a desert cooler is an example of convective mass transfer.
- A change of phase of a fluid can also cause mass transfer, such mass transfer is a combination of processes of diffusion and convection.

Some examples of mass transfer by phase change are,

(1) Hot flue gases leaving the chimney, rise by convection and then diffuse into atmospheric air

(2) Mass transfer from boiling water into air.

Terms	*Interpretation*
Heat energy	A form of energy in transit.
Conduction	Energy transfer into the medium due to existence of temperature gradient.
Convection	Energy transportation by moving fluid particles from hot region to cold region.
Radiation	Emission of energy in the form of electromagnetic waves by the surface.
Emissivity	A property of the radiating surface.
Thermal conductivity	Ability of the materials, which allows the heat conduction through them.
Heat transfer coefficient	A property of the fluid environment associated with heat convection.
Mass transfer	Movement of mass due to concentration difference in a mixture.

COMPARISON BETWEEN HEAT AND MASS TRANSFER

Analogy heat and mass transfer

Problem of mass transfer from a surface to flowing fluid (convection) is also solved in a similar way. It is only necessary to use appropriate variables

Heat transfer	Mass transfer
$\frac{T - T_f}{T_w - T_f}$ temperature	$\frac{y_A - y_{Af}}{y_{Aw} - y_{Af}}$ mass or molar fraction
a [m²/s] temperature diffusivity	D_{AB} [m²/s] diffusion coefficient
q heat flux [W/m²]	n_A mass flux [kgA/(m² s)]
$q = \alpha(T_w - T_f)$ heat transfer coef. [W/(m²K)]	$n_A = \beta(\rho_{Aw} - \rho_{Af})$ mass transfer coef. [m/s]

CONCENTRATIONS, VELOCITIES AND FLUXES

Mass concentration or mass density: The mass concentration of the component A within a multi component mixture is defined as mass of species A per unit volume of the mixture under consideration.

It is denoted by ρ_A and is expressed in kg/m^3. Mass concentration ρ_A

$$= \frac{\text{Mass of component A}}{\text{Volume of mixture}} = \frac{m_A}{V}$$

Molar concentration or molar density: The molar concentration of the component A is defined as the number of moles of species A per unit volume of mixture.

It is also called *molar density and denoted by C_A and expressed in* kg.mol/m^3.

The molar concentration,

$$C_A = \frac{\text{No. of moles of component A}}{\text{Volume of mixture}} = \frac{n_A}{V}$$

Number of moles of component

$$n_A = \frac{\text{Mass of component A}}{\text{Molecular weight of A}} = \frac{m_A}{M_A}$$

Therefore, molar concentration,

$$C_A = \frac{\text{Mass of component A}}{\text{Volume of mixture} \times M_A} = \frac{\rho_A}{M_A}$$

where M_A = molecular weight of component A.

CONCENTRATIONS, VELOCITIES AND FLUXES

Mass Fraction (m): The mass fraction is defined as the ratio of mass density or mass concentration of a component in a mixture to the total mass density of the mixture.

$$m_A = \frac{\rho_A}{\rho} = \frac{Mass density of component' A'}{Total mass density of mixture}$$

(Unit less Parameter)

Molar Fraction (x): The molar fraction (x) is defined as the ratio of molar concentration of a component in a mixture to the molar concentration of the mixture.

$$x_A = \frac{C_A}{C} = \frac{Molar Concentration component' A'}{Total Molar Concentration of mixture}$$

(Unit less Parameter)

Mass fraction: The mass fraction x_A *is defined* as the ratio of mass concentration of species A to the mass density ρ, of mixture.

$$x_A = \frac{\rho_A}{\rho}$$

Mole fraction: It is defined as the ratio of number of moles of component A to the total number of moles of mixture.
It is denoted by γ_A and expressed as :

$$\gamma_A = \frac{C_A}{C}$$

Basic quantities in mass transfer

	Mass quantities	*Molar quantities*	*Interrelationship*
Concentrations :			
Definition	$\rho = \frac{m}{V}$	$C = \frac{n}{V}$	$m = nM$
Species	$\rho_i = \frac{m_i}{V}$	$C_i = \frac{n_i}{V}$	$\rho_i = C_i M_i$
Fraction	$x_i = \frac{\rho_i}{\rho}$	$\gamma_i = \frac{C_i}{C}$	$\frac{x_i}{\gamma_i} = \frac{M_i}{M}$
Mixture	$\Sigma x_i = 1$ $\rho = \Sigma \rho_i$	$\Sigma \gamma_i = 1$ $C = \Sigma C_i$	$M = \Sigma \gamma_i M_i$
	$\rho = \frac{p}{RT} = \sum \frac{p_i}{RT}$	$C = \frac{p}{R_u T} = \sum \frac{p_i}{R_u T}$	$R = \frac{R_u}{M}$
Velocities :			
Species	u_i	u_i	
Average	$u = \Sigma x_i u_i$	$v = \Sigma \gamma_i u_i$	
Diffusional	$u_i - u$	$u_i - v$	
Fluxes :			
Absolute	$\mathbf{n}_i = \rho_i u_i$	$N_i = C_i u_i$	$n_i = N_i M_i = \rho_i u + j$
Bulk motion	$\rho_i u$	$C_i u$	
Diffusional	$J_i = \rho_i (u_i - u)$	$J_i = C_i (u_i - v)$	$N_i = C_i u + J_i$

Fick’s Laws of Diffusion

Fick’s laws of diffusion are mathematical statements describing how particles under random thermal motion tend to spread from a region of higher concentration to a region of lower concentration to equalize concentration on both the regions.

They state that **‘the rate of diffusion is directly proportional to both the surface area and concentration difference and is inversely proportional to the thickness of the membrane’**

A diffusion process that obeys Fick’s laws is called normal or Fickian diffusion; otherwise, it is called anomalous diffusion or non-Fickian diffusion.

Movement of particles (diffusion flux) from high to low concentration is directly proportional to the particle’s concentration gradient

$$J \propto \frac{d\varphi}{dx} \quad \text{or} \quad J = -D\,\frac{d\varphi}{dx}$$

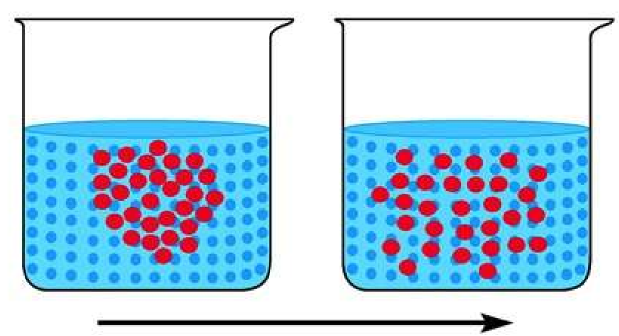
J = diffusion flux

D = diffusion coefficient or diffusivity

$d\varphi$ = change in concentration of the particle

dx = change in position

$\frac{d\varphi}{dx}$ = concentration gradient of the particle



Particles diffusing from high to low concentration

Comparison of Diffusive Mass Transfer & Conductive Heat Transfer

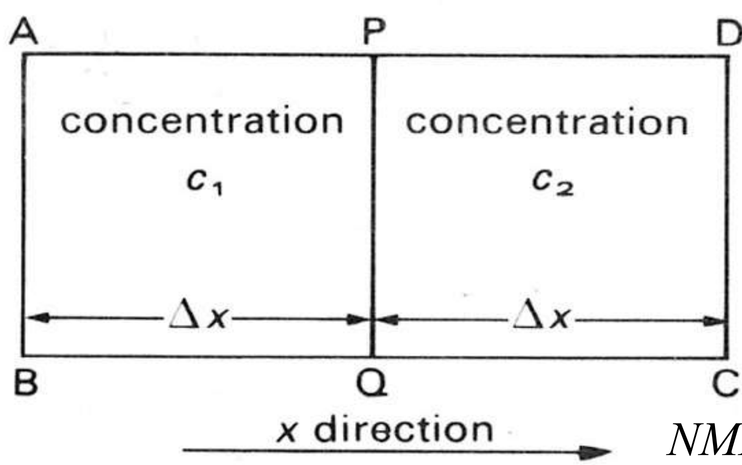
Sr. No.	Fick's Law	Fourier's Law
1.	Molecular diffusion is governed by the empirical relation : $\frac{\dot{m}_A}{A} = -D_{AB} \cdot \frac{d\rho_A}{dx}$ or $\frac{\dot{n}_A}{A} = -D_{AB} \cdot \frac{dC_A}{dx}$ a) $\frac{\dot{m}_A}{A}$ represents rate of mass flux b) $\frac{d\rho}{dx}$ represents the mass density gradient c) D_{AB} represents the diffusion coefficient for binary mixture	Fourier's law for unidirectional heat conduction is given as : $\frac{\dot{Q}}{A} = -k \cdot \frac{dT}{dx}$ a) (Q/A) represents the rate of heat flux b) $\frac{dT}{dx}$ represents the temperature gradient. c) k represents the thermal conductivity of material.
2.	Negative sign indicates that diffusion takes place in the direction of decreasing concentration.	Negative sign indicates that heat flows in the direction of decreasing temperature.
	cribes the transport of mass by diffusion.	It describes the transport of heat energy.
	transfer ceases when concentration gradient is	Heat transfer ceases when temperature gradient is zero.

Steady state, diffusion through a plane membrane

- Let us consider mass diffusion of fluids 'A' through a plain membrane of thickness L.

- The mass concentration of the fluid on opposite wall faces are C_{A1} & C_{A2} respectively.

- Considering diffusion along X-axis, then for steady state & unidirectional diffusion, 3 dimensional general mass equation reduces to equation (2).



Steady state, diffusion through a plane membrane

$$we know that;$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{g}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$Similar like;$$

$$\frac{\partial^2 C_A}{\partial x^2} + \frac{\partial^2 C_A}{\partial y^2} + \frac{\partial^2 C_A}{\partial z^2} + \frac{N_A g}{D_{AB}} = \frac{1}{D_{AB}}\frac{\partial C_A}{\partial t}$$

Steady state, one dimensional diffusion equation will be:

$$\frac{d^2 C_A}{dx^2} = 0 \ldots\ldots\ldots\ldots Eqn(2)$$

Steady state, diffusion through a plane membrane

$$Integrating\, Eqn(2)\, twice;$$

$$\frac{dC_A}{dx} = C_1$$

$$C_A = C_1 x + C_2 ....(3)$$

Boundary Conditions: ($C_A = C_1.x + C_2$)

1) At x=0; $C_A = C_{A1}$
2) At x=L; $C_A = C_{A2}$

Applying BC 1), we get $C_{A1} = C_1.0 + C_2$
Hence $C_2 = C_{A1}$

Applying BC 2), we get $C_{A2} = C_1.L + C_2$
Or $C_{A2} = C_1.L + C_{A1}$

$$\Rightarrow C_1 = \frac{C_{A_2} - C_{A_1}}{L}$$

$$Substituting\ C_1\ and\ C_2\ in\ Eqn..(3)$$

$$C_A = C_1 x + C_2$$

$$C_A = (C_{A_2} - C_{A_1})\frac{x}{L} + C_{A1}.........(4)$$

Mass transfer by diffusion is given by Ficks Law,

$$N_A = \frac{M_A}{A} = -D\frac{dC_A}{dx}...........(5)$$

Substituti ng C_A(*Eqn* 4) *in Eqn*..(5)

$$N_A = \frac{M_A}{A} = -D\frac{d}{dx}[(C_{A_2} - C_{A_1})\frac{x}{L} + C_{A1}]$$

$$N_A = \frac{M_A}{A} = -D\frac{d}{dx}[(C_{A_2} - C_{A_1})\frac{x}{L} + C_{A1}]$$

$$N_A = \frac{M_A}{A} = -D(C_{A_2} - C_{A_1})\frac{1}{L} + 0$$

$$N_A = \frac{M_A}{A} = \frac{D}{L}(C_{A1} - C_{A_2})$$

$$N_A = \frac{M_A}{A} = \frac{(C_{A1} - C_{A_2})}{(\frac{L}{D})}$$

L/D is known as Diffusional resistance plain membrane

The diffusion rate in the cylinder & spherical membrane is given by;

$$M_A = \frac{D(C_{A1} - C_{A_2})}{(\Delta x)} A_M$$

$$Where\ ;$$

$$For Cylinde\ r;$$

$$A_M = \frac{2\pi L(\Delta x)}{\ln \frac{r_2}{r_1}} = \frac{2\pi L(r_2 - r_1)}{\ln \frac{r_2}{r_1}}$$

$$For Sphere\ ;\ A_M = 4\pi r_2 r_1$$

Convective Mass Transfer

•The transport of material between a boundary surface and a moving fluid or between two non mixable moving fluids separated by a mobile interface.

•Convection is divided into two types:
1. Forced convection 2. Natural convection

- Compare between forced and natural convection mass transfer?

❖ **Forced convection**: In this type the fluid moves under the influence of an external force (pressure difference) as in the case of transfer of liquids by pumps and gases by compressors.
e.g: The Evaporation of water from an ocean when air blows over it.

❖ **Natural convection**: Natural convection currents develop if there is any variation in density within the fluid phase. The density variation may be due to temperature differences or to relatively large concentration differences.
e.g: The Evaporation of Alcohol.

The rate equation:

The rate equation for convective mass transfer (either forced or natural) is: $N_A = k_c \Delta C_A$

The rate equation for convective mass transfer (either forced or natural) is:

$$N_A = k_c \Delta C_A$$

Where;

1. N_A is the molar-mass flux of species A, measured relative to fixed spatial coordinates.
2. k_c is the convective mass-transfer coefficient .
3. ΔC_A is the concentration difference between the boundary surface concentration and the average concentration of the diffusing species in the moving fluid stream.

Mass transfer coefficient k_c= From the rate equation ($N_A = k_c \Delta C_A$) the mass transfer coefficient is the rate of mass transfer per unit area per unit driving force. It gives an indication to how fast is the mass transfer by convection.

Factor Affecting on mass-transfer coefficient is related to:
(1) The properties of the fluid,
(2) The dynamic characteristics of the flowing fluid and
(3) The geometry of the specific system of interest.

There are four methods of evaluating convective mass-transfer coefficients. They are:
(1) Dimensional analysis coupled with experiment.
(2) Analogy between momentum, energy, and mass transfer.
(3) Exact laminar boundary-layer analysis.
(4) Approximate boundary-layer analysis.

The molecular diffusivities of the three transport process (momentum, heat and mass) have been defined as:

1. Momentum diffusivity $\nu = \mu/\rho$
2. Thermal diffusivity $\alpha = k/\rho cp$
3. Mass diffusivity D_{AB}

It can be shown that each of the diffusivities has the dimensions of (L^2/T) hence; a ratio of any of the two of these must be dimensionless.

1. The ratio of the momentum diffusivity to the thermal diffusivity is designated as the Prandtl Number.
   $Pr = (Momentum\ diffusivity\ /\ Thermal\ diffusivity = \nu/\alpha = \mu c_p/k)$

2. The analogous number in mass transfer is Schmidt number given as which represents the ratio of the momentum diffusivity to mass diffusivity.
   $Sc = Momentum\ diffusivity/Mass\ diffusivity = \nu/D_{AB} = \mu\rho/D_{AB}$

3. The ratio of the diffusivity of heat to the diffusivity of mass is designated the Lewis number, and is given by
   $Le = Thermal\ diffusivity\ /\ Mass\ diffusivity = \alpha/D_{AB} = k/\rho c_p D_{AB}$

Note: Lewis number is encountered in processes involving simultaneous convective transfer of mass and energy.

4. The ratio of molecular mass transport resistance to the convective mass transport resistance of the fluid.

This ratio is generally known as the Sherwood number, S_h and analogous to the Nusselt number Nu, in heat transfer.

$$S_h = (molecular\ mass\ transport\ resistance) \ / \ (convective\ mass\ transport\ resistance)$$

$$S_h = k_c L / D_{AB}$$

Note: L is the characteristic length (it depends on the geometry)

Analogy heat and mass transfer

Analogical criteria

Heat transfer	Mass transfer
$Nu = \frac{\alpha D}{\lambda}$ Nusselt number	$Sh = \frac{\beta D}{D_{AB}}$ Sherwood number
$Pr = \frac{\nu}{a}$ Prandtl number	$Sc = \frac{\nu}{D_{AB}}$ Schmidt number
$Fo = \frac{at}{D^2}$ Fourier number	$Fo_D = \frac{D_{AB} t}{D^2}$ diffusional Fourier number
$Pe = \frac{uD}{a}$ Peclet number	$Pe_D = \frac{uD}{D_{AB}}$ diffusional Peclet number

Analogical correlations (valid for low concentrations, close to equimolar diffusion)

	Heat transfer	Mass transfer
Parallel flow around a plate	$Nu = 0.664 \sqrt{Re_L}\, Pr^{1/3}$	$Sh = 0.664 \sqrt{Re_L}\, Sc^{1/3}$
Flow around a sphere	$Nu = 2 + (0.4\sqrt{Re} + 0.06\, Re^{2/3})\, Pr^{0.4}$	$Sh = 2 + (0.4\sqrt{Re} + 0.06\, Re^{2/3})\, Sc^{0.4}$
Flow around cylinder	$Nu = (0.4\sqrt{Re} + 0.06\, Re^{2/3})\, Pr^{0.4}$	$Sh = (0.4\sqrt{Re} + 0.06\, Re^{2/3})\, Sc^{0.4}$

Que: The hydrogen gas diffuses through a steel wall of 2 mm thickness. The molar concentration of hydrogen at the interface is 1.5 kg-mol/m^3 and it is zero on the outer face. Calculate the diffusion rate of hydrogen, if its diffusivity coefficient is 0.3×10^{-12} m^2/s.

Solution: *Given : Diffusion of hydrogen through a steel wall.*

$L = 2$ mm $= 0.002$ m, $C_{A1} = 1.5$ kg-mol/m^3,

$D_{AB} = 0.3 \times 10^{-12}$ m^2/s, $C_{A2} = 0$

To find : Diffusion rate of hydrogen.

Assumptions :

(*i*) *Steady state diffusion.* (*ii*) *One dimensional diffusion without any* chemical reaction. (*iii*) *Constant properties.*

Analysis : The one dimensional molar diffusion is given by;

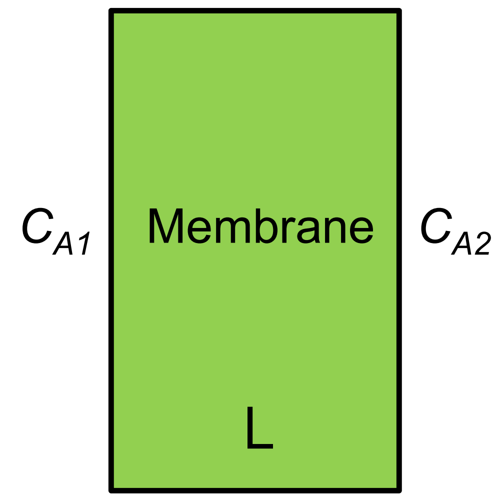
$$N_A = D_{AB}\frac{(C_{A1} - C_{A_2})}{(L)} = \frac{(0.3*10^{\wedge}-12)*(1.5-0)}{0.002}$$ **$= 1.5 \times 10^{-10}$ kg-mol/m^{2}s**

Que: Helium diffuses through a plane membrane of 3 mm thick . At the inner side the concentration of helium is 0.026 kg mole/m^3. At the outer side the concentration of helium is 0.008 kg mole/m^3. What is the diffusion flux of helium through the membrane. Assume diffusion coefficient of helium with respect to plastic by 1x10^{-9} m^2/s.

Solution: *Given: L=3 mm =0.003 m*

Concentration of Helium at inner side;

C_{A1}= 0.026 kg mole/m^3



Concentration of Helium at outer side;

C_{A2}= 0.008 kg mole/m^3

Diffusion Coefficient D_{AB}= 1x10^{-9} m^2/s.

Need to find= M_A/A =N (Diffusion Flux)

Need to find= M_A/A =N (Diffusion Flux)

We know that;

Molar Flux;

$$N = D_{AB}\frac{(C_{A1} - C_{A_2})}{(L)} = \frac{(1*10^{\wedge}-9)*(0.026 - 0.008)}{0.003}$$

$= 9 \times 10^{-9}$ kg-mol/m²s

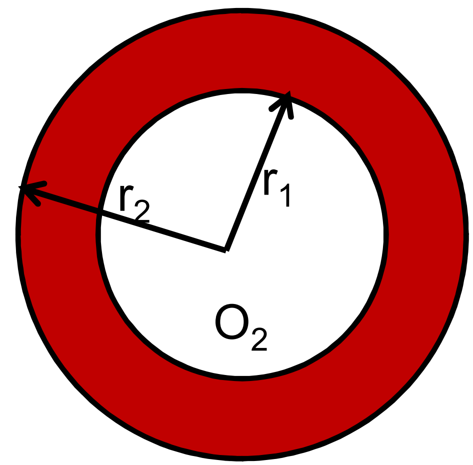
Home work;

Que: Gaseous Hydrogen is stored in a container of rectangular shape of 28 mm thickness . At the inner side the molar concentration of hydrogen is 1.4 kg mole/m^3. While at the outer side the concentration of hydrogen is zero. Calculate the molar diffusion flux of helium through the membrane. Assume diffusion coefficient of hydrogen through the steel, Take D_{AB}= 0.25x10^{-4} m^2/s.

Answer= **1.25×10^{-11} kg-mol/m^2s**

Que: Oxygen at 25°C and Pressure of 2 bar is flowing through a rubber pipe of inside diameter 25 mm and wall thickness 2.5 mm. The Diffusivity of O_2 in rubber is 3.12 x 10^{-3} kg mole/m^3 bar. Find the Loss of O_2 by diffusion per meter length of pipe.

Solution: *Given: T*= 25°C;



Inside Pressure P_1 = 2 bar;

Thickness t=2.5 mm =2.5 x 10^{-3} m

r_1,inner radius = 12.5 mm=12.5 x 10^{-3} m

r_2,outer radius = 12.5 +2.5 mm=15 x 10^{-3} m

Diffusion Coefficient D_{AB}= 0.21x10^{-3} m^2/s

Solubility= 3.12 x 10^{-3} kg mole/m^3 bar

Need to find= Loss of O_2 by diffusion per meter length of pipe.
Diffusion of O_2

We have;
for cylinder Molar Flux;

$$N_A = \frac{M_A}{A} = \frac{D_{AB}(C_{A1} - C_{A_2})}{(r_2 - r_1)}$$

$$M_A = \frac{D_{AB} * A * (C_{A1} - C_{A_2})}{(r_2 - r_1)} \ldots\ldots\ldots\ldots(Eqn.1)$$

Where ;

$$A = \frac{2\pi L(r_2 - r_1)}{\ln \frac{r_2}{r_1}} = \frac{2\pi * 1 * (0.015 - 0.0125)}{\ln(\frac{0.015}{0.0125})} = 0.086150 \ sq.m$$

Molar Concentration on inner side;

C_{A1}= Solubility x Inner Pressure =3.12 x10^{-2} x2 =6.24 x 10^{-3} kg mole/m^3

C_{A2}= Solubility x Inner Pressure =3.12 x10^{-2} x0 =0 (Assuming P_2=0)

Now substituting value of C_{A1}, C_{A2} & A in Eqn. (1);

$$M_A = \frac{D_{AB} * A * (C_{A1} - C_{A_2})}{(r_2 - r_1)}$$

$$= \frac{(0.21*10^{-9})*0.08615*[(6.24*10^{-3}) - 0]}{(0.015 - 0.0125)}$$

= 4.51 × 10^{-11} kg-mol/m^2s

Example. *Air at 1 atm and 25°C, containing small quantities of iodine, flows with a velocity of 6.2 m/s inside a 35 mm diameter tube. Calculate mass transfer coefficient for iodine. The thermo-physical properties of air are:*

$$\nu = 15.5 \times 10^{-6}\ m^2/s;\ D = 0.82 \times 10^{-5}\ m^2/s.$$

Solution. *Given*: $U = 6.2$ m/s; $d = 35$mm $= 0.035$m; $\nu = 15.5 \times 10^{-6}$ m^2/s; $D = 0.82 \times 10^{-5}$ m^2/s.

Mass transfer coefficient of iodine, h_m:

Schmidt number, $$Sc = \frac{\mu}{\rho D} = \frac{\nu}{D} = \frac{15.5 \times 10^{-6}}{0.82 \times 10^{-5}} = 1.89$$

Reynolds number, $$Re = \frac{\rho L U}{\mu} = \frac{dU}{\nu} = \frac{0.035 \times 6.2}{15.5 \times 10^{-6}} = 14000$$

Obviously the flow is *turbulent* and thus,

Sherwood number, $$Sh = 0.023\,(Re)^{0.83}\,(Sc)^{0.44}$$
$$= 0.023\,(14000)^{0.83}\,(1.89)^{0.44} = 84.07$$

Also, $$Sh = \frac{h_m d}{D}$$

$$h_m = \frac{Sh.D}{d} = \frac{84.07 \times 0.82 \times 10^{-5}}{0.036} = \mathbf{0.0197\ m/s.}$$